Referee Report

"Massless fermionic current of Schwinger pairs in 3D de Sitter spacetime"

Recommendation: Reject in the present form. The question addressed by the manuscript is worthwhile, and a correct computation of the massless fermionic current in dS_3 could be publishable. However, the displayed in-modes do not satisfy the manuscript's own Dirac system, the adiabatic counterterm has the opposite sign from that implied by the preceding equations, and the finite formula used for the plots is not the formula derived in Section 3 and Appendix B. These problems affect the central result rather than peripheral presentation. A substantially rewritten paper based on a fresh, independently checked derivation could be considered as a new submission.

Summary and scope

The manuscript studies a massless charged two-component spinor on the expanding Poincare patch of dS_3 in a prescribed homogeneous electric field. Equations (2.1)–(2.18) specify the conformally flat metric, a 2×2 gamma-matrix representation, and the potential $A_1 = -E/(H^2\tau)$. After the rescaling $\widetilde{\psi} = \Omega\psi$, the authors reduce the Dirac equation to Whittaker equations with

$$\lambda = \frac{eE}{H^2}, \qquad r = \frac{k_x}{k}, \qquad \kappa = \mathrm{i}\lambda r, \qquad \gamma = \mathrm{i}\lambda, \qquad \gamma_s = \gamma - \frac{(-1)^s}{2}.$$

They then propose early-time in-modes in equations (2.35) and (2.36), use these modes to obtain the cutoff current integral (3.3), and evaluate it by the Mellin–Barnes calculation in Appendix B. Section 4 constructs an adiabatic mode and subtracts the linear divergence (4.14), leading to the finite one-dimensional integral representation (4.16). Finally, Section 5 claims a regular linear response at weak field, $J \propto -\lambda$, a strong-field law $|J| \propto \lambda^{3/2}$, monotonicity without a sign change, and the absence of infrared hyperconductivity.

The narrow target is recognizable and potentially useful. Fermionic longitudinal currents were previously studied in dS_2 and dS_4, while the directly comparable dS_3 calculation in the cited literature concerns a scalar. If valid, a controlled interpolation for the massless fermionic current would fill a technical gap relevant to de Sitter QED and backreaction studies. The strong-field exponent itself is not new: the $E^{D/2}$ semiclassical scaling and the $D = 3$ scalar result are already given by Bavarsad, Stahl, and Xue, Phys. Rev. D 94, 104011 (2016). Likewise, the absence of fermionic infrared hyperconductivity was already found in other dimensions. The manuscript's real prospective contribution is therefore the correct dS_3 coefficient and full field-strength dependence. At present, those quantities are not established.

Major comments

1. **The displayed mode functions are inconsistent with the component equations.**

   This is the most serious issue. Equation (2.33) gives

   $$\gamma_1 = \gamma + \frac{1}{2}, \qquad \gamma_2 = \gamma - \frac{1}{2}.$$

   Accordingly, equation (2.28) assigns the Whittaker index $\gamma + 1/2$ to the first spinor component and $\gamma - 1/2$ to the second. Equations (2.35)–(2.38) do the reverse: for example, the upper entry of

(2.35) is $W_{\kappa,\gamma-1/2}$ and the lower entry is $W_{\kappa,\gamma+1/2}$. Thus the proposed spinor is not assembled from solutions of the two component equations as those equations are printed.

There is also a direct first-order check. At early times, equation (2.35) gives

$$\frac{U_2}{U_1} \longrightarrow \sqrt{\frac{1-r}{1+r}},$$

up to the branch chosen for the displayed square roots. In the same limit, the first-order Dirac relation later used in equation (4.10) gives

$$\frac{\mathcal{U}_2}{\mathcal{U}_1} \longrightarrow \mathrm{i}\sqrt{\frac{1+r}{1-r}}.$$

These are not the same solution. The endpoint $r = 1$ makes the failure especially transparent: equation (2.35) tends to a spinor proportional to $(1, 0)^{\mathsf{T}} e^{-\mathrm{i}k\tau+\mathrm{i}kx}$, while the early-time flat-space operator obtained from equation (2.22) maps that spinor to a nonzero vector. A phase convention cannot repair the exchanged component magnitudes.

Because equation (3.2) is constructed directly from these modes, this error propagates to the bare current, its ultraviolet sign, and every subsequent formula. The authors should rederive the two independent in-modes from the first-order Dirac equation, state all Whittaker and square-root branches, verify the modes by direct substitution, check the canonical normalization and Wronskian, and only then recompute the current. A numerical residual test at generic (λ, r, τ) would be a useful additional check. Merely swapping two indices in the typeset formula is not sufficient unless the current calculation is repeated, because exchanging the components reverses the σ_3 bilinear entering the longitudinal current.

2. **Equations (4.10)–(4.14) contain an independent counterterm sign contradiction.**

   Let
   $$q = kr + \lambda H\Omega, \qquad k_\perp = k\sqrt{1-r^2}, \qquad \omega^2 = q^2 + k_\perp^2.$$

   Equation (4.10) and the normalization (4.11) imply
   $$\left|\frac{\mathcal{U}_2}{\mathcal{U}_1}\right|^2 = \frac{(\omega+q)^2}{k_\perp^2} = \frac{\omega+q}{\omega-q}.$$

   Consequently, the integrand written in equation (4.13) is
   $$\frac{1-|\mathcal{U}_2/\mathcal{U}_1|^2}{1+|\mathcal{U}_2/\mathcal{U}_1|^2} = -\frac{q}{\omega}.$$

   Expanding this expression at large momentum and performing the angular integral gives
   $$\langle J^1\rangle_A = -\frac{eH^2}{4\pi}\,\Omega^{-1}\lambda\Lambda + O(\Lambda^0),$$

   whereas equation (4.14) has the opposite sign. This conclusion follows from the manuscript's own equations without making a choice about the disputed exact in-modes.

   The authors need to fix the sign starting from variation of the action (2.15), including their convention for the charge and F_{01}, and recompute both the exact and adiabatic bilinears with the same convention. This is not a harmless overall convention: the paper repeatedly interprets the sign as screening or antiscreening, and the corrected component ordering in comment 1 also changes the sign of equation (3.2). The claim below equation (4.14) that a "second adiabatic order" has been subtracted is additionally unclear, since the displayed construction retains only the leading instantaneous-frequency terms. The adiabatic orders assigned to A_μ, Ω, and their derivatives, and every term required for the current in $2+1$ dimensions, should be stated explicitly.

3. **The central finite formula changes sign between the derivation and the result used in the figures.**

   The second digamma contribution in equation (3.5) appears with a minus sign,

$$-(e^{2\pi\lambda} - e^{-2\pi\lambda r})\Psi(-\mathrm{i}\lambda - \mathrm{i}\lambda r).$$

   The independently displayed endpoint of Appendix B, equation (B.58), has the same minus sign. In equations (4.15) and (4.16), this is silently replaced by a plus sign. Subtracting equation (4.14), which is only a real term linear in Λ, cannot alter the sign of a finite digamma term. Figures 1 and 2 and the asymptotic discussion are said to use equation (4.16), so they are not based on the expression derived in Section 3 and Appendix B.

   The starting integral (3.3) is manifestly real, as is the expectation value of a Hermitian current. The appearance of an unexplained complex result in (3.5) or (4.15) cannot be resolved by the statement that "only its real part contributes." Instead, the branch choices and contour manipulations must yield a real expression analytically, or the authors must identify a precise physical prescription that produces an imaginary contribution and explain why it is to be discarded. The corrected closed expression should be checked directly against numerical integration of (3.3) at several finite cutoffs, followed by a controlled subtraction and $\Lambda \to \infty$ extrapolation. Until this sign discontinuity is resolved, neither the weak-field coefficient (5.5) nor the claimed monotonic interpolation is supported.

4. **Appendix B does not presently provide a reproducible derivation.**

   Several independent errors occur before equation (B.58):

   - Equation (B.4) gives the three-gamma Mellin–Barnes representation of $W_{\kappa,\gamma}(z)$ with $e^{+z/2}$ and an integration range written as $(-\infty, \infty)$. The standard representation corresponding to those three gamma factors has $e^{-z/2}$ and a vertical contour from $-\mathrm{i}\infty$ to $+\mathrm{i}\infty$.
   - Equation (B.5) contains p^{1-s-t}, but equation (B.7) prints p^{-s-t-1} while giving the antiderivative appropriate to p^{1-s-t}. The subsequent statement is also reversed: Λ^{2-s-t} grows, rather than vanishes, when $\mathrm{Re}(s+t) < 2$.
   - The right poles of $\Gamma(\mathrm{i}\lambda r - t)$ in equation (B.5) are $t = \mathrm{i}\lambda r + n$, not the $t = \mathrm{i}\lambda + n$ written in equation (B.12). Equation (B.15) later restores the r dependence without explaining the change.
   - In passing from equation (B.5) to (B.10), the outer r integral and its $(1+r)/\sqrt{1-r^2}$ weight disappear, and $\Gamma(-\mathrm{i}\lambda r - s)$ is changed to $\Gamma(-\mathrm{i}\lambda - s)$. It is not stated that the subsequent calculation is an integrand at fixed r, and the missing factors cannot be reconstructed unambiguously from the notation.
   - Equations (B.53)–(B.57) manipulate individually divergent quantities $\zeta(1, z)$. The pole cancellation under analytic continuation is not demonstrated, and equation (B.57) introduces an undefined term δ_E that is absent from the final result.

   These are not isolated typographical slips: they affect the exponential factor, contour domain, pole set, cutoff behavior, angular measure, and finite remainder. Appendix B should be rebuilt from a correct integral representation with an explicit fixed-r notation, valid contour strips, orientations and signs of residues, treatment of coincident or moving poles, and an explicit cancellation of all regulator-dependent constants. An independent numerical comparison with (3.3) is essential.

5. **The fermion content and the parity anomaly in $2+1$ dimensions must be specified.**

   Equation (2.13) chooses one irreducible two-component gamma-matrix representation. For a single charged two-component fermion in $2+1$ dimensions, a gauge-invariant regulator entails the parity anomaly and a parity-odd Chern–Simons response; the classic analyses include Niemi and Semenoff, Phys. Rev. Lett. 51, 2077 (1983), and Redlich, Phys. Rev. Lett. 52, 18 (1984). The manuscript neither fixes the regulator's parity-odd finite term nor discusses the inequivalent second gamma-matrix representation.

   This matters for what is meant by "the induced current." A single irreducible species can carry a transverse Hall/Chern–Simons current in addition to the longitudinal component computed here. A parity-invariant four-component theory can cancel the parity-odd responses by pairing the two inequivalent representations, but its longitudinal normalization is correspondingly doubled relative to one irreducible species. The authors should state which theory they mean, whether J^2 is zero, canceled, or omitted, and how gauge invariance is maintained. The final claim that both a parity-even Dirac mass and a parity-odd Haldane mass can simply be added to the stated two-component theory is also inaccurate without first introducing a reducible, doubled spinor structure.

6. **The renormalization condition and the support of the external field are physically consequential.**

   The potential (2.17) describes a constant physical electric field, but such a field is not a source-free Maxwell solution in an expanding de Sitter background. The split between a prescribed supporting source and an induced matter current must therefore be stated. This is especially important because the manuscript cites Bastero-Gil et al., JCAP 04 (2026) 040, which emphasizes that sustaining a constant de Sitter electric field requires additional photon/source dynamics and that an on-shell renormalization condition can alter the infrared current.

   The assertion after equation (4.14) that the counterterm contains no finite contribution is not, by itself, a physical renormalization condition. A finite local Maxwell counterterm shifts the current by a term proportional to $\nabla_\nu F^{\nu\mu}$; on this off-shell external background, that shift has the same linear-in-E structure as the headline weak-field term (5.5). The coefficient and even the interpretation of that linear response are therefore scheme dependent unless the gauge-field normalization and supporting source are fixed. The manuscript should list the allowed local finite terms, specify its renormalization condition, and compare the result with the alternative prescription discussed in its own recent reference. The absence of a $1/E$ infrared divergence may be robust, but the value and sign of the E coefficient require this additional argument.

7. **The current direction, normalization, and asymptotic formulas are mutually inconsistent.**

   For $e > 0$, $E > 0$, and hence $\lambda > 0$, both equations (5.3) and (5.5) give a negative current. The abstract says that the current is opposite to the external field, while Section 5.1.1 and the conclusion say that it flows along the field and screens it; the text also says that the current is positive. Figure 1 plots $|J|$, so it contains no information capable of resolving this conflict. Once equations (2.17) and (2.18) have fixed the orientation of F_{01}, the answer cannot be dismissed as an arbitrary sign convention for λ.

There is also a normalization problem. Equation (5.3) is dimensionally inconsistent as written and does not reduce algebraically to (5.4). Equation (5.4) omits the factor of Ω present in the plotted dimensionless physical current $|\Omega J|/(eH^2)$. The manuscript should consistently distinguish the coordinate component J^1 from the local orthonormal component $J^{\hat{1}} = \Omega J^1$, define the signed observable used in Maxwell's equation, and rewrite all asymptotic formulas in terms of that observable.

The numerical evidence should then be redone with the corrected signed expression. At minimum, the paper should show positive and negative λ, verify the expected field-reversal relation, report the sampled range and precision, explain the treatment of the endpoint singularities in the r integral, and give convergence tests in the cancellation-dominated regime shown in Figure 2. The quoted numbers 0.066 and 0.0262 differ by several percent from $1/16$ and $1/(4\pi^2)$; fit windows, fit functions, uncertainties, and convergence toward the true asymptotic regions are needed. A plot of an absolute value is not evidence for the absence of a zero or sign change.

Finally, $\lambda \gg 1$ at fixed H is not identical to a strict Minkowski limit $H \to 0$ at fixed E. The current accumulated over a Hubble time can scale as $1/H$ in the latter limit. The manuscript may reproduce the flat-space Schwinger rate or the $E^{3/2}$ scaling, but it has not shown a finite stationary flat-space current. The limiting prescription and the quantity being compared should be stated precisely.

8. The novelty and infrared interpretation need to be narrowed.

The manuscript repeatedly attributes the absence of infrared hyperconductivity to fermionic statistics or a "stabilizing role" of fermions. That conclusion does not follow from the comparison presented. A massless Dirac field is conformally invariant, whereas the massless minimally coupled scalar responsible for the comparison's strongest infrared behavior is not. The cited dS_3 scalar study already relates hyperconductivity to mass, curvature coupling, conformality, and tachyonicity; the more recent Bastero-Gil et al. analysis likewise finds that a conformally coupled scalar behaves much more like a fermion. A meaningful spin comparison should include, or at least discuss, the conformally coupled scalar in three dimensions ($\xi = 1/8$) under the same renormalization condition.

The priority statement should also be made precise. Equation (4.16) retains a numerical one-dimensional integral and is best described as a semi-analytic integral representation, not an entirely closed analytic current. If the calculation is corrected, the plausible new result is the longitudinal dS_3 interpolation and its coefficients. The $E^{3/2}$ exponent and the qualitative absence of fermionic infrared hyperconductivity are continuations of known results rather than new mechanisms. This narrower contribution could be appropriate for a journal such as Physical Review D if the foundational issues above are fully resolved.

Minor comments

1. Equations (2.20) and (2.22) contain extra or misplaced factors of Ω. The conformal rescaling should remove the spin-connection term and leave an unambiguous $-\sigma_3\partial_2$ operator. Equations (2.20)–(2.24) should be corrected and checked by direct multiplication.

2. "Weyl representation" is misleading in odd spacetime dimension. Equation (2.13) should be called an explicit irreducible Pauli-matrix representation, together with a discussion of the inequivalent second representation.

3. Equation (2.42) defines the in-vacuum only through $a_{\mathrm{in},\mathbf{k}}|0\rangle = 0$. It must also impose $b_{\mathrm{in},\mathbf{k}}|0\rangle = 0$, and equations (2.40)–(2.41) should state that all omitted anticommutators vanish.

4. The late-time modes (2.37)–(2.38) are not used. Their positive-frequency interpretation is delicate at $\lambda = 0$, where the stated late-time frequency vanishes. They should either be used in a Bogoliubov cross-check with carefully specified branches or removed.

5. Equation (3.1) should distinguish the operator $\widehat{j}^{\mu}$ from its expectation value and should use the curved matrix $\Gamma^{\mu} = \Omega^{-1}\gamma^{\mu}$ for a coordinate current. The origin of every power of Ω in equation (3.2) should then be shown. The paper should also demonstrate, rather than assume, the values of J^0 and J^2.

6. Equation (4.3) has an extraneous $d\tau'$ after the exponential. In equation (4.13), write $|\mathcal{U}_2/\mathcal{U}_1|^2$ directly. The limiting treatment of $k\sqrt{1-r^2} = 0$ in equation (4.10) should be explained.

7. Figure 1's embedded legend identifies the black curve as equation (4.14), which is the counterterm, while the caption identifies it as equation (4.16). Figure 2 should state explicitly whether it plots signed contributions or their magnitudes; a logarithmic plot of positive magnitudes obscures the claimed cancellation.

8. Appendix A, equations (A.15)–(A.18), uses k where κ appears to be intended and contains malformed exponential powers. These identities should be replaced by correctly transcribed standard formulas.

9. The strong-field saddle leading to equation (5.3) and the weak-field expansion leading to (5.5) should be displayed far enough to expose the expansion parameter, cancellations, and leading error term. The reported fitted "slopes" should be described consistently as coefficients or log-log exponents.

10. A thorough language and typesetting revision is needed. Examples include the duplicated date command, "Francee," "vaccume," "defined the vanishing," and "asymptotically expansion." These are secondary to the technical issues but presently impede a careful reading.

Conclusion

The paper asks a sensible question and may contain the outline of a useful result, but its main current formula is not presently derived from valid spinor modes or a controlled integral calculation. The mode-component mismatch, the counterterm sign, and the discontinuous digamma sign must be resolved before the numerical and physical conclusions can be assessed. I therefore cannot recommend publication or ordinary revision of this version. A new submission would need a complete rederivation, independent analytic and numerical checks, a specified parity and renormalization prescription, and a consistent signed physical-current analysis.