Referee Report

"Revisiting wormhole-induced global symmetry breaking"
arXiv:2607.10981

Recommendation: Reject in the present form. The proposed conclusion would be important if established, and a fundamentally reconstructed paper could merit consideration. However, the present manuscript does not derive the physical alpha-parameter average that it uses, its stationary-phase analysis omits data that determine which contribution dominates, and its central applications to axion quality and baby universes do not follow. There are also several definite algebraic and phase-structure errors in the examples intended to support the main claim. These problems affect the central result rather than matters that can be repaired by a limited revision.

Summary and overall assessment

The manuscript revisits the standard description of Euclidean wormholes in terms of bilocal interactions and Coleman alpha-parameters. Section II introduces a matter theory with a putative $\mathrm{U}(1)^{[p]}$ global symmetry, rewrites a wormhole-induced bilocal term by a complex Hubbard–Stratonovich integral, and then posits the Lorentzian ensemble in equation (9). Section III argues that, at large spacetime volume, the oscillatory alpha-integral localizes at stationary points or nonanalytic points of the vacuum energy. A Landau-type potential $U(f) - \lambda f$ is used to claim that the symmetric point $\alpha = 0$ dominates when the original matter theory is either in a conventional symmetric phase or in a spontaneously broken phase. Other critical points are said to be suppressed by $\exp[-\exp(2S_{\text{ins}})]$. Section IV then concludes that wormholes do not create the conventional axion-quality problem in a standard Peccei–Quinn model, that the same mechanism effectively selects a one-dimensional baby-universe sector, and that it realizes a multicritical-point principle.

The paper has a provocative physical target. Establishing that a controlled wormhole construction removes, rather than produces, dangerous Peccei–Quinn breaking would be highly significant. The manuscript is also commendably direct about the role of its vacuum-energy argument. Unfortunately, the argument currently mixes three inequivalent objects: a Hubbard–Stratonovich identity inside a path integral, a coherent transition amplitude summed over alpha-sectors, and a statistical or reduced-density-matrix average over superselection sectors. Stationary-phase cancellation can occur in the second object, but it is not probabilistic selection of an alpha-sector. The third object contains no interference between orthogonal sectors. A fixed member of the ensemble, meanwhile, continues to have the symmetry-breaking couplings. This distinction is decisive for every principal conclusion.

Even if equation (9) is provisionally accepted as the desired observable, the subsequent asymptotic analysis is not correct as written. The reduction from a complex alpha-plane to a one-dimensional half-line drops the polar Jacobian, equation (27) is not a normalized expectation value, and the relative powers of volume, Hessians, phases, and critical-manifold dimensions are omitted. The Landau-potential example and Appendix B also contain explicit errors. Consequently, the manuscript does not presently establish correctness, and its claims of broad generality and phenomenological significance are substantially stronger than the demonstrated results.

Major comments

1. **The physical meaning of the alpha-integral must be fixed before symmetry can be assessed.**

   Equation (6) is invariant under the global symmetry, and equation (7) is an auxiliary-field representation of that invariant bilocal interaction. At a fixed nonzero α_m, the corresponding local theory contains an explicitly symmetry-breaking source. If the weight and measure in equation (9) are rotationally invariant, the full integral is formally invariant under a simultaneous change of integration variables $\alpha_m \to e^{im\chi}\alpha_m$. This is an invariance of an ensemble that permutes its members; it is not automatically a global symmetry acting within one fixed theory. Conversely, if the weight is not rotationally invariant, the weight itself supplies explicit breaking.

   In the conventional baby-universe description, alpha-values label superselection sectors. An observer in one sector computes $\langle \mathcal{O} \rangle_\alpha$. An incoherent ensemble or reduced-state observable has the schematic form
   $$\int d\alpha\, P(\alpha)\, \langle \mathcal{O} \rangle_\alpha ,$$
   whereas the manuscript studies cancellations in a coherent quantity of the form
   $$\int d\alpha\, \omega(\alpha) Z_\alpha .$$
   These are not interchangeable. The author needs to derive the relevant expression from specified initial and final baby-universe states, or from a specified density matrix, and state whether the proposal concerns an annealed average, a quenched average, a transition amplitude, or a reduced state. The symmetry should then be tested operationally through Ward identities, charged correlators, factorization, and cluster decomposition. Without this step, localization of an integrated amplitude cannot show that a fixed universe has an exact global symmetry, and it does not answer the no-global-symmetry arguments invoked in Section I.

   There is also an internal problem after equation (25). A nonzero radial critical value of a rotationally invariant complex alpha-integral is an entire phase orbit. Integrating that orbit cannot leave *explicit* breaking in the averaged object. Selecting a point on the orbit would instead require spontaneous selection, boundary conditions, or postselection. The manuscript should sharply distinguish these possibilities.

2. **The Euclidean Hubbard–Stratonovich identity does not derive the Lorentzian prescription in equations (9)–(10).**

   Equation (7) is an ordinary Euclidean Gaussian identity. Equation (9) then changes to a Lorentzian functional integral, and equation (10) treats the alpha-integral as a coherent real-time amplitude. No continuation contour, $i\varepsilon$ prescription, baby-universe wavefunction, or boundary condition is given. Those choices are not technical details: they determine the large-volume asymptotics.

   In particular, equation (10) does not exhibit vacuum projection for general $|i\rangle$ and $|f\rangle$. The factors $e^{-iT\Delta E_n(\alpha)}$ have unit modulus, so excited states do not decay in real time. Finite-energy-density states can themselves carry extensive phases. Vacuum dominance requires additional hypotheses, such as Euclidean evolution followed by a controlled continuation or an explicit $i\varepsilon$ prescription. A Euclidean factor $e^{-V\rho(\alpha)}$, however, selects minima of ρ, not stationary points of an oscillatory phase. Since a source commonly lowers the vacuum energy as its magnitude grows, the Euclidean and Lorentzian prescriptions can favor qualitatively different regions.

The author should also separate the spatial volume from the evolution time in equation (10), specify the order in which they become large, and show that the overlaps and excitation terms are subextensive uniformly in alpha. Until this is done, the step from equation (10) to vacuum-energy localization is not justified.

3. **The stationary-phase calculation is incomplete, and equation (27) is algebraically not an expectation value.**

   For a one-dimensional nondegenerate saddle, the leading term contains not only $\omega(\lambda_c)$ but also the magnitude and signature of the Hessian:

$$I_c(V) \simeq \omega_c e^{-iV\rho_c - i\pi\,\mathrm{sgn}(\rho_c'')/4}\sqrt{\frac{2\pi}{V|\rho_c''|}}.$$

   For a kink with nonzero one-sided slopes, the coefficient instead contains the jump of the inverse slopes and scales as V^{-1}. Equations (13)–(17) suppress precisely these factors, although they decide the relative size and phase of competing contributions. Equation (16) twice writes $\partial\rho/\partial\rho$ where $\partial\rho/\partial\lambda$ is required; its final sign also does not agree with the displayed integration by parts. Equations (14) and (17) use an undefined λ_s rather than λ_c. The Riemann–Lebesgue theorem by itself states decay of the integral, not localization at a uniquely preferred physical theory.

   More seriously, equation (27) cannot be correct even schematically. Setting $\mathcal{O} = 1$ gives $1 + \sum_k \omega_k/\omega_0$, rather than one. With several critical contributions, the normalized result must have the form

$$\langle\mathcal{O}\rangle_M \simeq \frac{\sum_k A_k(V)e^{-iV\rho_k}\,\langle\mathcal{O}\rangle_k}{\sum_k A_k(V)e^{-iV\rho_k}},$$

   where $A_k(V)$ contains the weight, Hessian or derivative jump, critical-manifold volume, Maslov phase, and the appropriate power of V. Even an expansion about one dominant contribution involves $\langle\mathcal{O}\rangle_k - \langle\mathcal{O}\rangle_0$, not the additive expression in equation (27). Different ρ_k also produce persistent relative phases, so the normalized thermodynamic limit need not exist. Equations (22) and (25), which take logarithms of complex amplitudes without fixing a branch or prescription, do not cure this problem. Appendix A inherits the same missing normalization.

4. **The complex measure changes the dominance hierarchy and invalidates the one-dimensional comparison.**

   The fundamental variable in equation (9) is complex. Writing $\alpha_1 = re^{i\varphi}$ gives

$$d^2\alpha_1 = r\,dr\,d\varphi,$$

   and, with the definition below equation (18), the radial measure is proportional to $\lambda\,d\lambda$, not $d\lambda$. Absorbing φ into the matter field additionally requires a rotationally invariant weight and compatible states and observables. With several harmonics, one field redefinition cannot remove all independent phases.

   The omitted Jacobian is especially important at the claimed symmetric point. In a spontaneously broken phase, a small symmetry-breaking source generically gives $\rho(r) = \rho(0) - vr + \cdots$. The origin then contributes as

$$\int_0^\infty r\,dr\,e^{iVvr} \sim V^{-2},$$

   not with the one-dimensional endpoint scaling used in Section III A. A finite-radius kink scales as $\omega(r_c)V^{-1}$, while a smooth radial stationary orbit can scale as $\omega(r_c)V^{-1/2}$. Thus, for fixed S_{ins}, a nonzero critical orbit can eventually dominate in the strict $V \to \infty$ limit despite an extremely small but fixed Gaussian weight. The order of the $V \to \infty$ and $S_{\text{ins}} \to \infty$ limits is essential and is not discussed.

The same issue becomes more severe for the infinite product of complex α_m in equation (9): the measure requires a regulator, critical points can be manifolds with zero modes, and different codimensions carry different powers of volume. At minimum, the paper needs explicit two-dimensional alpha-integrals for representative symmetric and spontaneously broken potentials before claiming a general localization theorem.

5. **The phase classification following Figure 2 is not mathematically consistent.**

The first case assumes that $U(f)$ has one minimum and no other extrema and then asserts a unique solution of $U'(f) = \lambda$ for every $\lambda \geq 0$. This requires convexity and suitable growth of U', neither of which follows from the stated assumptions.

The second case contains a more direct error. For a smooth radial potential with a minimum at $f = 0$, equation (20) gives

$$U'_{\text{eff}}(0) = U'(0) - \lambda = -\lambda.$$

Hence $f = 0$ is not a local minimum for any $\lambda > 0$. The low-field minimum shifts to a nonzero branch $f_<(\lambda)$; a possible first-order transition is between this branch and a large-field branch, not between $f = 0$ and $v(\lambda_c)$. Equations (23)–(24) therefore give the wrong competing vacua and the wrong vacuum energy. Taken literally, equation (24) is constant on the whole interval $0 < \lambda < \lambda_c$, so that interval has no oscillatory suppression at all and the inverse left slope in equation (15) diverges. Equation (25) does not follow.

The third case also needs more than monotonicity: tilting a nonconvex radial potential can produce additional branch crossings. Moreover, equations (18)–(20) lose a factor of two because $\alpha^*\phi + \text{h.c.} = 2|\alpha| f \cos\theta$ with the stated definition of λ. A corrected analysis should retain the full complex source, solve all local branches, and compare their proper stationary-phase coefficients.

6. **The claimed double-exponential suppression is conditional, not generic, and Appendix B contains a definite scaling error.**

Equation (28) assumes a Gaussian tail, a critical physical source of order one so that $\alpha_c \propto e^{S_{\text{ins}}}$, and a particular normalization of alpha. Yet equation (9) declares ω model-dependent, and the text alternatively speaks of finite support. Compact support would remove the large-alpha critical point rather than suppress it; exponential or power-law tails give different answers; absorbing $\sqrt{c_m}$ into alpha changes the Gaussian width. The relation $S_{\text{ins}}(m) \simeq m S_{\text{ins}}(1)$ and positivity of all wormhole coefficients are also model assumptions, not consequences established here. Even for a Gaussian, weight ratios cannot be compared without the volume powers and phases discussed above.

Appendix B does not provide the advertised check. From equation (B6), writing $h = e^{-S_{\text{ins}}(5)}\alpha_5$, degeneracy requires

$$\frac{\lambda h^4}{\kappa^4} \sim \frac{h^6}{\kappa^5} \quad \Longrightarrow \quad h^2 \sim \lambda\kappa.$$

Thus the scaling is $\alpha_5 \sim \sqrt{\lambda\kappa}\, e^{S_{\text{ins}}(5)}$, not $\lambda\kappa e^{S_{\text{ins}}(5)}$ as stated in equation (B7). Numerical coefficients do not affect this square-root discrepancy. The assertion before equation (B2) that the global minimum has been shifted to zero is also not implemented in the explicit potential (B3). These points must be corrected before any conclusion is drawn from equation (B8).

7. **Section IV A does not analyze the axion-quality problem that it claims to solve.**

   A realistic PQ model contains the QCD anomaly potential that defines $\bar{\theta}$. Schematically one must analyze

   $$V(a;\alpha) = \chi_{\rm QCD}\left[1 - \cos(a/f_a + \theta)\right] + \sum_m \epsilon_m(\alpha_m)\cos(ma/f_a + \delta_m).$$

   The QCD term fixes a phase in axion field space. Consequently, the phase of α_m cannot be absorbed as below equation (18) without also shifting the physical QCD angle, and the minimized vacuum energy need not be a function only of $|\alpha_m|$. Equation (29) contains only the wormhole harmonics and therefore cannot compute either the displaced minimum or $\bar{\theta}$.

   The experimental quality bound concerns the value measured in one universe. A CP-odd observable can average to zero over phases even when a typical fixed alpha-sector violates the neutron-EDM bound. The relevant prediction is therefore a fixed-sector result or a well-defined probability distribution for $|\bar{\theta}| < 10^{-10}$, not cancellation in an unnormalized coherent amplitude. The paper should include the QCD susceptibility, radial PQ dynamics, all relevant complex harmonics and phases, and a normalized observable. Until such a calculation is supplied, the statements in the abstract, Section IV A, and Section V that the axion-quality problem does not arise must be removed. At most the present toy model suggests a possible question for further study.

8. **The baby-universe conclusion is incompatible with the unitary evolution displayed in equations (30)–(31).**

   For orthogonal alpha-states, an alpha-dependent vacuum energy changes branch phases, $c_i \to c_i e^{-iE_0(\alpha_i)T}$, but not their magnitudes. After tracing over the baby-universe Hilbert space, the weights remain $|c_i|^2$; relative phases cancel from the reduced density matrix. Stationary-phase interference between different alpha-values therefore cannot turn equation (30) into the product state (32). Such interference would require projection onto, or postselection of, a coherent final baby-universe state, and that operation must be stated explicitly. Ordinary unitary evolution cannot erase the entanglement claimed in equation (30).

   This is not a minor interpretive issue: it directly contradicts the proposed support for the baby-universe hypothesis. The discussion should be withdrawn unless the author supplies a dynamical map or measurement protocol that actually produces equation (32). Equation (31) also writes the bra incorrectly, and the sentence that follows should use $\dim \mathcal{H}_{\rm BU} > 1$, not ≥ 1.

9. **The claimed extension to general higher-form symmetries is presently only formal.**

   The calculation in Section III B is an ordinary complex-scalar Landau analysis. Spontaneous breaking of a higher-form symmetry is diagnosed by extended charged operators and depends on spacetime dimension, form degree, infrared fluctuations, and gauge constraints. It is not generically represented by a single local radial potential $U(f)$. No concrete higher-form instanton, determinant, induced alpha-measure, or higher-form phase transition is calculated.

   There is already a definitional problem in equation (3): the shift should be by a closed spacetime p-form with quantized periods, not by an element described as belonging to $H^p(C_p;\mathbb{Z})$. The normalization and periodicity of the parameter, the allowed range of p, and the distinction between electric and magnetic generalized symmetries should be stated. The manuscript should either restrict its principal analysis to a well-defined $p = 0$ model or provide a genuine brane-field derivation and compare it with the standard formulation of generalized global symmetries.

10. **The multicritical example in Figure 3 and equation (35) is incorrect.**
For the real scalar potential

$$U(\phi) = \frac{\lambda_2}{2}\phi^2 + \frac{\lambda_3}{3!}\phi^3 + \frac{\lambda_4}{4!}\phi^4,$$

degeneracy between the origin and a nonzero stationary point gives

$$\phi_* = -\frac{2\lambda_3}{\lambda_4}, \qquad \lambda_2 = \frac{\lambda_3^2}{3\lambda_4}.$$

Thus the first-order locus is a positive parabola in the (λ_3, λ_2) plane, not the line shown in Figure 3. At $\lambda_2 = \lambda_3 = 0$, a real ϕ^4 theory has $\mathbb{Z}_2$ symmetry, not $\mathbb{Z}_4$; indeed $\lambda_3 = 0$ is the symmetry-enhanced locus. The assertion that the ensemble produces emergent $\mathbb{Z}_4$ is therefore false. The further statement about eliminating the quadratic-divergence problem is not derived from the displayed model and should not be presented as an implication of the current calculation.

Correctness, novelty, and significance

The new element appears to be the application of earlier Coleman/"big fix" and multicritical-point ideas to wormhole-induced symmetry-breaking sources, together with a proposed extension to higher-form symmetries. The underlying stationary-phase mechanism is not new, as the manuscript itself notes. The higher-form extension currently consists of notation and a mean-field analogy, rather than a controlled new result. The axion application would be important, but its significance depends entirely on resolving the fixed-sector versus ensemble issue and performing the QCD calculation described above.

The literature discussion should therefore be substantially deepened. The manuscript cites, but does not engage, modern work on factorization and the meaning or absence of low-energy ensemble averaging, as well as the longstanding Fischler–Susskind and Polchinski criticisms of wormhole sums. The relation to the baby-universe density-matrix interpretation, to the Marolf–Maxfield framework, and to recent axion-wormhole analyses should be discussed at the level of assumptions and observables rather than included only in citation lists. For higher-form symmetry, the standard generalized-symmetry framework and its dimensional restrictions should be the starting point.

A potentially publishable project could take one of two forms. The stronger route would construct a controlled gravity/baby-universe model, derive the Lorentzian alpha-state and contour, and demonstrate localization in normalized observables accessible within one universe. A more modest route would present a finite-dimensional or scalar toy model as a possible interference mechanism, with the complex measure and all saddle coefficients treated exactly and without claims that it restores a global symmetry, solves axion quality, or proves the baby-universe hypothesis. The present manuscript lies between these options and consequently overstates both correctness and scope.

Minor and presentational comments

1. With the conventional measure, equation (7) requires an overall $1/\pi$. More importantly, equation (8) has the wrong source coefficient: the Hubbard–Stratonovich identity gives $\sqrt{c_m}e^{-S_{\text{ins}}}\alpha_m^*\mathcal{O}_m +$ h.c., not $c_m e^{-2S_{\text{ins}}}\alpha_m^*\phi^m +$ h.c. Equation (9) returns to the former instanton scaling, so the three equations are inconsistent.

2. Equation (26) omits the factor $1/\sqrt{c_1}$ implied by the definition of λ and represents a complex critical orbit as a single real point. Both the normalization and the phase degeneracy matter for the later suppression estimate.

3. The assumption of a smooth finite-support weight below equation (11) should be reconciled with the Gaussian tail used in equations (28) and (B8). These are physically different hypotheses, not interchangeable examples within the claimed proof.

4. Appendix A suppresses the alpha-dependent interaction in its notation for S in equation (A1). It must also specify the order of the source, thermodynamic, and spontaneous-symmetry-breaking limits. Functional differentiation of an incorrectly normalized saddle sum does not establish equation (A3).

5. The manuscript should consistently distinguish explicit breaking, spontaneous breaking, invariance of an ensemble, and invariance under a spurionic transformation of couplings. Several conclusions currently move between these notions.

6. A careful proofread is needed. Examples include “spacial,” “perfuming,” “Aa a result,” and repeated subject–verb and article errors. Figure captions should state the parameter choices and explain the plotted phase boundaries.

Conclusion

The manuscript asks an interesting and consequential question, but the claimed answer is not supported by the present formalism. The alpha-sector observable is not defined, Lorentzian interference is treated as physical sector selection, the complex measure and normalized saddle sum are omitted, and the concrete potential examples contain errors. Because the axion-quality and baby-universe conclusions depend directly on these points, I cannot recommend publication after an ordinary major revision. I recommend rejection in the present form, while encouraging resubmission of a fundamentally reformulated and substantially narrower analysis if the physical alpha prescription and corrected calculations can be supplied.