# Referee Report

*Quantum Horizon and Quantum Membrane Paradigm from Black Hole Quantum Mechanics*

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## Summary of the paper

The manuscript develops an ambitious extension of the author's proposed matrix quantum mechanics of black holes. The starting point is a spin-$J = (N-1)/2$ fuzzy-sphere block, interpreted as a horizon, together with a half-filled sea of fundamental fermionic degrees of freedom. The paper advances three main claims. First, the coherent-state line bundle of the spin-$J$ representation is interpreted as an intrinsic Berry monopole of Chern number $2J = N-1$, and the fundamental fermions are identified with monopole lowest-Landau-level (LLL) states. Second, guiding-center motion of these states is used to infer Hall transport, while a phenomenological Langevin equation is used to add an Ohmic response. Third, the horizon block is coupled to a commuting environmental block through off-diagonal matrix links. One extremal link mode becomes tachyonic, and its proposed condensate is argued to lock the horizon gauge field to the boundary value of the exterior Maxwell field. The resulting surface admittance contains longitudinal, Hall, and polarization terms and leads, after assuming a Schwarzschild exterior, to a frequency- and helicity-dependent reflection coefficient.

This is an imaginative synthesis. A controlled microscopic realization of these ideas would be interesting to the black-hole, matrix-model, and quantum-Hall communities. Several ingredients are also useful in isolation. In particular, the coherent-state construction in equations (4.16)–(4.23), after a sign convention is repaired, is the standard geometric-quantization description of the spin-$J$ Hilbert space; the fixed-$M$ calculation leading to the extremal eigenvalue in equation (6.35) is a concrete result; and the helicity algebra leading to equation (6.76) is reasonable once the assumed admittance is granted.

The central claims, however, are not established by the present derivation. The microscopic current is not projected to the claimed guiding-center current, the thermal scaling is evaluated in the wrong time frame, the multi-block fermionic gauge representation is inconsistent, and the proposed condensate is neither localized nor shown to be a self-consistent nonlinear saddle. These problems directly affect the Hall response, the charge identification, the locking mechanism, and the reflection coefficient.

## Correctness, novelty, and significance

The application of the coherent-state/LLL correspondence to this particular black-hole proposal is potentially useful, but the correspondence itself is standard rather than a new physical monopole carried by every fuzzy sphere. The genuinely novel part would be the combination of a microscopic horizon response with a dynamically generated link condensate. At present, that part rests on unproved replacements and parametrically large neglected effects. The manuscript therefore does not yet demonstrate a quantum membrane paradigm in the journal sense of a derived, self-consistent effective description.

The literature discussion is broad concerning the classical membrane paradigm, the author's previous matrix model, and echo phenomenology, but it should engage more directly with the standard Haldane-sphere/monopole-harmonic and geometric-quantization literature, projected LLL response and Kubo calculations, and bifundamental Higgsing in quiver or matrix systems. That comparison is needed both to delimit the novelty and to identify the calculations required for a controlled effective theory.

## Recommendation

**I recommend rejection in the present form, with encouragement to resubmit after a substantial reconstruction.** This recommendation is not based on presentation or on a request for additional qualifications. The missing steps are the principal dynamical links in the argument, and several internal inconsistencies change the advertised large-$N$ predictions. A publishable revision would need one consistent gauged multi-block theory, a microscopic response calculation, and a backreacted nonlinear condensate solution.

## Major comments

1. **The Berry/LLL construction has an explicit sign inconsistency and its physical status is overstated.**

   Equations (4.19)–(4.23) are not mutually compatible as written. Equation (4.19) gives $\psi \mapsto e^{-i\beta}\psi$ and $\mathcal{A} \mapsto \mathcal{A} + d\beta$, for which the covariant derivative is $D = d + i\mathcal{A}$. With equation (4.23), $\mathcal{A}_{\bar{z}} = iJz/(1 + |z|^2)$, and with the wavefunction in equation (4.21),

   $$\psi = P(z)(1 + |z|^2)^{-J},$$

   one obtains $D_{\bar{z}}\psi = -2Jz\psi/(1 + |z|^2)$, rather than the zero asserted in equation (4.22). The holomorphic coordinate, charge, or connection convention must be conjugated or reversed, and the corrected sign must then be propagated through the $u/d$ Berry curvatures and the Hall response.

   More conceptually, Appendix C computes the familiar Berry bundle of spin coherent states over their parameter sphere. This is a kinematical line bundle, not by itself a propagating spacetime magnetic field. The assertion that a rank-changing transition must release a physical monopole does not follow from Chern-number conservation: changing the representation changes the Hilbert space and the line bundle. A conservation theorem would require a fixed-Hilbert-space embedding and a smooth family of projectors, or an explicit conserved topological current through the tunneling solution. The paper should either supply such a construction or narrow the claim to the standard coherent-state geometric interpretation.

2. **The Hall current is not derived from the microscopic matrix current.**

   The current obtained from the Yukawa coupling is equation (4.42),

   $$j^A \sim \zeta^A u^\dagger d^\dagger + \bar{\zeta}^A du,$$

   which is off-diagonal in the two radial-spin bands and creates or annihilates a particle-hole pair. Equation (4.53), by contrast, is a diagonal point-particle guiding-center current, and equation

(4.54) postulates its minimal coupling to $b_A$. The paragraph following equation (4.58) says that virtual inter-band mixing makes these descriptions equivalent, but no projection or integration over the opposite band is performed.

This is a central dynamical step, since $P_\pm \sigma_T^A P_\pm = 0$. Starting from the finite-$N$ action, the authors should perform a controlled Schrieffer–Wolff/adiabatic projection or compute the retarded current kernel. Such a calculation must recover equation (4.54), determine possible frequency, gradient, and contact terms, and keep track of the exact $N+1$ and $N-1$ band degeneracies and the spin-frame contribution to the Berry curvature. For a partially filled, macroscopically degenerate LLL, the assumed uniform ensemble and the conditions under which a DC Hall coefficient exists must also be stated. As written, equations (4.55)–(4.63) are plausible LLL phenomenology but do not follow from the displayed matrix current. In addition, multiplying equation (4.55) by the actual inverse of $\mathcal{F}_{AB}$ gives a minus sign relative to equation (4.58); the inverse-tensor convention should be defined and the sign checked.

3. **The FIDO/asymptotic-time mismatch changes the thermal scaling and suppresses the claimed Hall effect.**

   Equations (3.16)–(3.18) define the local FIDO time by $\tau \sim t/N$, and the fermion, guiding-center, and Langevin equations are all written in $\tau$. The estimate preceding equation (4.72), however, inserts the asymptotic Hawking scale $T_H \sim 1/R$ and a decay rate $\Gamma \sim 1/R$ directly into this local-time dynamics. At a stretched horizon a Planck proper distance away,

$$T_{\text{FIDO}} = \frac{T_H}{\sqrt{f_{\text{sh}}}} \sim \frac{1}{l_P}, \qquad \Gamma_{\text{FIDO}} \sim \frac{1}{l_P}.$$

   If the manuscript's own estimate $\eta \sim T^2$ is used, then

$$\eta \sim l_P^{-2}, \qquad \mathcal{F} = \frac{J}{R^2} \sim \frac{1}{N l_P^2}, \qquad \gamma_T = \frac{\eta}{\mathcal{F}} \sim N,$$

   not $c_T/N$ as stated in equation (4.72). Equation (4.71) then still gives an order-one longitudinal conductivity on the strong-damping branch, but its Hall term is suppressed by $1/N^2$ relative to the ideal result, contrary to equation (4.75). This materially alters one of the paper's principal predictions.

   Even apart from this frame error, equation (4.64) is a phenomenological Langevin ansatz. The statement $P_{\text{diss}} = \eta v$ is dimensionally and mechanically incorrect; frictional power is $\eta v^2$. The manuscript acknowledges that neither KMS nor fluctuation–dissipation has been established, and the coefficient $c_T = 1/(2q^2)$ is explicitly conjectured only so that $\sigma_{xx} = 1/(4\pi)$. Equation (6.66) later inserts this value as if it had been derived. A Kubo calculation in a specified interacting thermal state is required; otherwise the universal membrane coefficient and all predictions that use it must be presented as assumptions.

4. **The multi-block fermionic representation and Gauss-law argument are inconsistent.**

   The projector-weighted actions (5.8) and (5.29) are new model definitions rather than consequences of equation (1.1), and the fixed projectors hard-wire a preferred block decomposition. More seriously, equation (5.7) treats the node fermions as fundamentals, $\psi_i \mapsto U_i \psi_i$, while treating the off-diagonal fermions as bifundamentals, $\eta^{ij} \mapsto U_i \eta^{ij} U_j^\dagger$. Cross Yukawa terms such as those in equation (5.16) are not invariant under these simultaneous rules. Treating the entire boldface fermion by conjugation would repair those terms but would turn the diagonal fermions into adjoints, eliminating the fundamental-index structure used for the entropy and Berry construction. Conversely, the covariant derivative in equation (5.58) acts only on the left and is incompatible with bifundamental links. The kinetic term also changes from $i\psi^\dagger\dot{\psi}$ in equations (1.1) and (4.1) to $i\dot{\psi}^\dagger\psi$ in equations (5.1), (5.8), and (5.29), which differs by a sign up to a boundary term.

Consequently, the Gauss-law discussion in Section 5.3 is not derived from a well-defined gauge symmetry. If the group is $SU(N)$, the constraint is traceless and cannot be traced to obtain equation (5.66). If it is $U(N)$, the trace constraint enforces total gauge neutrality and requires an explicit normal-ordering/background-charge prescription. Moreover, in Gaussian Maxwell normalization one has $\nabla \cdot E = 4\pi\rho$ and $Q = (4\pi)^{-1}\int E_r\, d\Sigma$, whereas equations (5.68)–(5.69) omit these factors and do not explain how the unit matrix gauge charge becomes the Yukawa-defined charge $q = a_2/(2a_0)$. The authors must define a single gauge group and representation assignment, prove invariance of the full action, and rederive the node constraints and asymptotic flux from that action before identifying $Q = q(r - s)$.

5. **The single tachyonic link mode does not establish a Planck-thick, rotationally invariant condensate.**

   The calculation of the extremal $M = J + 1$ mode and

   $$k_-(\xi) = 2(\xi - J)^2 - 4(J + 1)$$

   in equation (6.35) is a useful concrete result. It is nevertheless only one direction in the full link field space. The manuscript does not show that it is the global lowest mode, classify the other tachyons, impose the Gauss constraints, or solve the nonlinear equations for the diagonal blocks and links.

   There are several immediate obstacles to the claimed shell. First, the mode is tachyonic throughout

   $$|\xi - J| < \sqrt{2(J + 1)}.$$

   Even under the manuscript's semiclassical identification $r \simeq 2l_P\xi$, this is a coordinate-width band of order $\sqrt{N}\, l_P$, not $l_P$ as asserted after equation (6.46). Second, $\xi_a$ is the eigenvalue of the shifted operator

   $$Ly_a = \frac{x_a}{2l_P} + Lp_a$$

   in equation (6.20), not an eigenvalue of position. A joint $\xi_a$ eigenstate is delocalized in $x$, so equations (6.45)–(6.48) cannot treat $x$ and $p$ as simultaneous c-numbers, set $p = 0$, and infer a real-space shell. This conclusion also conflicts with equations (3.14)–(3.16): a Planck proper-distance stretched horizon has Schwarzschild coordinate offset of order $l_P/N$, not $l_P$.

   Third, the passage from one rank-one mode to a uniform condensate is not justified. Spin coherent states are nonorthogonal and overcomplete, while the quartic potential is nonlinear in $W$. Cross-mode/star-product terms must be evaluated in an explicit finite-$N$ matrix configuration; separate single-mode energies cannot simply be summed to obtain equation (6.53). Equation (6.44) also reverses the horizon/environment ket-bra orientation relative to the correctly oriented $W : N_H \times N_E$ mode in equation (6.34). Equation (6.18) is dimensionally ill-typed inside $\mathrm{Tr}_H$: $h^\dagger h$ and $W^\dagger W$ are environmental-block matrices, whereas the prior expression (6.12) correctly contains $hh^\dagger$ and $WW^\dagger$.

   Finally, backreaction cannot be neglected. Equations (6.13) and (6.43) give a physical condensation energy of order $-N/l_P$ for even one rank-one minimum, comparable to the black-hole mass.

Populating the $N$ angular cells used to motivate equation (6.53) naively gives order $-N^2/l_P$, larger than the background energy by a factor of $N$. A full normalized saddle, including all cross terms and the response of both diagonal blocks, is indispensable. Without it, equations (6.53)–(6.55) do not demonstrate locking around the assumed background.

6. **The boundary effective action is not a consistent truncation of the horizon theory.**

   Equation (6.56) drops the factor $l_P^{-2}$ multiplying the magnetic/angular term in equation (6.3), and equation (6.58) correspondingly defines a dimensionally inconsistent $\Box = \partial_A^2 - \partial_\tau^2$ while $A$ is a unit-sphere index. In addition, the normal scalar cannot simply be "dropped" in passing from equation (6.3) to equation (6.56): equation (6.3) contains the linear mixing $-8f\varphi_H$ inside the potential, so $\varphi_H = 0$ is not a consistent truncation for nonzero $f$. It must be retained or integrated out, after which the magnetic kernel and polarization response should be recomputed.

   The finite locking stiffness should likewise be kept until the response kernel is obtained. Equation (6.63) follows only for $\omega, |p| \ll \sqrt{N}/l_P$, but the subsequent discussion advertises a reflection coefficient for general frequency. The precise FIDO normalization in equation (3.17) was chosen rather than derived, so the numerical polarization coefficient $D$ is also not fixed until the map between matrix time and local proper time is established.

   There is a further sign inconsistency in equation (6.38). With the stated convention $e^{(+)} = (e^{(1)} + ie^{(2)})/\sqrt{2}$, one finds $e_a^{(+)} e_b^{(+)*} = (P_{ab}^T - i\epsilon_{abc}n_c)/2$, not the printed plus sign. The following sentence, which gives $D_{ab} = -i\epsilon_{abc}n_c$, actually uses the corrected sign. This should be repaired throughout the link-helicity calculation.

7. **The reflectivity is conditional on unproved inputs and uses the wrong frequency variable.**

   The frequency in equations (6.62)–(6.66) is conjugate to FIDO time $\tau$. Equations (6.69)–(6.79) then switch to Schwarzschild time $t$ without applying $\omega_\tau = \omega_t/\sqrt{f_{\text{sh}}} \sim N\omega_t$. In particular, the polarization term in a Schwarzschild-time admittance acquires the corresponding redshift. The stated frequency dependence and the large-$N$ reflectivity therefore do not follow as written.

   More broadly, Section 3 derives a regulator-dependent flat Cartesian Maxwell action, with $V_3 = 24R^2L$ chosen to reproduce the desired Gaussian normalization, whereas Section 6.3 imports a Schwarzschild metric and tortoise coordinate that the manuscript explicitly does not derive. For a charged horizon, the exterior should be Reissner–Nordstrom rather than Schwarzschild beyond leading order in small $Q$. The reflection formula is thus conditional on an assumed geometry, an unconstructed condensate, an unproved projected current, and a conjectured value of $\sigma_{xx}$. It should not be described as a non-phenomenological prediction until these inputs are controlled. The approximation in equation (6.79) also requires $|\sigma_H| \ll |\omega D|$ (and the low-frequency regime), not merely "small charge"; its apparent divergence as $\omega \to 0$ is outside that expansion.

## Minor comments and concrete corrections

1. The entropy formula (4.12) has the wrong leading coefficient for natural logarithms. With $M = N^2$,

$$\log\left[\binom{M}{(M+n)/2}^2\right] = 2M\log 2 - \frac{n^2}{M} + O\left(\frac{n^4}{M^3}, \log M\right),$$

not $2N^2[1+O(n^2/N^4)]$. The exact $N+1$ and $N-1$ band degeneracies should also be included when discussing subleading entropy.

2. The statement in Section 7 that LLL coherent packets resolve area $l_P^2$ is inconsistent with Appendix B. A spin-$J$ fundamental LLL has $N$ orbital cells, so equation (B.7) gives $\Delta A \sim 4\pi R^2/N \sim N l_P^2$ and magnetic length $\ell_B \sim \sqrt{N}\, l_P$. The $N$ flavors add internal degeneracy, not additional spatial orbitals. Relatedly, the physical Berry field $J/R^2$ after equation (4.55) is of order $1/(N l_P^2)$, not "of order $N$."

3. The Reissner–Nordstrom energy term in equation (4.10) is inserted as the classical energy of a charged shell rather than derived from the matrix Hamiltonian. The text should distinguish this consistency target from a result of the model and include the Maxwell normalization and gravitational backreaction.

4. In equation (6.77), the amplitude should be written with $|D_\pm|$ unless a sign range is imposed. Negative $D_\pm$ otherwise gets folded inconsistently into the stated phase.

5. In equation (B.10), the summation variable is $l$ but the coefficient contains an undefined $r$. Both occurrences should be $l$.

6. Equation (1.1) is followed by the statement that the model has "three" dimensionless coefficients while only $a_0$ and $a_2$ are listed. Equation (2.1) also appears to contain a malformed $J^\mu A_\mu$ term. These and the numerous grammatical and encoding errors should be corrected in a careful final copy-edit.

## Required path to a publishable result

The paper could become significant if the authors (i) formulate one gauge-invariant multi-block theory with consistent fundamental and link representations; (ii) derive the projected Hall and dissipative response from finite-$N$ correlation functions in a specified state and in one time frame; (iii) construct a normalized rotationally covariant link saddle, including its full spectrum, Gauss constraints, radial profile, vacuum energy, and backreaction; and (iv) only then derive the low-frequency boundary kernel on a clearly stated exterior geometry. Those results would permit a meaningful assessment of whether the proposed quantum membrane is correct, novel, and predictive.