Referee Report

Anomaly mediation in Seiberg–Witten theories

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Recommendation: Reject in the present form.

Summary of the paper

The manuscript studies anomaly-mediated supersymmetry breaking (AMSB) in four-dimensional $\mathcal{N} = 2$ $SU(2)$ gauge theories with $N_F = 0, 1, 2, 3$ massless hypermultiplets. Its first central observation is that, in the perturbative ultraviolet description, the AMSB scalar and gaugino masses can be packaged, after an $SU(2)_R$ rotation exchanging the two adjoint fermions, as an $\mathcal{N} = 1$-preserving adjoint mass. The authors then argue that instanton contributions to the nonperturbative beta function spoil the equality of the scalar and fermion masses and hence break the residual supersymmetry completely.

The manuscript next applies the conformal-compensator formula directly to the exact Seiberg–Witten effective action. On the Coulomb branch it constructs the potential in equation (3.17) from the exact Kahler potential and presents numerical slices intended to show that the moduli are driven to the singular points. Near a singularity, explicit dyon hypermultiplets are added and the scalar potential (3.23) is analyzed. The authors claim an exact global minimum at $a_D = 0$, with a dyon condensate given by equation (3.40). This condensate differs in magnitude and phase from the one obtained by replacing the Seiberg–Witten deformation parameter by the perturbative AMSB adjoint mass. The analysis is extended to the singularities and Higgs branches for $N_F = 1, 2, 3$. Finally, the paper argues that the small-$m_{3/2}$ AMSB vacua must evolve toward the Seiberg–Witten-deformed vacua as $m_{3/2}$ passes through the strong-coupling scale, although this purported change is said not to be a phase transition.

Overall assessment

The question is interesting and potentially useful for assessing how much information small-AMSB calculations retain about strongly coupled nonsupersymmetric limits. The perturbative reorganization of the AMSB terms after an $SU(2)_R$ rotation is a neat observation, and it is worthwhile to examine whether a nonperturbative beta function leaves an observable imprint on the soft spectrum. The attempt to follow the same deformation through the Coulomb, dyonic, and Higgs descriptions is also ambitious.

However, the principal results are not established. There is a basic integrated-in versus integrated-out inconsistency in the dyon effective theory, and the algebraic proof of positivity in Section III D contains dimensionally and numerically incorrect steps. These are not peripheral errors: they invalidate the claimed exact condensate, the factor-of-two comparison with the Seiberg–Witten deformation, the loop-stability argument, and most of the later discussion of how the vacua evolve. In addition, the Coulomb-branch potential depends on a Kahler representative without a sufficient matching prescription, and the claimed large-$m_{3/2}$ endpoint is assumed rather than derived by anomaly-mediated threshold matching. Repairing these points requires a new analysis rather than a local revision. For this reason I do not think the manuscript is suitable for publication in its present form.

Major comments

1. **The near-singularity action double-counts the light dyon contribution.**

   Equation (2.42) retains the logarithmic Seiberg–Witten Kahler potential and the associated coupling $g_{\text{eff}}(a_D)$ obtained from the exact prepotential. That action is the effective action in which the monopole hypermultiplet has already been integrated out. The manuscript then adds explicit fields $M, \widetilde{M}$ while continuing to use the same logarithm and the same $g_{\text{eff}}(a_D)$. This counts the light-hypermultiplet loop both through the Seiberg–Witten logarithm and through the explicit dynamical fields.

   The required matching is discussed directly in M. Luty and R. Rattazzi, JHEP 11 (1999) 001, especially their Section 4.4 and equations (4.38)–(4.41). When the monopoles are integrated in, their one-loop contribution must be removed from the Seiberg–Witten coefficient functions. The resulting Wilsonian coupling depends on an independent renormalization scale and is nonsingular at $a_D = 0$. This reference is present in the bibliography of the manuscript but is not cited or used.

   This issue changes the core physics of Section III D. The paper's selection of the condensate relies on $g_{\text{eff}}(a_D) \to 0$ while $\partial g_{\text{eff}}/\partial a_D$ diverges at the origin. In a correctly integrated-in theory, the coupling must instead be run to a physical threshold. Once a dyon condensate forms, the dual photon and charged matter become massive and the running stops at the Higgs scale, not at $|a_D| = 0$. Consequently, the flat potential at $a_D = 0$, the adiabatic prescription below equation (3.25), the divergent term used after equation (3.38), and the assertion that loop AMSB vanishes at the vacuum are artifacts of an inconsistent effective action unless shown otherwise in a properly matched calculation.

   The authors must reconstruct the dyon EFT with the light fields integrated in, specify the Wilsonian scale and threshold matching, and then redo the minimization. All condensates, vacuum energies, and masses should be recomputed in that framework.

   There is also a sharp quantitative cross-check. In the standard pure-$SU(2)$ normalization, equation (4.53) of Luty and Rattazzi gives the leading monopole expectation value in terms of the two soft fermion masses. Mapping the AMSB trajectory with $m_\chi = 0$ and $m_\lambda/g^2 = m_{3/2}/(4\pi^2)$ appears to reproduce the manuscript's "expected Seiberg–Witten" condensate, rather than the "actual" value smaller by a factor of two. The authors must reconcile this result explicitly. It strongly suggests that the advertised universal factor of 1/2 is a consequence of the missing integrated-in threshold rather than new physics.

2. **The central positivity proof is algebraically and dimensionally incorrect.**

   Let $v = m\widetilde{m}$ and impose the D-flat condition used by the authors. Then

   $$|m|^2 = |\widetilde{m}|^2 = |v|, \qquad |m|^2 + |\widetilde{m}|^2 = 2|v|.$$

   With the change of variables in equation (3.28),

   $$v = \frac{1}{\sqrt{2}} \left(\Delta - m_{3/2}^\dagger \partial K \right),$$

the first term of equation (3.23), using its printed normalization, becomes

$$\sqrt{2}\,|a_D|^2\left|\Delta - m_{3/2}^{\dagger}\partial K\right|,$$

not

$$\sqrt{2}\,|a_D|^2\left|\Delta - m_{3/2}^{\dagger}\partial K\right|^2$$

as printed in equation (3.29). Since both Δ and $m_{3/2}^{\dagger}\partial K$ have mass dimension two, the displayed first term in (3.29) has mass dimension six, whereas every other term in the potential has mass dimension four. The coefficient of $|\Delta|^2$ in (3.29) is also inconsistent with the factor $g_{\text{eff}}^2/2$ in (3.23).

There is a separate factor error in the quadratic argument. For

$$Q(x) = g_{\text{eff}}^2 x^2 - 2|m_{3/2}a_D|x,$$

the stationary point is $x = |m_{3/2}a_D|/g_{\text{eff}}^2$, not $2|m_{3/2}a_D|/g_{\text{eff}}^2$ as stated after equation (3.33). Equations (3.34)–(3.36) consequently do not follow. Moreover, the superpotential $W = \sqrt{2}A_D M\widetilde{M}$ gives $2|a_D|^2(|m|^2 + |\widetilde{m}|^2)$, whereas equations (2.53), (3.22), and (3.23) use coefficient one; Appendix A later switches to coefficient two.

Thus the claimed nonnegativity and uniqueness of the global minimum have not been proved, and equation (3.40) is not shown to hold to all orders. The same invalid derivation is invoked for equation (5.4), the deformed Higgs branches, and all entries called "actual condensates" in the later tables. The full potential must be rederived with consistent normalizations and minimized anew, including a proof of boundedness and a search for additional stationary or degenerate configurations.

3. **Appendix A does not provide a valid canonical or RG-improved treatment of the mass gap.**

   The proposed redefinition $a_D = \widetilde{g}(\widetilde{a}_D)\widetilde{a}_D$ is not holomorphic because $\widetilde{g}$ depends on $|\widetilde{a}_D|$. It therefore is not a supersymmetry-preserving change of Kahler coordinates, contrary to the criterion stated immediately before the redefinition. Likewise, rescaling a gauge field by a field-dependent function is not an ordinary canonical normalization: it changes the gauge transformation and produces additional derivative and connection terms. These effects are not included in equations (A8)–(A11). The appendix therefore cannot establish either the stated loop masses or the stability of $a_D = 0$.

   The scaling of the purported mass gap is also misstated. From $|m\widetilde{m}| \sim |m_{3/2}\Lambda|$, a canonically normalized dyon expectation value scales as $\sqrt{|m_{3/2}\Lambda|}$, so a Higgs mass is parametrically $g_D\sqrt{|m_{3/2}\Lambda|}$, not $m_{3/2}g_D$. More fundamentally, g_D must be evaluated self-consistently at this mass threshold. The manuscript repeatedly states that the AMSB and Seiberg–Witten-deformed spectra differ, but it never displays or diagonalizes a bosonic and fermionic mass matrix. A revised analysis should compute the physical spectrum and vacuum energy explicitly; these are also the natural invariant quantities with which to compare the two deformations.

4. **The Coulomb-branch AMSB potential is not defined in a physically invariant way.**

   Equation (3.15) changes under $K \to K + f(u) + \overline{f(u)}$, as the authors acknowledge in Appendix B. The proposed rule of choosing a representative that approaches the classical Kahler potential at infinity does not by itself fix this freedom uniquely, nor does the manuscript show that it follows from matching the conformal compensator through the strongly coupled theory. Holomorphic additions that respect the asymptotics can still change (3.15), its critical points, and the condensate proportional to ∂K. Physical vacua cannot depend on an arbitrary Kahler representative.

Relatedly, Λ is treated as inert when the compensator formula is applied to $K(u, \Lambda)$, even though the dynamical scale is generated by the same renormalization-group anomaly to which AMSB couples. The composite coordinate u also does not have the canonical Weyl weight of an elementary chiral field. Merely rewriting u as u/Λ does not determine the compensator dependence of either quantity. The missing RG-spurion matching can change the terms attributed to the exact Kahler potential.

This issue should be resolved by deriving the compensator dependence from a specified sequestered supergravity or rigid-spurion setup and matching it from the ultraviolet to the Seiberg–Witten variables, including the transformation of the compensator under Kahler transformations. At minimum, the authors must demonstrate that the potential, vacuum energies, and claimed condensate comparison are invariant under every residual Kahler transformation compatible with their matching conditions. An assertion of a preferred gauge based only on asymptotic behavior is not enough.

Even within the selected representative, the conclusion that all Coulomb moduli roll to singularities is conditional. Appendix B proves a statement only if the exact Kahler potential is real convex. The plotted one-dimensional slices in Figures 2–4 do not establish global convexity or exclude an off-axis saddle of K, especially for $N_F = 3$, where no discrete rotation covers the full plane. The manuscript should provide either an analytic proof or a reproducible two-dimensional numerical analysis of all critical points and Hessians, with branch cuts, duality frames, precision, and convergence under control.

5. **The ultraviolet instanton argument and the large-$m_{3/2}$ endpoint require threshold matching.**

The possibility that the nonperturbative beta function splits the perturbative $\mathcal{N} = 1$ multiplet is interesting. However, the calculation in equations (3.11)–(3.14) leaves the instanton coefficient unspecified, fixes $\theta = 0$, and does not explain the coupling scheme in which the nonperturbative beta function and $\gamma_\Phi = -2\beta/g$ may simultaneously be inserted into the standard AMSB formula. Since nonperturbative definitions of the physical coupling are scheme sensitive, the authors should formulate this in terms of the complex coupling, state the matching scheme, and identify a scheme-independent mass splitting or Ward identity. The beta function for θ should also be retained when making claims at general phase.

More importantly, Sections IV and VI assume that, after the heavy scalar and one adjoint fermion are integrated out, the remaining theory is supersymmetric $\mathcal{N} = 1$ SYM or SQCD. Ordinary anomaly mediation is ultraviolet insensitive: below a threshold its soft terms are determined by the beta functions and anomalous dimensions of the low-energy theory. The low-energy $\mathcal{N} = 1$ beta function is not the $\mathcal{N} = 2$ beta function, so one would generally expect an AMSB mass for the surviving gaugino unless the rotated supercharge and the compensator threshold enforce a nontrivial cancellation. No such matching calculation is given. The authors must integrate out the adjoint multiplet in superspace, including its compensator dependence, and demonstrate which soft operators remain. Without this step, the assumed endpoint, the use of the supersymmetric gaugino-condensate relation in equations (4.5)–(4.7), and the entire decoupling comparison are unsubstantiated.

6. **The proposed change of vacua and the absence of a phase transition are conjectures, not results.**

   Sections IV and VI repeatedly describe the interpolation argument as "vague," "indirect," or based on what is "unlikely." Decoupling at asymptotically large $m_{3/2}$ does not determine the path of vacua through the incalculable region $m_{3/2} \sim \Lambda$, and it does not imply equation (6.8) at finite $m_{3/2}$. Nor does equality of the massless particle content and ordinary global-symmetry-breaking pattern prove that two theories are in the same phase. Vacuum-energy nonanalyticities, discrete theta data, line operators, mixed anomalies, domain-wall physics, or topological sectors could distinguish them.

   The claim in the abstract that the vacua "must change," the statement that the change is not a phase transition, and the assertion of a common universality class should therefore be removed or explicitly labeled as conjectures unless supported by an invariant diagnostic. The paper should distinguish what is calculated in a controlled regime from what is inferred. It should also identify an observable meaning for the phases and magnitudes in Tables IV and VIII. The local operator $m\widetilde{m}$, the phase of the special coordinate, and ∂K are sensitive to field rephasings, duality-frame conventions, normalization, and Kahler transformations. A claimed mismatch becomes physical only after it is tied to an invariant spectrum, vacuum energy, correlation function, anomaly, or order parameter.

   The $N_F = 3$ discussion already supplies a concrete counterexample to an analysis based only on massless particles and ordinary symmetries. Table III appears to reverse the light-dyon multiplicities: the $(1, 0)$ singularity at the origin carries four light monopoles transforming in the $\mathbf{4}$ of $SU(4)$, while the $(2, -1)$ singularity carries a singlet, not vice versa. Condensation of the magnetic-charge-two singlet leaves a $\mathbb{Z}_2$ gauge sector and an infrared topological field theory. This structure is emphasized in the cited recent analysis from which the local periods are taken, but it is absent here. The $N_F = 3$ Higgs-branch assignment and every claim about phases or universality must be revisited with the correct multiplicities, line operators, one-form data, and residual TQFT.

7. **The relation to the prior soft-breaking literature needs to be reassessed.**

   The manuscript usefully explains its overlap with the more recent work on the Seiberg–Witten deformation plus AMSB. However, the directly relevant analysis of general soft breaking near monopole points by Luty and Rattazzi is omitted from the discussion despite appearing in the bibliography. Earlier work on softly broken $\mathcal{N} = 2$ theories is cited collectively, but the manuscript does not compare its potential, integrated-in matching, or vacuum scaling with those results. Once the effective-action issues above are corrected, the authors should state precisely which result is new beyond that literature. The potentially novel parts appear to be the special perturbative $SU(2)_R$ reorganization and its proposed nonperturbative AMSB interpretation; these should be isolated from conclusions already known for generic soft deformations.

Minor comments

1. The charge convention is inconsistent. The text defines (n_m, n_e) and the BPS mass as $|an_e + a_D n_m|$, so the monopole becoming massless at $a_D = 0$ has charge $(1, 0)$. Section II C 4 and Table III instead label it $(0, 1)$, while Section II C 5 returns to $(1, 0)$. All tables and duality-frame statements should be checked.

2. Equation (3.13) contains $\exp[-(-8\pi^2/g^2)]$ in its last term. The sign should be corrected and all terms in the instanton expansion rechecked.

3. The nondimensionalization in equation (3.19) differentiates with respect to a quantity written as $\Lambda_0^2 u$, whereas the surrounding text states that the dimensionless coordinate is u/Λ_0^2. The displayed equation is dimensionally inconsistent as written.

4. Below equation (5.6), taking $m_{3/2} \to 0$ is described as equivalent to taking $C \to \infty$. These are different limits; both may suppress a ratio, but they probe different regions of the moduli space and should not be identified.

5. The footnote attached to Table VIII selects between two discrepant formulas partly because one of them preserves a universal factor of 1/2. That is circular evidence for the factor being tested. The derivative of the local prepotential and every table entry should be independently recalculated after the integrated-in matching is fixed.

6. The figures should state how the periods were analytically continued, how branch cuts were treated, and how extrema were located. One-dimensional curves are illustrative but should not be presented as a global numerical test.

7. The manuscript would benefit from a thorough proofreading. Examples include "gauge theries," "theory theory," "an remnant supersymmetry," and several inconsistent conjugations of $m_{3/2}$ and ∂K between equations (3.40), (5.4), and the text below equation (5.6).

Conclusion

The manuscript addresses a worthwhile problem, but the two main controlled claims—the exact dyonic vacuum at small $m_{3/2}$ and the supersymmetric endpoint at large $m_{3/2}$—are not presently derived from a consistent matched effective theory. The integrated-in dyon action and the algebra of the scalar potential must both be rebuilt before the condensates, spectrum, and interpolation can be assessed. Because those changes may qualitatively alter the paper's conclusions, I recommend rejection in the present form, while encouraging the authors to reconsider the problem in a substantially revised analysis.