

Referee Report

“The Hitchhiker’s Guide to Tensionless String Limits in 4d SCFTs”

Recommendation: Major revision

Summary and overall assessment

This manuscript studies overall weak-coupling limits of large- N four-dimensional superconformal gauge theories and seeks a microscopic string-theory explanation for the three universality classes previously identified from the CFT distance exponent, the ratio a/c , and the large- N Hagedorn temperature. Its main strategy is to engineer the theories in Table 1 by Type IIA Hanany–Witten configurations and to organize them according to a parent brane system. Class I is associated with the critical Type IIB string, Class II with the double-scaled background of two NS5-branes familiar from the proposed dual of $\mathcal{N} = 2$ superconformal QCD, and Class III with a three-NS5-brane background. For the last case the authors write the CHS theory as a linear-dilaton factor times an $\mathcal{N} = 2$ minimal model and identify the normalizable worldsheet deformations corresponding to collinear NS5-brane positions. The paper also discusses interpolating quivers, extensions to multiple gauge factors, and more speculative “quasiconformal” examples.

The organizing idea is attractive and potentially important. In particular, the systematic collection of brane realizations, the relation between partial weak-coupling limits and subsets of NS5-branes, and the attempt to connect the $k = 3$ Hagedorn class with a concrete deformation of the CHS worldsheet go substantially beyond a short classification exercise. The manuscript is ambitious, contains many useful constructions, and could become a valuable reference for the high-energy theory community.

At present, however, several central steps are either internally inconsistent or remain conjectural at precisely the points where the manuscript claims to have established a common perturbative string description. There are explicit errors in the gauge-coupling dictionary, in the worldsheet central-charge check, and in one chiral spectrum. More importantly, the Class III theories involving “glued” NS5-branes rely on an unestablished four-dimensional brane rule and sit at a coincident-brane discriminant locus whose residual strong-coupling throat is not analyzed. The argument that flavor branes and parent/daughter operations are subleading also does not apply as stated to the Veneziano systems with N_f/N fixed. These are not merely matters of wording: they affect the correctness and scope of the proposed explanation of the universality classes. I therefore recommend major revision. The principal issues are detailed below.

Major comments

- 1. The relation between the NS5 background and the actual holographic string needs to be stated and justified more sharply.**

Sections 2.2–2.3 and 4.2–4.3 first remove the D4-branes, write an exact worldsheet for the resulting double-scaled NS5 background, and then argue that restoring the large number of color and flavor branes should generate an AdS_5 throat. Even for $\mathcal{N} = 2$ superconformal QCD this last step is a proposal rather than a known backreacted solution. For the new Class II and Class

III examples, neither the backreacted background nor its worldsheet is constructed. Thus the exact statement established by equations (4.1), (4.2), and (4.9)–(4.20) concerns the ambient NS5 system before the leading D-brane backreaction, whereas the manuscript often presents it as the worldsheet of the string propagating in the holographic dual.

The distinction matters because the proposed AdS₅ region is generated by the very branes that have been omitted, and their backreaction need not leave all features of the ambient closed-string spectrum unchanged. The revision should separate clearly: (i) exact facts about the double-scaled NS5 worldsheet; (ii) the brane engineering of the low-energy gauge theory; and (iii) the conjecture that a suitable D-brane backreaction produces a holographic throat whose tensionless sector is governed by the same worldsheet. Ideally the authors should provide a quantitative check linking these stages, for example by tracking the string tension in five-dimensional Planck units, identifying the sector that yields the CFT Hagedorn growth, or matching protected data. Without such a check, the new non-Einstein AdS/CFT pairs and the identification of their tensionless strings should be presented as proposals, not as derived dualities.

2. The large- N argument does not cover the $N_f \sim N$ flavor sectors used in the constructions.

Section 4.1 correctly notes that a boundary is suppressed only in the quenched regime $N_f \ll N$. The principal Class II and Class III examples are instead Veneziano systems. Superconformal QCD has $N_f = 2N$, and the linear Hanany–Witten parents contain order- N semi-infinite D4-branes (or, after duality, order- N flavor branes). In this regime the factor $N_f g_s$ is order one at fixed 't Hooft coupling, so worldsheets with flavor boundaries and the flavor-brane backreaction contribute at leading order. They cannot be discarded by the argument given for a fixed number of probe branes.

Orientifold/orbifold planar equivalence may still relate appropriate neutral sectors, but it requires a precise matching of couplings, an unbroken projection symmetry, and a statement of which operators are retained. It is not enough to say that all added ingredients are subleading. The authors should formulate the parent/daughter relation for each class and distinguish order-one changes in flavor multiplicity from the order- N flavor sector already present in the parent. They should also explain why the leading Hagedorn spectrum relevant here lies in a common neutral closed-string sector despite the leading flavor contribution. This is essential to the inference in Sections 4.2 and 4.3 that a common NS5 system by itself establishes a common emergent string.

3. The gauge-coupling dictionaries in equations (2.5) and (4.5) are inverted relative to the Type IIA interval formulas.

Equation (2.5) states

$$\frac{g_1^2}{g_2^2} = \frac{\beta}{1 - \beta},$$

whereas equation (2.7) gives $g_1^{-2} \propto \beta$ and $g_2^{-2} \propto 1 - \beta$. The latter implies

$$\frac{g_1^2}{g_2^2} = \frac{1 - \beta}{\beta}.$$

This is also the relation required by the subsequent limit: $g_2 \rightarrow 0$ with g_1 fixed corresponds to $\beta \rightarrow 0$ in the scaling used in equations (2.7)–(2.8), while the printed equation (2.5) would send the ratio in the opposite direction.

The same inversion occurs between equations (4.5) and (4.6). From equation (4.6), $g_i^{-2} \propto \beta_i$ and $g_0^{-2} \propto 1 - \beta_1 - \beta_2$, so one obtains

$$\frac{g_i^2}{g_0^2} = \frac{1 - \beta_1 - \beta_2}{\beta_i}, \quad i = 1, 2,$$

rather than the ratios printed in equation (4.5). Since the identification of the decoupled gauge factor, the shrinking intervals, and the double-scaling limit rests on this dictionary, the authors should correct the equations and then check every subsequent assignment of g_i , β_i , and τ_i .

4. The worldsheet central-charge check in equation (4.16) is incorrect.

For the supersymmetric Kazama–Suzuki cosets in the conventions used in equation (4.15),

$$c\left[\frac{SL(2, \mathbb{R})_k}{U(1)}\right] = 3 + \frac{6}{k}, \quad c\left[\frac{SU(2)_k}{U(1)}\right] = 3 - \frac{6}{k}.$$

The manuscript assigns the minus sign to both factors. In particular, at $k = 2$ the minimal model has $c = 0$, but the cigar has $c = 6$, not zero. The correct cancellation is $c(\mathbb{R}^{5,1}) = 9$, the two cosets sum to 6, and the superghost contribution is -15 . Equation (4.16) should be corrected, and the surrounding discussion should state explicitly whether the levels are supersymmetric or bosonic levels. Since the $k = 2$ reduction and the $k = 3$ worldsheet are central to the paper, it would also be useful to spell out the $\mathbb{Z}_k$ projection and GSO prescription sufficiently to show that the deformed theory has the intended critical Type II spectrum.

5. The “glued” NS5 configurations require a separate strong-coupling analysis.

The equal-spacing $k = 3$ configuration and the configuration with $\tau_1 = 0$ are treated in Section 4.3.2 as the same worldsheet CFT connected by marginal deformations. This conclusion is not established. Double-scaled little string theory is weakly controlled when the NS5-branes are separated by amounts scaled with g_s , so that the Liouville/Landau–Ginzburg potential caps the strong-coupling region. If two NS5-branes remain exactly coincident, the position polynomial has a repeated root: the background lies on a discriminant locus, and one normally expects a residual singular throat associated with the coincident pair. Merely observing that both λ_0 and $\lambda_{1/2}$ are nonzero does not show that the dilaton is bounded or that perturbation theory is governed everywhere by $g_{\text{eff}} = g_s/\tau$.

The authors should analyze the full $k = 3$ Landau–Ginzburg potential at the repeated-root locus, determine whether it has a runaway or an additional noncompact branch, and compute the maximal effective string coupling. If the glued locus is singular, they must explain what additional scaling or resolution defines the intended theory. This issue directly affects theories III.4–III.8 and I.8, and it prevents the present argument from identifying their limit with the smooth equally spaced three-NS5 background simply by continuity in marginal parameters.

There is also an immediate conflict with the coupling dictionary. Equation (4.6) gives $g_1^{-2} \propto \beta_1$ and $\tau_1 \propto \beta_1$, while the glued construction sets $\beta_1 = \tau_1 = 0$. It therefore has $g_1 = \infty$, not the finite g_1 assumed in equation (4.8); locally, g_s/τ_1 is likewise uncontrolled. The authors cannot use the outer separation $\tau = \tau_1 + \tau_2$ as the only effective-coupling scale without resolving this contradiction.

6. The four-dimensional theory associated with the $\theta \rightarrow 0$ glued-NS5 limit has not been derived.

Section 3.3.3 explicitly describes the construction of theory III.4 as heuristic. Starting from the Kutasov configuration and taking the angle to zero certainly makes a previously integrated-out adjoint light, but this does not by itself establish the claimed SCFT. One must integrate that field back in and specify the complete superpotential. In particular, the original $\text{Tr } X^3$ interaction, the couplings of the new adjoint to the flavors, and possible mixed interactions between the two adjoints determine the R -charges, beta functions, chiral ring, and conformal manifold. Field content alone is insufficient to identify a four-dimensional $\mathcal{N} = 1$ SCFT.

More basically, at $\theta = 0$ the isolated NS5 and the glued stack have the same kappa-symmetry projector. The displayed brane array then preserves eight supercharges; a path-dependent “memory” of having approached from nonzero angle cannot by itself retain only four. The Kutasov superpotential reviewed in Appendix A.5 relies on a relative NS5 rotation. Therefore an additional discrete datum, flux, boundary interaction, or other freezing mechanism is required if the endpoint is truly to remain $\mathcal{N} = 1$. The manuscript must identify that ingredient rather than infer it from the desired matter content.

The gap propagates to theories III.5–III.8, where the projections of the two adjoints are inferred using phrases such as “this suggests” and “we expect,” and to I.8, whose T-dual is assigned an unknown partially frozen singularity. The revision should give the superpotentials, anomaly checks, NSVZ/Leigh–Strassler marginality conditions, and an open-string or field-theory derivation of the orientifold projections. If such a derivation is not presently available, these models should be labeled conjectural and should not carry the same evidentiary weight as the standard $\mathcal{N} = 2$ Hanany–Witten constructions.

There is also a direct sign contradiction that must be resolved. Section 3.3.3 uses $O8^+ \rightarrow SO$ and $O8^- \rightarrow USp$ on the D4 sector, while Appendix A.2.3 states $O8^- \rightarrow SO$ and $O8^+ \rightarrow USp$. These opposite assignments interchange the proposed derivations of III.7 and III.8. A consistent Chan–Paton calculation, including the four ND directions, is required.

7. Equation (3.8) gives the wrong chiral multiplicities and is gauge-anomalous as printed.

Table 1 and the text immediately before equation (3.8) require $2N - 4$ fundamentals and $2N + 4$ antifundamentals for theory II.5. Equation (3.8) instead prints $2(N - 4)$ and $2(N + 4)$. This is not an innocuous rewriting: together with one symmetric and one conjugate antisymmetric tensor, the printed spectrum has a nonzero $SU(N)^3$ anomaly. In standard normalization the tensor contribution is $(N+4) - (N-4) = 8$, while the fundamental imbalance is $2(N-4) - 2(N+4) = -16$, leaving -8 . With the intended multiplicities the imbalance is -8 and the anomaly cancels. The equation and every later use of this spectrum should be corrected.

8. The normalization of the CFT-distance exponent in equation (2.1) is not consistent with equation (2.2).

Equation (2.1) writes $\gamma_J \sim \exp[-d/\beta]$ with $\beta = \dim G_{\text{free}}$. Near a weakly coupled cusp, however, the coefficient of the Zamolodchikov metric is proportional to $\dim G$, so the geodesic distance scales with $\sqrt{\dim G} \log(1/g^2)$. Consequently the decay exponent scales as the inverse square root of $\dim G$ (up to the stated convention-dependent numerical factors), not as its inverse. The square-root scaling is also needed to obtain the $\alpha \propto \sqrt{c/\dim G}$ dependence in equation (2.2), after accounting for $m^2 \sim \gamma_J$ and the conversion to Planck units. The authors should rederive equations (2.1)–(2.2) in one set of conventions and correct the definition of β .

9. The quasiconformal extension should be separated from results about conformal manifolds.

Section 5.3 considers theories with a vanishing one-loop coefficient but no established interacting fixed point or exactly marginal gauge coupling. Such theories do not presently supply a conformal manifold, a Zamolodchikov distance, or a fixed-radius AdS dual. Therefore terms such as “infinite-distance limit,” “tensionless string,” and “new universality class” do not yet have the same meaning as in Sections 2–5.2. A one-loop cancellation and a free-field Hagedorn temperature are interesting hints, but they do not establish an emergent-string limit. The material can remain as a useful outlook if the claims are explicitly reformulated as conjectures and the RG assumptions are stated. Otherwise, the authors would need to demonstrate the relevant fixed points or controlled walking regimes and define the gravitational observable being classified.

The flagship example makes the problem especially concrete. For $SU(N)$ with $N_f = 3N$ vector-like pairs, the NSVZ numerator is $3N - N_f(1 - \gamma_Q) = 3N\gamma_Q$. Since $\gamma_Q < 0$ at weak coupling, the gauge beta function has the IR-free sign: the free point is approached in the infrared and the theory has a UV Landau pole. This contradicts the description in Section 5.3 of these models as perturbations of a free UV fixed point with asymptotically AdS behavior.

Novelty and significance

The paper’s most convincing new contribution is the synthesis: standard Hanany–Witten, orientifold, and orbifold ingredients are organized so that partial weak-coupling limits correspond to retaining a cluster of k NS5-branes, and the recently observed Hagedorn classes are related to this integer k . The use of the known normalizable CHS deformations to write the collinear $k = 3$ background is also a worthwhile step. These features are interesting enough for publication in a journal such as JHEP or PRD if the logical status of the conclusions is made precise.

The novelty is less secure for the glued-NS5 and partially frozen constructions. The manuscript itself notes that their gauge-theory interpretation and Type IIB dual are not established in the literature. In that setting, a proposed rule can be valuable, but it must be supported by nontrivial checks: complete superpotentials, anomalies, marginality, moduli-space dimensions, and agreement under dual descriptions. At present the results are inferred mainly because they reproduce the desired matter table. This is not yet enough to support the advertised complete construction of every simple-group SCFT or the assertion that every member has the same perturbative closed-string background.

The literature positioning of Section 4.3.2 also needs revision. The statement that the coupled line-distribution deformation has apparently not been studied should be compared with Prezas and Sftos, “Supersymmetric moduli of the $SU(2) \times \mathbb{R}_\phi$ linear dilaton background and NS5-branes” (arXiv:0804.3062), especially its analysis of NS5-branes on a line, and with Petropoulos and Sftos, “NS5-branes on an ellipsis and novel marginal deformations with parafermions” (hep-th/0512251), including its bar/line degeneration. These papers study exactly marginal CHS operators and nonfactorizable deformations closely related to the present construction. The genuinely new point may be the proposed four-dimensional Class III interpretation, which should be stated as such. Likewise, the proposed cigar orientifolds in Section 4.2 should be compared with the exact crosscap literature on $\mathcal{N} = 2$ Liouville/cigar models, including arXiv:hep-th/0703151, hep-th/0703148, and hep-th/0409039.

The manuscript should also distinguish more carefully what follows from the earlier CFT calculations and what is independently explained here. Equality of a/c , the distance exponent, and the Hagedorn temperature within the three simple-group classes is input from the cited classification papers. The new claim is that a common parent NS5 system explains those equalities. That explanation will be substantially stronger once the parent/daughter conditions and the leading Veneziano flavor sector are treated explicitly.

Minor comments and requested corrections

1. In Appendix A.2.1 an O4-plane along 01236 is said to reflect 56789. Its transverse coordinates are 45789; the main text later uses the latter action. The orientifold definition should be corrected.
2. In Section 3.2.2, after equation (3.7), the exactly conformal SO theory is called “our original theory II.5”; it is II.4. Similarly, the interpolating example near equations (3.10)–(3.11) says that one limit recovers III.3, although the setup was described as gluing III.2 to II.3.
3. In the interpolating model of equation (3.9), obtaining the symmetric $SU(N)$ theory from the $SU(N' + 1)$ factor requires $N' = N - 1$, not $N' = N + 1$ as stated.
4. In Section 3.3.2 the III.3 construction is called the $k = 1$ case of the formula in Appendix A.2.2. The configuration has three NS5-branes; the intended statement appears to be $p = 1$ in the $k = 2p + 1$ formula.
5. Appendix A.2.2 states that for $k = 2p + 1$ there are “ $2k$ remaining” NS5-branes after the stuck brane. This should be $2p = k - 1$.
6. In Appendix A.3.1 the second representative for the meson y contains an extra/missing index, and the hypersurface relation is later written as $zyw = t^2$ although the defined variables are x, y, z, t . Both should read consistently with $xyz = t^2$.
7. The equation defining the unit S^5 in Section 4.3.1 is missing an absolute-value bar on the z_1 term. It should be $|z_1|^2 + |z_2|^2 + |z_3|^2 = 1$.
8. The scaling sentence after equation (4.20) sends τ to a multiple of R . This appears to be a typographical error for a rescaling of τ itself. The authors should present the transformation cleanly and include the factors of ℓ_s before going to string units.
9. Section 5.1 has “ $g_1 \rightarrow$ ” with the limiting value omitted. The statement should read $g_1 \rightarrow 0$.
10. The conclusion that a simple gauge group “cannot involve more than 3 NS5s” is stronger than what has been shown. The appendix itself discusses stacks of an arbitrary number of glued NS5-branes, and more elaborate projections can reduce the number of gauge factors. The authors should either prove this claim within a precisely delimited class of exactly conformal constructions or attribute the threefold result to the field-theory classification rather than to a general brane-counting obstruction.

11. A careful proofreading pass is needed. There are repeated slips such as “worldsheet”/“woldsheet,” “thee”/“three,” “descomposes,” and mismatches between class labels. Given the large number of constructions, a compact summary table listing for each theory its parent, quotient, mobile NS5 count, flavor scaling, and status (established or conjectural) would materially improve reliability and readability.
12. The simultaneous Baume–Mantegazza paper is now arXiv:2607.12014 rather than merely “to appear.” Its affine/finite ADE and dual-Coxeter organization is complementary to the present brane account. The authors should explain where the NS5-number classification reproduces that structure and delimit its scope beyond A-type linear quivers.

Recommendation

The manuscript contains a strong organizing idea and several potentially significant new constructions, but the current version does not yet establish its central claims with the standard of correctness expected for publication. I recommend **major revision**. A publishable revision should, at minimum, correct the explicit equations and spectra; resolve the O8 projection contradiction; derive or clearly downgrade the glued-NS5 gauge theories; analyze the repeated-root worldsheet locus; and replace the quenched flavor argument with a valid treatment of the Veneziano parent/daughter systems. If these points are addressed, the work would be interesting and useful to the community.