

Referee Report

“Perusing confining pseudoreal theories: a story of emerging massless spin-1 bosons”

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Recommendation: Reject in the present form. The classification and conditional phase survey are potentially useful, but the paper’s defining conclusion—one or two exactly massless composite spin-1 particles—does not follow from the analysis. One of the central operator identities is incompatible with the chiral charges and field content, and the remaining tumbling, anomaly, and current arguments do not establish a massless pole or select the asserted vacuum.

Summary and overall assessment

The manuscript studies minimal asymptotically free gauge theories containing Weyl fermions in pseudoreal representations. Starting from the authors’ classification of fifteen one-species models and four two-species symplectic models, it first uses a conjectured all-orders beta function to divide the theories into likely conformal, nonconformal, and indeterminate cases. For the last two groups it applies a most-attractive-channel (MAC) tumbling analysis, catalogues ordinary discrete and one-form symmetries, and examines low-dimension gauge-invariant operators. The central proposal is that an adjoint bilinear channel generically yields a residual Abelian gauge factor and that the current in equation (11) supplies a gauge-invariant description of the corresponding massless vector. The claimed exceptions are the one-species Spin(11) model, which is said to be fully gapped, and the two-species Sp(6) model, which is said to contain two free massless vectors and one Nambu–Goldstone boson.

The survey has attractive features. It brings a finite list of unusual asymptotically free theories into a common notation, records useful branching rules and anomaly data, identifies concrete lattice and functional-RG targets, and highlights the Spin(11) and two-species Sp(6) models as qualitatively distinct cases. If a robust mechanism for the advertised photons were given, the result would be both novel and interesting for the high-energy theory community.

At present, however, the logical chain does not support that conclusion. The conformal-window step is an extrapolation of an ansatz precisely where the bilinear used in its unitarity bound vanishes. The tumbling score chooses a representation but neither proves bilinear condensation nor fixes an adjoint orbit. The anomaly analysis constrains possible infrared realizations but does not force the particular condensates assumed. Most importantly, the local current in equation (11) is not thereby a massless particle, and equation (12), which is intended to connect the chiral four-fermion order parameter to that current, cannot be the stated Fierz identity. These are not peripheral issues: they remove the only proposed gauge-invariant bridge to the headline result.

The old composite-vector literature cited by the authors establishes that such mechanisms can be contemplated under additional dynamical assumptions, not that they occur in the present purely fermionic theories. Likewise, the earlier SU(6) anomaly studies support interesting candidate symmetry-breaking patterns, but they do not establish a photon. The genuinely new part of the paper is therefore also the part that presently lacks a derivation. A viable future paper could either provide new nonperturbative evidence for a Maxwell sector or recast the tables explicitly as conjectural phase scenarios. The current categorical claims are not suitable for publication in a journal such as PRD or JHEP.

Major comments

1. Equations (4), (11), and (12) are algebraically incompatible.

Equation (4) defines

$$X_R = (\psi_r \psi_r)_R (\psi_r \psi_r)_R.$$

With the manuscript's normalization of the anomalous chiral symmetry, $q_A(\psi_r) = 1$, this operator has $U(1)_A$ charge 4 and contains four left-handed fields. By contrast, equation (11), $A_\mu = \psi_r^\dagger \sigma_\mu \psi_r$, is neutral and contains one ψ_r and one $\psi_r^\dagger$. Consequently $A_\mu A^\mu$ contains two fields of each chirality and has zero chiral charge. A Fierz rearrangement can alter spinor and gauge-index contractions, but it cannot change chirality content or global charge. Thus equation (12),

$$X_R = A_\mu A^\mu + \dots,$$

cannot hold as an operator identity. The anomaly does not cure this algebraic mismatch.

If the intended left-hand side were $(\psi\psi)_R^\dagger (\psi\psi)_R$, it would be neutral and could contain current-current structures after Fierzing, but it would no longer be the charged order parameter in equation (4). If the intended operator were $X_R^\dagger X_R$, it would contain eight fermions. The authors must write all Lorentz and gauge projectors, coefficients, and Hermitian conjugates explicitly and preserve the symmetry charges term by term. In its present form, the identity on which the gauge-invariant vector interpretation rests is false.

There is a second, independent problem with the inference after equation (12). Even a valid relation involving the scalar $A_\mu A^\mu$ would not imply a one-point function $\langle A_\mu \rangle$, nor would either statement imply a massless pole. A nonzero vector one-point function would in fact select a spacetime direction and break Lorentz invariance, an ingredient of some Bjorken-type models that is neither claimed nor analyzed here.

2. A current operator does not establish a massless spin-1 state.

The ‘‘Gauge-invariant operators’’ discussion identifies equation (11) as the lowest-dimension gauge-invariant vector operator and then assumes that it ‘‘may feature a zero mode.’’ Operator dimension and Lorentz quantum numbers do not determine the mass of the lightest state in its spectral density. Conserved currents routinely interpolate massive vector resonances; here the proposed $U(1)_A$ current is not even conserved because of the gauge anomaly. There is therefore no Ward identity in the manuscript that protects a zero mass.

The appropriate test would be a transverse $1/p^2$ pole with nonzero residue in a specified gauge-invariant two-point function, together with a low-energy Maxwell action and the associated emergent gauge redundancy. The authors should also identify an infrared field-strength operator and explain the electric and magnetic one-form symmetries expected of a free Maxwell sector. None of these structures follows from the existence of A_μ .

The asserted two-fermion operator basis is also incomplete. Every pseudoreal representation has an antisymmetric invariant tensor Ω_{AB} , so the operator

$$T_{\alpha\beta} = \Omega_{AB} \psi_\alpha^A \psi_\beta^B$$

is a nonvanishing gauge singlet transforming as a self-dual Lorentz tensor $(1,0)$ and carrying chiral charge 2. It is another dimension-three, two-fermion spin-1 operator, with field-strength rather than vector-potential quantum numbers. Its existence directly contradicts the claim before equation (11) that only one two-fermion operator can be constructed. It does not by itself prove a

photon either, but it must be included in any complete operator and spectral analysis and makes the proposed one-to-one association between A_μ and the tumbling vector untenable.

The argument that a vector cannot be massive because there is no light scalar to provide a longitudinal polarization is incorrect. A generic composite massive vector is a Proca particle; its longitudinal polarization is part of the massive spin-1 representation and requires no separate light scalar. A Higgs field becomes necessary only after an exact gauge redundancy has already been demonstrated, which is precisely what is missing. The QCD analogy is also misleading: the ω is a massive state associated with an isoscalar vector current, whereas the anomalous axial-current channel contains massive axial-vector resonances as well as the pseudoscalar η' . The absence of an anomaly-induced contribution to one particular vector mass is not a theorem of masslessness. The Weinberg–Witten reference is a constraint that an emergent description must evade; it is not positive evidence for the claimed state.

3. The tumbling construction is not a derivation of a gauge-invariant Coulomb phase.

The scalar bilinear $(\psi_r\psi_r)_R$ is gauge covariant. Elitzur’s theorem precludes treating its expectation value as an ordinary gauge-invariant order parameter. Conversely, a nonzero expectation value of the gauge-singlet four-fermion operator X_R in equation (4) need not factorize into bilinear one-point functions and does not by itself select a residual subgroup. A Hubbard–Stratonovich or gauge-fixed mean-field description may be a useful ansatz, but its effective action, stationary points, and gauge-invariant content must be established rather than assumed.

Likewise, the score in equation (6) is a one-gauge-boson-exchange estimate at strong coupling. It can rank candidate channels within that approximation; it cannot show that a condensate necessarily forms, that a channel with negative Δ “cannot possibly” condense, or that subsequent strong dynamics is vector-like and fully gapped. The paper needs a controlled limit, gap equation, effective potential, semiclassical deformation, lattice evidence, or another nonperturbative argument. Fradkin–Shenker complementarity does not by itself turn a gauge-fixed adjoint Higgs ansatz with a residual long-range U(1) into a proven gauge-invariant phase. In particular, a surviving compact U(1) may be Coulombic or may be removed by further charged condensation or monopole dynamics. This fate must be analyzed rather than inferred from equation (5).

4. Neither the adjoint channel nor Table I selects exactly one Abelian factor.

Equation (6) distinguishes representations, not the orbit of a condensate inside the adjoint. Different adjoint directions have different centralizers. A generic adjoint element preserves a maximal torus, while special degenerate directions yield the regular maximal subgroups displayed in Table I. An effective potential is required to choose among these possibilities. No such vacuum-alignment calculation is given.

This omission produces an internal conflict with the main conclusion. The text following equation (5) explicitly allows full abelianization and as many vectors as the rank of the gauge group. The SU(6) path B can undergo a further tumbling step and generate a second Abelian factor; the analogous Spin(12) path B inherits that ambiguity. Table I itself lists several competing paths, and the next-to-MAC Spin(11) channel in equation (8) has a photon although the MAC path in equation (7) does not. The statement in the summary that one-species theories yield “exactly one” vector is therefore not a result of the analysis. The authors must either derive the orbit and cascade dynamically or present every phase conditionally, including its relative vacuum energy and stability.

5. The conformal-window partition is more uncertain than the paper acknowledges.

Equations (1)–(3) use a proposed all-orders beta function, not an exact result for these non-supersymmetric theories. More seriously, the manuscript correctly notes that for one pseudoreal Weyl species the gauge-singlet scalar bilinear vanishes and its anomalous dimension is ill-defined. It then nevertheless uses the inequality derived from that bilinear at $n_r = 1$ to label models “likely” conformal, and extrapolates from $n_r = 2$ to call other $n_r = 1$ models outside the window. The monotonicity and operator-continuity assumptions behind this step are not supplied. The nonvanishing four-fermion operator has a different dimension for which equation (3) gives no bound.

For the two-species theories, the fixed-point numerator provides one relation among several anomalous dimensions. Applying the one-species inequality to each representation is not justified. At minimum the authors should derive the aggregate necessary unitarity condition from equations (1) and (2), state what additional assumption fixes individual anomalous dimensions, and show the calculation for all four rows. Table I should list T_r , C_r , C_G , β_0 , the numerical threshold, and comparisons with any available two-loop, semiclassical, supersymmetric, or lattice evidence. Until then, “out” and “likely” are model-dependent indications rather than established phase assignments. This uncertainty propagates to the set of theories on which the tumbling conclusion is based.

6. The anomaly analysis is overinterpreted as mandatory symmetry breaking.

Instantons explicitly reduce the anomalous continuous $U(1)_A$ to an exact discrete subgroup, as encoded by equation (9). A mixed ’t Hooft anomaly of that exact subgroup is conceptually different: it does not explicitly break the symmetry, but constrains its possible infrared realizations. Spontaneous breaking is one solution. Gapless bosonic degrees of freedom, a topological sector, symmetry extension, or combinations of these can also match an anomaly. The absence of local gauge-invariant fermionic operators does not eliminate those alternatives. Indeed, the proposed Maxwell sector would itself carry generalized symmetries and would have to be incorporated into the matching.

The statements around equations (9) and (10) and in Table II should therefore separate: (i) exact instanton breaking of the continuous symmetry; (ii) a mixed anomaly of the remaining exact symmetry; and (iii) a hypothesized spontaneous breaking pattern chosen under extra assumptions such as the absence of topological order. The global form of each gauge group, the allowed spin or spin- G structures, counterterm conventions, and the full anomaly coefficients should be stated. The “strong anomaly” criterion cited later is likewise a dynamical proposal; it does not generically require the ’t Hooft vertex itself to acquire a factorized expectation value.

7. The two-species $Sp(6)$ conclusion requires several unsupported steps.

Equations (13) and (14) describe one possible adjoint-tumbling ansatz and leave one Abelian gauge factor. The second photon is obtained only after additional assumptions. The existence of the mixed four-fermion object in equation (15) does not show that a condensate in the attractive $r_2 r_2$ channel induces a bilinear condensate in the repulsive $r_1 r_1$ channel. A gauge-singlet four-fermion expectation value again need not factorize. Equation (16) is a schematic decomposition of the six-fermion ’t Hooft operator, not a proof that the MAC and next-to-MAC must both acquire aligned bilinear expectation values.

Even if both auxiliary bilinears are introduced, their relative orientation must be determined by an effective potential. Equation (17) gives one possible common stabilizer, not a unique endpoint. The authors should provide the normalized $U(1)_A$ and $U(1)_V$ charges, all anomaly coefficients, the bilinear effective potential, the common-orbit analysis, the complete fermion mass matrix, and

the subsequent dynamics of the $SU(2)$ and compact Abelian factors. Finally, two independent fermion currents do not imply two photons. Spontaneous breaking of the exact $U(1)_V$ instead guarantees a longitudinal scalar pole associated with the Nambu–Goldstone boson; a transverse massless vector pole is a separate dynamical claim that must be shown.

8. There are definite numerical and representation-theory inconsistencies in the catalogue.

For the $Sp(8)$ theory with $r = \mathbf{48}_{A3}$, Table I and the Appendix give the adjoint-channel score as $\Delta = 23/2$. Table II gives the instanton remnant $\mathbb{Z}_{14}$, so in the manuscript’s normalization $T_{\mathbf{48}} = 7$. Using

$$C_2(r)d(r) = T(r)d(G), \quad d(G) = 36, \quad d(r) = 48,$$

one obtains $C_{\mathbf{48}} = 21/4$. Since $C_{\text{Adj}} = 5$, equation (6) gives

$$\Delta = 2C_{\mathbf{48}} - C_{\text{Adj}} = \frac{21}{2} - 5 = \frac{11}{2},$$

not $23/2$. This correction does not change the ordering against the displayed **315** channel, but it is a useful internal check.

There is also a dimension mismatch in the $Sp(4)$ model with $r = \mathbf{16}_{(1,1)}$. The symmetric square must have dimension $16 \times 17/2 = 136$, whereas the channels listed in Table I and the Appendix, **10** + **35** + **81**, sum to only 126. The standard C_2 decomposition contains a second copy of the adjoint **10**. This multiplicity matters: the “basis” in equation (4) requires multiplicity indices, and two independent adjoint bilinears need not have the same vacuum alignment. The authors should provide a reproducible table of dimensions, Dynkin indices, Casimirs, complete tensor products with multiplicities, branching rules, and all Δ values, and recheck every entry.

Novelty and significance

The finite model catalogue and its uniform comparison are useful. The classification of the models themselves, however, comes from the authors’ companion work, while the beta-function ansatz, MAC criterion, tumbling, discrete-anomaly constraints, and composite-vector ideas are established ingredients. The main new claim is their simultaneous realization in these specific UV-complete purely fermionic theories and the near-universality of the resulting photon. That claim would be significant, but it cannot be credited as a result without the missing spectral and vacuum-selection analysis.

The paper should compare its assumptions much more sharply with the cited literature. Earlier $SU(6)$ work constrains candidate discrete phases but does not predict a photon. Bais–Frere and Bjorken-type constructions, and later induced-vector models, employ additional current-current interactions, Lorentz-breaking assumptions, compositeness conditions, classical gauge structures, supersymmetry, or elementary adjoint Higgs fields. These are precedents for possible mechanisms, not a theorem that applies to equations (4), (11), and (12). A revised work should state which mechanism is actually realized, identify its assumptions, and provide a quantitative observable that distinguishes the proposed Coulomb phase from confinement or a topological phase.

Minor and presentation comments

1. State the exact scope of “all pseudoreal theories.” The apparent completeness assumes simple simply connected gauge groups, irreducible pseudoreal matter, minimal one- or two-species content,

and particular global assumptions concerning the new $SU(2)$ anomaly. Product groups, other global forms, and nonminimal flavor content are not included.

2. Define the representation suffixes F , $S3$, $A3$, $A5$, and $R11$ in Table I, as well as “N-MAC” and the precise meaning of the “SUSY?” column. For even-dimensional Spin groups, specify which chiral spinor and which center charge are used.
3. Table II contains the two models marked “?” in Table I, so its caption should not describe every entry simply as likely outside the conformal window. Also, the prose says that $d_r \bmod T_r$ appears in the second column, whereas the relevant anomaly information is in the last column.
4. Equation (10) needs a careful statement of the mod- $X/2$ versus mod- X condition, including fermion-parity and spin-structure subtleties. The table should distinguish an exact symmetry, an anomalous symmetry, and a conjectured unbroken subgroup rather than denoting each transition by the same arrow.
5. In equation (11), specify the two-component spinor convention; many standard conventions require $\bar{\sigma}^\mu$ in $\psi^\dagger \bar{\sigma}^\mu \psi$. Also distinguish an interpolating current from a canonically normalized one-particle vector field.
6. Equations (15) and (16) place expectation-value brackets inside objects that are alternately called operators and condensates. Define the local operators first, with all gauge contractions, and denote their expectation values separately.
7. The final combined $Sp(6)$ row of Table I leaves the Δ column blank even though two scores, $7/2$ and $5/2$, drive the proposed scenario. Those values and their channel assignments should be shown.
8. Avoid describing the full infrared theory as “gapped” while also claiming one or more massless particles. “A gapped non-Abelian sector plus a putative Maxwell sector” would express the intended scenario more precisely.
9. The heading “Two-species $Sp(6)$ theory” is formatted differently from the other unnumbered headings. More importantly, the abstract and conclusion should distinguish proven representation-theory facts from conjectural phase assignments and nonperturbative predictions.

Recommendation

I recommend rejection in the present form. The manuscript contains a useful taxonomy and several interesting conditional scenarios, but the asserted massless-vector spectrum is neither protected nor derived, equation (12) is inconsistent with the operator charges, the number of Abelian factors is not dynamically selected, and the anomaly and conformal-window arguments are used beyond what they establish. Repairing these points would require a new central dynamical argument rather than a localized clarification. A substantially recast paper that presents the tables as conjectures and supplies concrete lattice/FRG tests could be worthwhile; a stronger version would need direct evidence for the transverse massless pole and a gauge-invariant Maxwell effective theory.