Referee Report

Carrollian limit of NS–NS and Heterotic Supergravity

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Recommendation: Reject in the present form, with encouragement to resubmit after a complete rederivation.

Summary and overall assessment

The manuscript studies an ultra-relativistic, or Carrollian, contraction of the bosonic NS–NS and heterotic string effective actions. With $w = 1/c \to \infty$, the authors decompose the relativistic metric into a Carroll clock and a degenerate spatial metric, split the Kalb–Ramond field into a two-form $b_{\mu\nu}$ and a one-form A_μ, and shift the dilaton so that the measure is intended to cancel the leading w^2 terms in the Lagrangian. They then identify the finite part of the Levi–Civita connection, introduce associated non-metricities, and present a purported finite NS–NS action. The construction is extended to a non-Abelian gauge potential decomposed into ${a_\mu}^i$ and χ^i, including a Yang–Mills Chern–Simons modification of the three-form. A central claim is that the Green–Schwarz transformation of the Carrollian one-form can be trivialized by the redefinition in equation (3.11), while the transformation of the two-form remains nontrivial. The manuscript also discusses equations of motion obtained both by expanding the parent relativistic equations and by constrained variation, and it argues that a curvature-squared term can remain finite in a double scaling of α'.

The subject is timely, and a correct target-space derivation of Carrollian NS–NS and heterotic dynamics would be of genuine interest. In particular, it could complement the non-relativistic NS–NS and heterotic limits cited by the authors, the non-Riemannian Double Field Theory descriptions, and the recent Carroll-string literature. The proposed distinction between the one-form and two-form realizations of the Green–Schwarz mechanism could be a noteworthy result.

Unfortunately, the present calculations do not establish these claims. The determinant scaling used to fix the dilaton is reversed; the heterotic gauge transformations do not preserve the field decomposition from which they are said to follow; the displayed dilaton equation is not the Euler–Lagrange equation of the stated action; and the variational appendix contains explicit index and measure errors. These are not matters of presentation. They affect the defining limit, gauge invariance, and dynamics of the proposed theory. A corrected manuscript would require a comprehensive rederivation rather than a routine revision.

Major comments

1. **The determinant and dilaton scaling in equation (2.8) are inconsistent with equations (2.1)–(2.2).**

   In a frame adapted to the clock, the metric ansatz reads

$$\hat{g}_{\mu\nu} = \mathrm{diag}(-w^{-2}, h_{ij}), \qquad \hat{g}^{\mu\nu} = \mathrm{diag}(-w^2, h^{ij}).$$

It follows immediately that

$$\sqrt{-\hat{g}} = \frac{\Omega_c}{w}, \qquad \sqrt{-\hat{g}}\, e^{-2\hat{\phi}} = \Omega_c\, w^{-(2\alpha+1)} e^{-2\varphi}.$$

Equation (2.8) instead uses $w^{-2\alpha+1}$, as though $\sqrt{-\hat{g}} = w\Omega_c$. Requiring the measure to scale as w^{-2} therefore gives $\alpha = 1/2$, not $\alpha = 3/2$. With the value stated in the manuscript, the actual measure is of order w^{-4}, and its product with the claimed order-w^2 Lagrangian coefficient vanishes as w^{-2} rather than giving the nonzero actions (2.40) and (3.17).

This error must be corrected before any other power counting can be trusted. The replacement $\hat{\phi} = \frac{1}{2}\ln w + \varphi$ may leave the formal coefficient of the finite action unchanged, because the logarithmic shift has no spacetime derivative, but that conclusion has to be demonstrated by repeating the expansion. The authors should also discuss the physical regime: even the corrected positive shift sends $g_s = e^{\hat{\phi}}$ to strong coupling as $w \to \infty$. The resulting construction is at best a formal tree-level contraction unless an additional coupling or background scaling is specified.

2. **The heterotic gauge transformations (3.7)–(3.10) do not preserve the stated spatial decomposition.**

   Equation (3.1), together with $a_\mu{}^i \tau^\mu = 0$, implies

$$\chi^i = \tau^\mu \hat{A}_\mu{}^i, \qquad a_\mu{}^i = P_\mu{}^\nu \hat{A}_\nu{}^i, \qquad P_\mu{}^\nu = \delta_\mu{}^\nu - \tau_\mu \tau^\nu.$$

   Projecting the parent transformation $\delta \hat{A}_\mu{}^i = \partial_\mu \lambda^i + f^i{}_{jk} \lambda^j \hat{A}_\mu{}^k$ then gives

$$\delta\chi^i = \tau^\mu \partial_\mu \lambda^i + f^i{}_{jk} \lambda^j \chi^k,$$
$$\delta a_\mu{}^i = P_\mu{}^\nu \partial_\nu \lambda^i + f^i{}_{jk} \lambda^j a_\mu{}^k.$$

   The manuscript instead assigns the full derivative to $\delta a_\mu{}^i$ and omits the longitudinal derivative from $\delta\chi^i$. Its rule therefore gives

$$\tau^\mu \delta a_\mu{}^i = \tau^\mu \partial_\mu \lambda^i,$$

   so a generic gauge transformation leaves the constrained field space. The displayed rules would be consistent only after the unstated and physically substantial restriction $\tau^\mu \partial_\mu \lambda^i = 0$.

   The same problem affects the one-form inherited from the Kalb–Ramond field. Since $A_\mu \tau^\mu = 0$, the intended transformation in equation (3.10) has a nonzero contraction proportional to $(\tau \cdot \partial \lambda^i)\chi_i$. Consequently, the claimed gauge invariance of $\bar{A}_\mu = A_\mu - \frac{1}{2} a_\mu{}^i \chi_i$ does not follow from a constraint-preserving projection of the parent symmetry. With the correctly projected transformations, $\delta(a_\mu{}^i \chi_i)$ also contains a term proportional to $a_\mu{}^i \tau^\nu \partial_\nu \lambda_i$.

   This issue strikes the main novelty of the heterotic section. The authors must derive the complete transformations from the parent fields for unrestricted local gauge parameters, show that all algebraic constraints are preserved, verify closure, and then reassess equations (3.11)–(3.17). A reduced symmetry with time-independent gauge parameters would require an explicit derivation and physical justification; it cannot be introduced implicitly.

3. **The Kalb–Ramond decomposition is either overcomplete or incompatible with the listed transformations.**

   The manuscript repeatedly calls $b_{\mu\nu}$ a spatial two-form but never imposes $\tau^\mu b_{\mu\nu} = 0$. Without that condition, equation (2.6) has more variables than the parent two-form: in ten dimensions an unconstrained $b_{\mu\nu}$ has 45 components and a spatial A_μ has 9, whereas $\hat{B}_{\mu\nu}$ has only 45. Indeed,

$$b \longrightarrow b + 2\tau \wedge \Lambda, \qquad A \longrightarrow A - \Lambda$$

for spatial Λ leaves $\hat{B}$ unchanged. This is an additional local redundancy that is not included in Section 2.1.1.

If $b_{\mu\nu}$ is intended to obey $\tau^{\mu} b_{\mu\nu} = 0$, the ordinary two-form shift and the Green–Schwarz transformation must be projected, with compensating transformations of A_{μ}, in order to preserve spatiality. The transformations currently written in Sections 2.1.1 and 3.1.1 do not do this. The authors should define the independent field space unambiguously, state every constraint and redundancy, and derive gauge-invariant projected curvatures and Bianchi identities before presenting the actions. This is also needed for a reliable degree-of-freedom count.

4. **Equation (4.2) is not the dilaton Euler–Lagrange equation, and the subsequent expansion inherits the error.**

   For the string-frame action displayed in equation (3.2), varying the dilaton gives, in the normalization of the manuscript,

$$\hat{R} + 4\hat{\nabla}^2\hat{\phi} - 4(\partial\hat{\phi})^2 - \frac{1}{12}\bar{H}^2 - \frac{1}{4}\hat{F}^2 = 0.$$

   Equation (4.2) instead sets $-2\hat{\mathcal{L}} = 0$. The object $\hat{\mathcal{L}}$ is not defined so as to contain the integration-by-parts term, and equations (4.10)–(4.14) confirm that the authors have expanded the unvaried action density: the Laplacian is absent and the dilaton kinetic term has the action sign rather than the Euler–Lagrange sign.

   Other coefficient equations also fail direct checks. Relative to equation (4.3), the signs of the dilaton-gradient terms in equations (4.16) and (4.17) are reversed. In the Yang–Mills equation, the order-w^0 term $-h^{\nu\rho}(\hat{\nabla}_{\nu}\hat{F}_{\mu\rho})^{(0)}$ is missing from equation (4.20) and appears spuriously in equation (4.21), which therefore mixes different orders. The notation also alternates between $\hat{H}$ and $\bar{H}$ in a section meant to cover the heterotic theory.

   The full relativistic equations should be corrected first and then expanded afresh. The result should be an explicit independent system in the Carrollian variables, not a list that leaves most curvature and derivative coefficients in hatted, unevaluated form.

5. **The constrained variational analysis is incomplete and contains explicit algebraic errors.**

   Appendix A does not provide a reliable second derivation of the dynamics. For the action used there, the beginning of the dilaton variation should be

$$-2\Omega_c e^{-2\varphi} L + 8\partial_{\mu}\Big(\Omega_c e^{-2\varphi}\tau^{\mu}\tau^{\nu}\partial_{\nu}\varphi\Big),$$

   whereas equation (A.1) starts with $+\Omega_c e^{-2\varphi} L$ and also changes φ to an undefined ϕ in the second term. Equation (A.2) inexplicably changes several total-derivative factors from $e^{-2\varphi}$ to $e^{+2\varphi}$. More decisively, equation (A.3) contains the term

$$4\partial_{\beta}\Big(\partial_{\mu}\tau^{\mu}\,\tau_{\alpha}\tau^{\beta}h^{\rho\sigma}\Omega_c e^{-2\varphi}\Big),$$

   which leaves ρ and σ uncontracted in what should be a vector equation with only free index α.

   There is also a structural problem. The constraints enforced in equation (4.26) are redundant, so the associated multipliers are not uniquely determined without accounting for the reducibility of the constraint set. Their contributions are never included in the appendix equations or eliminated.

Moreover, only the truncation $H = F = 0$ is varied, and the comparison with the expanded parent equations is explicitly deferred. Varying the finite coefficient of a singular expansion is not automatically equivalent to setting every positive, zero, and negative power in equations (4.5)–(4.22) to zero; subleading fields and constraint equations can matter.

The authors should either carry out the multiplier elimination and demonstrate equality of a projected independent set of equations, or substantially narrow the paper's claims. In its present form the manuscript has not derived the NS–NS and heterotic dynamics by two complementary methods.

6. **The covariance and action formulas require checks in a well-defined set of geometric variables.**

   Equation (2.17) contains the undefined object $\tau^{\rho\sigma}$, presumably intended to be $\tau^\rho\tau^\sigma$. More broadly, the paper should explain whether the exact split fixes the usual local Carroll/Milne-type redundancy in the inverse fields or whether an additional boost symmetry is present. The constrained variation treats $\tau_\mu, \tau^\mu, h_{\mu\nu}, h^{\mu\nu}$ as independent off shell while using formulas for the connection and measure that rely on the constitutive relations; a consistent off-shell extension has not been specified.

   The very long expressions in equations (2.38), (3.15), and (3.16) are difficult to verify and obscure the relevant invariant structure. Some simple reductions are encouraging: for an adapted flat clock, the leading NS–NS three-form terms can be organized into the expected electric square, and the Yang–Mills terms reduce to a square built from the temporal field strength and $D_i\chi$. The authors should make such structures manifest by introducing projected field strengths, extrinsic curvature, acceleration, and standard Carroll tensors. At minimum they should demonstrate invariance and consistency in the adapted flat-clock, Abelian, $\chi = 0$, and pure dilaton-gravity limits. The coefficient discrepancy between the sentence before equation (2.38), which states $-\frac{1}{4}(\tau \cdot \partial\varphi)^2$, and the coefficient -4 in equation (2.40) must also be resolved.

7. **The "heterotic" truncation and the α' ordering are not consistently specified.**

   The action (3.2) retains F^2 and the Yang–Mills Chern–Simons modification of H, but suppresses their α' normalization and omits the Lorentz Chern–Simons and curvature-squared terms tied to them in the standard heterotic effective action. Section 5.2 then rescales only α' associated with a curvature correction. The authors need to state precisely which truncation and derivative scheme they are using and explain how the gauge sector scales in the same limit. Calling the displayed result the complete bosonic heterotic theory is otherwise misleading.

   This issue also affects the assumption $\bar{H} = 0$. In heterotic theory the Green–Schwarz Bianchi identity relates $\mathrm{d}\bar{H}$ to gauge and gravitational characteristic forms. Thus $\bar{H} = 0$ is not a generic consistent truncation unless the relevant forms satisfy the required equality. The manuscript should impose and discuss this condition rather than use $\bar{H} = 0$ as an unrestricted route to a curvature-squared statement.

8. **Section 5.2 establishes much less than the claimed finiteness of four-derivative corrections.**

   The narrow observation that a curvature-squared term can survive a double scaling is plausible after correcting the dilaton exponent: a w^{-2} measure, $\alpha' = \alpha'_c/w^2$, and an order-w^4 curvature square can combine to give an order-one term. However, equations (5.1)–(5.3) inspect only selected powers of $\hat{\mathrm{Riem}}^2$ after setting the three-form to zero. They do not analyze the complete bosonic or heterotic correction, including torsionful curvature, gauge, three-form, dilaton, and field-redefinition-dependent terms.

There is also a gap internal to the displayed computation. Equation (2.26) allows a coefficient $\hat{R}^{(4)}$, but equations (5.1)–(5.3) silently contain only $\hat{R}^{(2)}$, $\hat{R}^{(0)}$, and $\hat{R}^{(-2)}$. If $\hat{R}^{(4)}$ vanishes by identities following from the leading connection, that fact and the responsible projections should be proved before the power count is called complete. Similarly, the asserted vanishing of all w^{10}, w^{8}, and w^{6} contractions should be supported by transparent tensor identities rather than only the statement that a long computation was performed.

Finally, $\alpha' \to \alpha'_c/w^2$ is a double scaling of the fundamental string length, not merely a Carroll contraction at fixed theory. Its relation to the worldsheet limit and to the strong-coupling dilaton scaling should be explained. Until the full invariant is treated, the conclusion should be restricted to a diagnostic scaling of a single Riem^2 term.

9. **The physical interpretation and comparison with existing work remain too qualitative.**

   Section 5.1 cites the relevant non-relativistic NS–NS and heterotic constructions, including the formulations of Bergshoeff and collaborators and the Double Field Theory approach of Lescano and Osten. Yet the central claim that the present theory interpolates between different Green–Schwarz realizations is not supported by a field-and-symmetry dictionary, and the gauge inconsistency above presently invalidates the comparison. A revised paper should show concretely how its variables, redundancies, connection, and curvatures differ from a non-Riemannian DFT parametrization or a shifted connection.

   The manuscript also presents one scaling as "the" Carrollian limit, whereas the cited near-horizon string literature distinguishes electric and magnetic Carroll branches. The chosen scaling of the two-form and gauge fields should be classified relative to those branches, and alternative consistent scalings should at least be discussed. The near-horizon and worldsheet sections contain no calculation: there is no nontrivial solution, linearized spectrum, background-field coupling, or beta-function comparison. The manuscript properly calls the worldsheet relation conjectural in Section 5.3, but the introduction and conclusion sometimes state it more strongly.

   Finally, the paper alternates between ten-dimensional supergravity actions and the claim that the NS–NS construction applies in arbitrary dimension, including the $D = 26$ bosonic string. The ten-dimensional superstring and 26-dimensional bosonic-string interpretations should be clearly separated, including any central-charge term away from the relevant critical dimension. A corrected nontrivial solution or an explicit match to one of the cited worldsheet or near-horizon branches would materially strengthen the significance of the work.

Minor and presentation comments

1. Equation (3.10) contains a malformed antisymmetrization bracket and should be rewritten unambiguously.

2. The left-hand side of equation (3.16) reads $[\bar{H}]^{(2)}$ rather than $[\bar{H}^2]^{(2)}$.

3. The sentence introducing equation (3.17) says $w \to 0$, contrary to the $w \to \infty$ limit used throughout the manuscript.

4. The gauge algebra, invariant inner product, trace convention, and normalization of f_{ijk} should be stated. These conventions are essential for the Chern–Simons coefficients and for comparison with the heterotic gauge groups.

5. The notation for relativistic and Carrollian three-forms should be made uniform. In particular, Section 4 alternates between $\hat{H}$, H, and $\bar{H}$ in equations that are presented as one system.

6. Equations (5.1)–(5.3) mix hatted and unhatted inverse metrics. The contraction and the meaning of each expansion superscript should be defined once and used consistently.

7. The reference to the Carrollian worldsheet paper should include its title and complete bibliographic information. A "work in progress" citation cannot replace a consistency calculation required by the present manuscript.

8. The manuscript would benefit from a careful technical and language edit. There are numerous malformed formulas, encoding problems, typographical errors, and stray manually inserted citation numbers. These problems are particularly serious in a paper whose main evidence consists of long component expressions.

Novelty, significance, and recommendation

The proposed target-space extension of Carrollian gravity to the universal string fields and a non-Abelian heterotic sector is potentially original and well motivated. If the finite action, constraint-preserving symmetries, and equations of motion can be established, the result could be suitable for a journal such as JHEP or Physical Review D. The manuscript also identifies several promising connections to non-Riemannian geometry, near-horizon strings, and higher-derivative limits.

At present, however, the defining dilaton scaling is wrong, the heterotic gauge contraction does not preserve its own constraints, and neither route to the equations of motion is correct or complete. The Green–Schwarz trivialization and the claim of a complete finite heterotic action therefore remain unproved. Correcting these points will require revisiting essentially every central result in Sections 2–5 and Appendix A. I consequently recommend rejection in the present form, while encouraging the authors to submit a substantially rederived manuscript if the corrected construction continues to support the proposed conclusions.