# Referee Report

*Large $N$ conformal bootstrap and chirality*

arXiv:2607.11332

**Recommendation: Reject in the present form, with encouragement to submit a complete technical article after substantial reconstruction.** The proposed result is potentially interesting: an exact $d$-dimensional $O(1/N^3)$ anomalous dimension for a nonsupersymmetric scalar chiral theory could separate the graph-orientation constraint from supersymmetry in the transcendental content of large-$N$ critical exponents. In its present conference-proceedings form, however, the manuscript does not display that anomalous dimension, the bootstrap equations that determine it, or the perturbative data claimed to check it. The principal cancellation is therefore not independently assessable. In addition, equations (4)–(6) contain a factor-of-$N$ inconsistency that changes the order of the reconstructed beta-function term, and the discussion of reference [22] places the first possible $\gamma^5$ issue at the wrong loop order. These points affect the logical framework and the headline conclusion, not merely the presentation.

## Summary and overall assessment

The manuscript asks why non-Riemann-zeta or multiple-zeta structures found in the $O(N)$ scalar model at $O(1/N^3)$ are absent from the corresponding Wess–Zumino exponent. Section 2 reviews the reorganization of perturbative beta functions in $1/N$, the universal critical theory, and the conformal-bootstrap integral $I(\mu)$. Equation (12) recalls a cubic chiral superfield interaction. Section 3 then introduces the complex scalar theory (13), whose vertices have uniformly oriented edges, and states that only the two topologies represented in Figure 2 enter its bootstrap condition at the required order. A two-coupling six-dimensional completion is given in equation (15). The author reports that the resulting coefficient $\eta_3$ contains neither $I(\mu)$ nor $\widehat{\Omega}(\mu)$ and gives its values at $\mu = 3/2$ and $\mu = 5/2$ in equation (16). The scalar result is compared with the earlier fermionic calculation summarized around equation (17), and Section 4 attributes the observed cancellations to chirality rather than supersymmetry.

There is a worthwhile question here. References [8] and [10] provide supersymmetric and fermionic comparators, while the scalar chiral $O(1/N^3)$ result appears to be new. A closed expression for $\eta_3(\mu)$, together with a transparent account of its period basis and perturbative consequences, would be useful to the large-$N$ and multiloop communities. The current text is instead an announcement of the result. The only displayed new numbers are the two special-dimensional evaluations in equation (16), and the sole independent check is assigned to an unpublished calculation in reference [19]. This falls short of the self-contained evidence needed to judge correctness, novelty, and significance for a journal such as PRD or JHEP.

## Major comments

1. **Equations (4)–(6) are inconsistent in their $N$ counting.**

   Let $x = b_{11}Ng$. The $i = 1$ term in equation (4) is

$$Ng^2 \frac{1}{N}\, \beta_1(x) = g^2 \beta_1(x),$$

which is $O(N^{-2})$ when $g = O(N^{-1})$. Combining equations (4) and (5), including the $O(1/N)$ displacement of the fixed point, gives

$$\beta_1'(x) = -\frac{2b_{11}}{x^2}\,\omega_1(x).$$

Consequently the corresponding reconstructed contribution is

$$-2b_{11}g^2 \int_0^{b_{11}Ng} \mathrm{d}\xi\, \frac{\omega_1(\xi)}{\xi^2},$$

up to the integration constant fixed by the perturbative boundary condition. Equation (6) instead prints an additional factor of $N$. Its integral term is then $O(N^{-1})$, at the same order as the leading beta function, rather than the $O(N^{-2})$ contribution defined in equation (4). The displayed remainder is correspondingly inconsistent as well: after retaining the $i = 1$ term, the next absolute order is $O(N^{-3})$ under the stated scaling.

This error should be corrected and the derivation given explicitly. It is especially important because equations (1)–(6) are presented as the bridge between exact critical exponents and all-loop perturbative information. For the multi-coupling theory (15), the manuscript must also replace the one-coupling derivative in equation (5) by the appropriate stability-matrix eigenvalue when discussing correction-to-scaling exponents.

2. **The central $O(1/N^3)$ result and its derivation are absent.**

   The paper never writes the convention

$$\eta = \frac{\eta_1}{N} + \frac{\eta_2}{N^2} + \frac{\eta_3}{N^3} + O(N^{-4}),$$

   nor does it display the claimed general function $\eta_3(\mu)$. Figure 2 is a pictorial consistency condition without its analytic equation. The graph amplitudes, group and symmetry factors, regulator poles, amplitude combinations, and cancellation that remove $I(\mu)$ and $\widehat{\Omega}(\mu)$ are all omitted. The two values in equation (16) cannot establish a functional cancellation in arbitrary $d$: they provide only two evaluations of an undisclosed formula.

   The complete closed form of $\eta_3(\mu)$ is the minimum information needed for publication. The author should also provide the bootstrap equations and enough intermediate quantities to reproduce the result. If the expression is too long for the main text, it can be organized in a defined basis of gamma and polygamma functions with an electronic ancillary file, but the coefficients of $I(\mu)$ and $\widehat{\Omega}(\mu)$ and the mechanism by which they vanish must be visible in the article. The lower-order coefficients should be stated so that normalization and the meaning of $N$ can be checked against prior work.

3. **The claimed four-loop validation is not available for scrutiny, and the fixed point of equation (15) is underspecified.**

   After equation (15), the author states that four-loop field anomalous dimensions, beta functions, and an operator-mixing matrix agree fully with the large-$N$ exponents. No one of these quantities is shown; reference [19] is only "paper in preparation." This check is particularly important because it is the sole independent test of the new bootstrap calculation. An inaccessible future calculation cannot carry that evidentiary burden.

   Equation (15) has two couplings and can have more than one perturbative fixed-point branch. The paper should give $g_{1*}$ and $g_{2*}$ to the orders needed for the comparison, identify which branch realizes the universal theory (13), state its large-$N$ scaling, and discuss the domain in which the fixed point is real. At least a table should compare the $\epsilon$ expansion of $\eta_1$, $\eta_2$, and $\eta_3$ with the four-loop calculation coefficient by coefficient. The beta functions and anomalous dimensions should be included or placed in a public ancillary file. The phrase "dimension two operators" also needs correction: in six dimensions a scalar has canonical dimension two, whereas quadratic mass operators have canonical dimension four. If "operators quadratic in the fields" is intended, that should be stated.

4. **The chain relating equations (12), (13), and (15) is not defined sufficiently to support the advertised comparison.**

   Equation (12) is the interaction of a single cubic chiral superfield. It does not display the $O(N)$ singlet/vector superfield structure that would lead to the fields $\sigma$ and $\phi^i$ in equation (13). Equation (13), in turn, suppresses the coupling normalization and absorbs all dimension-dependent singlet dynamics into an unspecified function $f_\sigma$, while equation (15) introduces two independent couplings and explicit $\sigma^3$ interactions. The reader is not shown precisely which supersymmetric parent is being decomposed, which component or auxiliary fields are retained, or in what sense the nonsupersymmetric cubic theory is "the bosonic sector" of that parent. After the auxiliary fields are eliminated, the literal on-shell bosonic Wess–Zumino component theory contains quartic F-term interactions; equation (13) is instead a universal or Hubbard–Stratonovich-type cubic representation. That distinction should be made explicit.

   The actual $O(N)$ superpotential should be written, followed by the relevant component reduction and the large-$N$ assignments of every field and coupling. The author should say whether barred fields are complex conjugates or independent Euclidean integration variables, specify their global charges, and explain the factors of $1/2$ and $1/6$ in equation (15). The role of $f_\sigma$ in each critical dimension and the universality relation between the four- and six-dimensional descriptions should also be stated. In particular, the local scalar completion (15) is organized near six dimensions, whereas the fermionic Yukawa theory (17) is organized near four; they are related universal theories with a common directed topology, not literally two sectors of a single fixed point. Without this information, it is difficult to decide which properties follow from graph orientation, which follow from the chosen fixed-point branch, and which may still be inherited from the supersymmetric construction.

5. **The conclusion that chirality is the structural cause of the cancellation is stronger than the argument given.**

   The directed vertices imply a bipartite-type constraint on allowed graphs and exclude closed odd-edge structures. This is a useful observation, but neither "odd-edge subgraph" nor the precise orientation rule is defined mathematically. More importantly, the paragraph following equation (12) expressly acknowledges that allowed graphs can still contain $I(\mu)$ and that its contribution may cancel only in their sum. Section 4 nevertheless elevates the observed absence to the claim that chirality is the driving reason.

   The scalar calculation, if correct, demonstrates that full supersymmetry is not necessary for the cancellation. It does not by itself prove that the orientation constraint is sufficient, or that no other structural feature of equations (13) and (15) is responsible. The paper should identify the two Figure 2 contributions individually and show the algebraic identity that cancels their $I(\mu)$ and $\widehat{\Omega}(\mu)$ coefficients. It should then formulate the scope of the statement: is it a theorem for a class of oriented cubic theories, an observation about the matter-field exponent at one order, or a conjecture supported by the scalar and fermionic examples? A comparison with a minimally modified nonchiral theory would make the causal inference considerably more convincing.

6. **The transcendentality statement and the phrase "to all orders in perturbation theory"**

**require precise qualification.**

The manuscript alternates among "multiple zetas," "multiple zeta values," and the earlier literature on non-zeta periods. These are not interchangeable without a convention. Expansions of polygamma functions already generate ordinary Riemann zeta values, and products of such values can be represented in a multiple-zeta basis. The older result in reference [8] is more specifically phrased as the survival of only the Riemann-zeta series and the cancellation of non-zeta transcendentals associated with nontrivial knot periods.

The author should define the excluded class, list the complete functional basis remaining in $\eta_3$, and prove from its $\epsilon$ expansion which periods cannot occur. Merely noting the absence of $I(\mu)$ and $\widehat{\Omega}(\mu)$ is not a proof unless it is also shown that these exhaust all sources of the unwanted periods at this order. In fact $\widehat{\Omega}$ is itself a polygamma combination whose expansion produces ordinary zeta values, so its absence is not independently synonymous with absence of depth-greater-than-one periods. In addition, the result is exact in loop order only within the fixed $O(1/N^3)$ coefficient of a critical exponent. It is not an unrestricted statement about the full wave-function renormalization. The manuscript should identify the towers of perturbative coefficients constrained by the result, the coupling and subtraction scheme used in that mapping, and the orders in $1/N$ that remain uncontrolled.

7. **The fermionic comparison and the $\gamma^5$ outlook contain an unsupported and apparently incorrect loop-order statement.**

The discussion around equation (17) should distinguish clearly between the general non-Abelian expression, in which $I(\mu)$ is present, and the final Abelian chiral-XY limit, in which its coefficient vanishes. The Weyl Lagrangian also needs explicit flavor indices and spinor contractions if it is to support a detailed comparison with the scalar calculation.

More concretely, Section 4 says that reference [22] suggests a $\gamma^5$ problem at six loops and that its large-$N$ counterpart would first appear at $O(1/N^4)$. The discussion surrounding Figure 3 of reference [22] says instead that the four-loop candidate integrals vanish, that the first nonzero contribution can occur at five loops, and that a $\gamma^5$ prescription is required from five loops onward for the general model. The current statement should therefore be corrected. Any proposed mapping between that five-loop threshold and a particular $1/N$ order must be derived by explicit graph counting; it does not follow from reference [22] as cited.

8. **The novelty and journal-level significance cannot be evaluated until the new information is made usable.**

References [6], [8], and [10] already contain the conformal-bootstrap framework and the ordinary scalar, supersymmetric, and fermionic $O(1/N^3)$ comparators. The apparent advance here is the scalar chiral coefficient and the inference drawn from it. Yet neither the coefficient nor new perturbative predictions are supplied. As a result, the present eight-page text is suitable as a conference progress report but not as a self-contained journal article.

A full submission should state which high-loop coefficients are predicted beyond existing calculations, provide numerical or analytic checks in representative dimensions, and compare the new scalar chiral formula directly with the formulas in references [6], [8], and [10]. It should also moderate the title and conclusion unless a general orientation-based cancellation mechanism is proved. With those additions the calculation could be technically valuable; without them, the novelty remains an assertion rather than a reviewable result.

## Minor and presentation comments

1. The abstract should read "the bosonic sector of the $O(N)$ Wess–Zumino model."

2. Equation (10) should define $\Pi(\Delta)$ analytically. Figure 1 alone does not specify propagator indices, normalization, or the precise regulator placement needed to reproduce $I(\mu)$.

3. After equation (11), define $\psi$ as the digamma function and state that primes denote derivatives with respect to its argument.

4. Before equation (13), "two doublets of fields" should be replaced by "two conjugate pairs" unless a genuine doublet symmetry is intended.

5. The statement following equation (12) that the component Lagrangian contains "scalar Yukawa interactions" is unclear. If scalar–fermion Yukawa terms and scalar self-interactions are meant, they should be named separately.

6. The single-coupling notation in equations (1)–(6) should be explicitly identified as pedagogical and then generalized before applying it to equation (15). In a two-coupling problem, beta functions are a vector and correction-to-scaling exponents are stability-matrix eigenvalues.

7. The citation to reference [9] does not by itself substantiate the broad claim in Section 1 about the principle of maximal transcendentality. A direct reference and a statement of the version of that principle being used would help.

8. The statement in Section 1 that supersymmetry, gauge symmetry, and chirality are "strictly only present" in integer dimensions is too broad. Gauge symmetry itself is routinely continued in dimensional regularization; the delicate points concern dimension-specific Lorentz/chiral algebra and supersymmetric degree-of-freedom matching. The sentence should identify the actual obstruction.

9. Equation (16) should state explicitly whether $\eta_3$ is the coefficient of $N^{-3}$ for $N$ complex fields or for the corresponding number of real components. This convention is essential when comparing with the $O(N)$ literature.

10. Equation (16) should be supplemented by the near-six-dimensional $\epsilon$ expansion that is said to agree with the four-loop calculation; the values at $d = 3$ and $d = 5$ do not test that comparison.

11. The phrase "the universal theory resides in all spacetime dimensions" should be softened. The critical formulas can be analytically continued in $d$, but existence, reality, and unitarity of a fixed point are dimension- and $N$-dependent.

12. In Section 4, replace "neither the scalar or fermion sector" by "neither the scalar nor the fermion sector." The manuscript would also benefit from standardizing adjectival forms such as "all-orders," "two-loop," "four-loop," and "six-dimensional."

13. The date in the conference block should be "12–17 April 2026." Reference [12] has an extraneous comma after A.N. Vasil'ev, and journal-volume formatting should be standardized.