Referee Report

$T\overline{T}$ correlators from tensionless strings

Summary

The paper develops a worldsheet framework for correlation functions in the proposed single-trace $T\overline{T}$-deformation of the tensionless $k = 1$ AdS_3/CFT_2 duality. The starting point is the exact tensionless correspondence between pure NS-NS strings on $AdS_3 \times S^3 \times \mathbb{T}^4$ at $k = 1$ and the symmetric orbifold of $\mathbb{T}^4$. The authors aim to go beyond the spectral and partition-function tests of the deformed duality by defining physical vertex operators and computing their tree-level two-point functions.

The manuscript has several interlocking components. After reviewing the tensionless string toolkit, the authors introduce an auxiliary string theory obtained by adjoining to the tensionless worldsheet theory a zero-central-charge sector built from $X^{\pm}$, a (ψ, π) system, and an additional $(\beta^{(g)}, \gamma^{(g)})$, $(b^{(g)}, c^{(g)})$ ghost system. This auxiliary sector is chosen so that the Berkovits–Vafa topologically twisted $\mathcal{N} = 4$ structure with $\mathsf{c} = 6$ is preserved. The proposed spacetime dual is the symmetric orbifold of $\mathbb{T}^4 \times \mathcal{A}_0$, with $\mathcal{A}_0$ a non-unitary free-field theory of vanishing central charge. The paper supports this auxiliary duality by matching the single-cycle torus partition function, constructing DDF operators for the auxiliary spacetime fields, and explaining how the original $\mathrm{Sym}^N(\mathbb{T}^4)$ sector embeds into the enlarged theory.

The deformation is then implemented through the worldsheet $\beta\bar{\beta}$ perturbation corresponding to the single-trace $T\overline{T}$ deformation. By adding the auxiliary bosons and using the field redefinition

$$\gamma_\mu = \gamma - \mu X^-, \qquad \bar{\gamma}_\mu = \bar{\gamma} + \mu X^+,$$

together with the corresponding definitions of $X^{\pm}_\mu$, the local worldsheet action is rewritten in free-field form. Motivated by this rewriting, the authors propose deformed $\mathcal{N} = 2$ and $\mathcal{N} = 4$ generators and use them to define physical states. They construct explicit deformed vertex operators for the spacetime ground state and the spacetime $\mathcal{R}$-symmetry generators. A key output is the mass-shell shift

$$m = m_0 + \mu^2 p\bar{p},$$

which parallels earlier $k > 1$ AdS_3 and TsT analyses and is central to the final correlator.

The paper also contains a boundary-field-theory analysis of $T\overline{T}$-deformed correlators. The authors show that the two-point function proposed in earlier work by Cui and collaborators reproduces the known leading logarithmic contributions to the position-space perturbative result after Fourier transformation. They further observe that replacing the undeformed conformal weights in a momentum-space CFT correlator by $h_i^0 + \mu^2 p_i \bar{p}_i$ gives a simple solution of Cardy's Callan–Symanzik equation, although they correctly stress that Cardy's equation alone does not determine a unique correlator.

The final worldsheet calculation defines tree-level correlators in the deformed Berkovits–Vafa topological string, including the additional $Z\overline{Z}$ screening insertion needed for the auxiliary fields. For the deformed ground-state vertex operator, background-charge conservation is used to show that only one term in the expansion of the two vertex operators contributes. The correlator then factorizes, after stripping off the momentum-conservation delta function, into a tensionless-string part

and an auxiliary free-field part. The resulting exact tree-level momentum-space two-point function is proportional to

$$\frac{2^{-4m}\Gamma(1-2m)}{\Gamma(2m)}(p\bar{p})^{2m-1}, \qquad m = m_0 + \mu^2 p\bar{p},$$

and therefore selects precisely the gamma-function prefactor appearing in the Cui proposal.

Evaluation

This is a technically ambitious and interesting paper. The problem addressed is timely: a controlled holographic definition of non-conformal, non-local $T\overline{T}$-deformed theories is an important target, and the $k = 1$ tensionless string is arguably the natural setting where the phrase "single trace" can be made sharp. The manuscript is also unusually concrete for this subject. It does not merely argue by analogy with $k > 1$ strings, but gives an explicit deformed worldsheet algebra, explicit physical vertex operators, a prescription for correlators, and a nontrivial two-point computation.

The logical structure is broadly convincing. The auxiliary construction is well motivated by the need to keep a Berkovits–Vafa $\mathcal{N} = 4$ topological string structure while introducing the $X^{\pm}$ variables familiar from earlier treatments of $T\overline{T}$-like deformations. The matching of partition functions and the DDF-operator construction provide meaningful evidence that the auxiliary sector is not an arbitrary bookkeeping device. The deformation by field redefinition gives a plausible route to the deformed BRST current, and the subsequent checks, including the physicality of the deformed vertex operators and the preservation of the expected $\mathcal{N} = (4, 4)$ spacetime supersymmetry algebra, make the proposal more robust.

The main result, namely the recovery of the gamma-function two-point function from a first-principles $k = 1$ worldsheet computation, is significant. It sharpens the status of a proposal that previously arose in $k > 1$ or TsT-based analyses where the relation to a genuine symmetric-orbifold single-trace deformation is less direct. It also provides a concrete bridge between three otherwise somewhat separate strands of the literature: the exact tensionless AdS_3 string, Cardy's equation for $T\overline{T}$-deformed correlators, and the large-momentum/JT-gravity-motivated behavior emphasized by Aharony and Barel.

I do not see an obvious fatal flaw in the manuscript. At the same time, several central points are presently supported by plausible algebraic checks rather than by a fully first-principles derivation. This is acceptable for a paper opening a new framework, but the paper should be clearer about which statements are established, which are prescriptions, and which are checks of the prescriptions. The current version is often careful about this, but a few claims, especially surrounding the deformed BRST current, the selection of vertex-operator normalization, and the comparison with perturbative $T\overline{T}$ correlators, would benefit from sharper qualification.

Recommendation

I recommend publication after minor revision. The paper contains new and publishable results, and it should be of interest to researchers working on AdS_3 holography, tensionless strings, irrelevant deformations, and exact $T\overline{T}$ observables. The requested revisions below are mostly clarifications and strengthening of the presentation; I do not think they require changing the main calculation or the main conclusion.

Major Comments

1. **Status of the deformed $\mathcal{N} = 4$ algebra and BRST current.**

   The most important conceptual step is the proposal of the deformed $\mathcal{N} = 2$ and $\mathcal{N} = 4$ generators in Section 4.2. The manuscript is honest that the deformed BRST current is not derived directly from a first-principles gauge-fixing analysis of the deformed action, but rather from the free-field rewriting and the field redefinition involving γ_μ and $X^\pm_\mu$. This is a reasonable strategy, and the subsequent checks are strong. However, because the rest of the paper depends on this algebra, the authors should make the evidence for closure and consistency more transparent.

   In particular, it would be helpful to spell out which OPEs of the deformed $\mathcal{N} = 2$ algebra were checked explicitly and whether any subtleties arise from the fact that γ_μ and $\bar{\gamma}_\mu$ are no longer holomorphic and antiholomorphic for $\mu \neq 0$. The paper states that the generators satisfy the topologically twisted $\mathcal{N} = 2$ OPEs and then extends to $\mathcal{N} = 4$, but the reader would benefit from a short list of the nontrivial checks, especially those involving Q_μ, $\pi\partial X^+_\mu$, and the auxiliary ghost term $\gamma^{(\mathrm{g})}b^{(\mathrm{g})}$. Since the authors note that BRST currents are defined up to total derivatives, they should also explain whether these ambiguities could affect integrated correlators or only local representatives of the same charge.

   The global aspects of the field redefinition should also be clarified. The text correctly notes that the deformation is not trivialized because the boundary conditions and zero modes are not trivialized. Since the final two-point result depends crucially on the zero-mode/momentum structure, a concise statement of how the zero modes of $X^\pm$, γ, and $\bar{\gamma}$ are treated in the deformed algebra would make the argument cleaner.

2. **Role of the auxiliary $\mathcal{A}_0$ sector and the embedding of the physical subsector.**

   The auxiliary duality in Section 3 is plausible and useful, but its physical status should be stated more sharply. The proposed boundary theory $\mathrm{Sym}^N(\mathbb{T}^4 \times \mathcal{A}_0)$ contains a non-unitary zero-central-charge sector with infinitely many ground states. The paper gives good evidence that this sector cancels in the partition function and that the original $\mathrm{Sym}^N(\mathbb{T}^4)$ subsector can be embedded by imposing null-gauging-like conditions. However, the discussion would be strengthened by a clearer separation between three statements: the auxiliary string is a computational enlargement, the enlarged theory has its own symmetric-orbifold interpretation, and the desired $T\overline{T}$-deformed $\mathrm{Sym}^N(\mathbb{T}^4)$ observables are obtained from a specific subsector of the enlarged theory.

   The identification of the q-dependent auxiliary ground states and the conclusion $f(q) = q$ are elegant, but the logic is somewhat compressed. The two-point and n-point background-charge arguments show the correct selection rule for the classes of states considered. It would help to say explicitly whether this is meant as a complete identification of the auxiliary ground-state degeneracy or only as the identification needed for the correlators computed in the paper.

   Similarly, the null-gauging discussion at the end of Section 3.5 is important because it explains how the auxiliary excitations are removed. The authors state that a detailed worldsheet realization would require a picture-independent formulation and is challenging. That limitation is acceptable, but it should be connected explicitly to the later two-point computation: why are the vertex operators used in Section 6 guaranteed to lie in the intended $\mathbb{T}^4$ subsector after deformation, and in what sense do the auxiliary fields decouple rather than merely factorize in this special correlator?

3. **Normalization freedom of deformed vertex operators.**

   The footnote in Section 4.3 noting the freedom to multiply the deformed vertex operator by a smooth function of $\mu^2 p\bar{p}$ is important. Such a multiplication would change precisely the momentum-dependent prefactor U in the final two-point function, up to the constraints imposed by the $\mu \to 0$ limit and by leading logarithms. Since selecting the gamma-function U is one of the paper's central claims, the normalization issue deserves more discussion in the main text.

   The authors argue that their chosen representative is natural because it is not obtained by adding a sum of descendants to an undeformed physical operator. This is suggestive, but the criterion is not fully explained. Is there a canonical normalization inherited from the undeformed x-basis two-point function? Is the normalization fixed by the requirement that the $\mathcal{R}$-symmetry generator have a standard Ward identity? Is it fixed by a choice of local worldsheet representative in the small Hilbert space? Or is the claim more modest, namely that the most direct physical representative gives the Cui normalization while other operator-renormalization schemes could differ by finite momentum-dependent factors?

   Clarifying this point would also improve the comparison with perturbative $T\overline{T}$ calculations. The leading logarithmic terms are scheme-independent, but finite terms and lower powers of logarithms are not. The final string result is more specific than the leading logarithmic comparison, so the manuscript should state what physical or worldsheet principle selects this finite normalization.

4. **Exact result versus leading-logarithmic perturbative agreement.**

   Section 5 makes a useful comparison between the momentum-space two-point function with the Cui gamma-function prefactor and the known position-space leading logarithmic perturbative expansion. The derivation is correct in spirit and clarifies why the shifted conformal weight $h^0 + \mu^2 p\bar{p}$ is natural. However, the manuscript should be careful not to imply that this proves equality with the full all-order renormalized position-space two-point function in a fixed scheme. The text usually states this distinction, but some summary passages could be read as stronger.

   I suggest adding a compact paragraph after the Fourier-transform discussion emphasizing the following: the comparison establishes agreement of the universal leading logarithms; Cardy's equation and the leading logarithms do not fix the full renormalized correlator; the worldsheet result should be regarded as a particular holographic prescription for the finite completion of the correlator. This would make the relation between Sections 5 and 6 clearer and would preempt a natural objection from readers familiar with scheme dependence in $T\overline{T}$ perturbation theory.

5. **Details of the two-point calculation in Section 6.3.**

   The final computation is concise and likely correct, but it is central enough that a few intermediate details should be added. The statement that background-charge conservation eliminates all but one of the 81 possible terms is plausible, especially in light of the warm-up calculation, but a short table of the relevant ρ, u, $i\chi$, and auxiliary ghost charges for the three pieces of Ψ^L and Ψ^R would make the selection rule immediately verifiable.

The factorization into tensionless and auxiliary contributions after removing the momentum-conservation delta function is also a key step. The footnote notes that zero modes prevent full factorization before the double-bracket convention is applied. Since this is exactly where the $|z_1 - z_2|^{-4\mu^2 p\bar{p}}$ factor from the auxiliary fields cancels the shifted worldsheet scaling from the tensionless part, a slightly more explicit explanation would improve confidence in the cancellation.

Finally, the Fourier transform of $|x|^{-4m}$ is analytically continued through regions where the integral diverges and has poles. The footnote points to a renormalization prescription in the literature. Given that the gamma-function prefactor is the headline result, the authors should briefly state which analytic continuation or subtraction prescription is being adopted and whether contact terms at $p = 0$ are being suppressed. This is especially relevant for the $m_0 = 0$ ground-state limit, where the distributional identity at the end of Section 6.3 is used.

6. **Scope of the symmetry analysis.**

   The global symmetry check in Section 4.4 is a useful consistency test. The authors identify translations, $\mathfrak{su}(2)$ $\mathcal{R}$-symmetry generators, and supercharges satisfying the expected two-dimensional $\mathcal{N} = (4, 4)$ algebra. They also state that they have not found DDF operators for the spacetime rotation generator or for the outer $\mathfrak{su}(2)$ generators. This is not a problem for the main result, but the implications should be stated more explicitly. Does the absence of these DDF representatives merely reflect a limitation of holomorphic worldsheet-current methods, as in the undeformed tensionless theory, or could it signal that the realized spacetime algebra is only a subalgebra of the expected one? A short clarification would help calibrate the strength of this check.

Minor Comments

- Section 3.1 contains a few typographical errors: "forth order pole" should be "fourth order pole", and "anti-holomoprhic" should be "anti-holomorphic".
- In the DDF-operator discussion for the auxiliary ghosts, "separetely" should be "separately".
- In Section 6.1, "precesence" should be "presence".
- The phrase "Before turning into the technical details" in Section 4.1 would read better as "Before turning to the technical details".
- In Section 4.4, "the complicated expressions for $\mathcal{Q}^{+-}$ and $\mathcal{Q}^{--}$ is" should be "are".
- The notation M' and M' appears in several forms. A uniform convention would improve readability.
- The paper uses both boundary and worldsheet symbols with very similar names, for instance $X^{\pm}$ and $\mathcal{X}^{\pm}$, $G^{\pm}$ and $\mathcal{G}^{\pm}$, and several ghost systems. A short notation table near the beginning of Section 3 would be helpful for readers.
- In Section 5, the assumptions under which the undeformed correlator can be analytically continued in the weights h_i should be stated in the paragraph introducing the shifted-weight solution, not only in the footnote.

- The final result is presented for the ground state and $\mathcal{R}$-symmetry operators, with $m_0 = 0$ and $m_0 = 1$. It would be useful to state explicitly whether the same derivation is expected to apply to other untwisted-sector operators once their deformed vertex operators are known, or whether new complications are expected.

Conclusion

The paper makes a substantive contribution to the study of single-trace $T\overline{T}$ deformations and tensionless AdS_3 strings. Its main worldsheet calculation is novel, technically nontrivial, and significant. The recommended revisions are intended to clarify the status of the proposed deformed topological-string structure, the role of the auxiliary sector, and the normalization and renormalization choices entering the final two-point function. Once these points are addressed, the paper should be suitable for publication in a high-energy theory journal.