Referee Report

"Superconductive or superfluid condensation in curved spacetime"

Summary and overall assessment

The manuscript begins with the standard in/out mode construction for quantum fields in curved spacetime, reviews finite two-mode bosonic and fermionic squeezing transformations, and applies this formalism to the Unruh and Hawking states. It then makes several much stronger claims: that every field-theoretic Bogoliubov transformation is implemented by a unitary squeezing operator; that the product of this operator with an ordinary scattering matrix proves unitarity in an arbitrary curved spacetime and during black-hole evaporation; that the fermionic and bosonic squeezed vacua are respectively genuine superconducting and superfluid condensates; and that a reverse-engineered mean-field Hamiltonian produces a physical gravitational gap. The final sections compute an entropy, attempt to recover the black-hole area law, and propose a positive-energy evaporation model and an endpoint mass estimate.

There is a sound and potentially pedagogical finite-mode core. In particular, most of the two-oscillator algebra in Sections III.A and III.B is standard, and the occupation numbers in equations (102) and (124) reproduce the familiar Fermi–Dirac and Bose–Einstein factors. The algebraic resemblance between a two-mode fermionic squeezed state and a BCS variational state can also be a useful observation if it is stated narrowly.

The principal conclusions, however, do not follow. The central unitary-implementability theorem is false in an infinite-dimensional quantum field theory; the proposed S-matrix proof assumes the unitarity it claims to establish and never supplies a map from the initial state to the complete late-time exterior state; the BCS/BEC terminology is not supported by the physical diagnostics of superconductivity or superfluidity; and several later calculations contain independent normalization, dimensional, and algebraic errors. These are not matters that can be repaired by polishing the presentation. They invalidate the claimed novelty and the logical spine of the paper.

Recommendation: reject. A substantially different and much shorter paper might be possible if the author restricts the discussion to regulated finite-mode squeezed states and presents the BCS comparison explicitly as a formal analogy. Such a paper would have to abandon the claimed general unitarity theorem, the resolution of the information-loss problem, and the asserted derivation of a physical high-temperature superconducting phase.

Major comments

1. **The unitary-implementability theorem in Section III.C is false for quantum fields.**

   The manuscript correctly constructs unitary squeezing operators for a finite two-mode system. It then asserts that the general field-theoretic result follows by taking an infinite product over momenta. This step is precisely where the finite-dimensional argument ceases to apply. A canonical Bogoliubov transformation on the one-particle space is implementable by a unitary operator on the chosen Fock representation only under an additional Shale–Stinespring condition. In the notation of Section II, the off-diagonal coefficient must be Hilbert–Schmidt: $\mathrm{Tr}(\beta^\dagger\beta) = \sum_{i,j} |\beta_{ij}|^2 < \infty$, with the corresponding integral and distributional qualifications in the continuum. The canonical identities in equations (8) and (12) do not imply this condition.

The cancellation displayed in equation (120) proves unitarity only for a finite product. It neither proves convergence of the infinite product nor shows that the resulting vector lies in the original Fock space. Equations (105), (112), and (157) use exactly the products whose existence is in question. The issue is physical rather than technical: ordinary Minkowski and Rindler Fock representations in infinite volume are a standard example in which the naive global unitary equivalence fails. Modewise normalization does not cure the volume divergence or the vanishing of the vacuum overlap.

This omission also contradicts the manuscript's own particle-number formula. Equation (18) gives the number in one out-mode; implementability requires control of the sum over all out-modes as well. The author must replace the theorem with the correct necessary and sufficient condition, specify the one-particle spaces and regulators, and verify the condition in every claimed application before taking any thermodynamic or continuum limit. Without that analysis, the statements that the in- and out-Hilbert spaces are always unitarily equivalent and that a global $S_{\rm sq}$ exists are incorrect.

2. **Section V is circular and does not address black-hole information loss.**

   Equation (132) introduces "the unitary S-matrix of the theory." Equations (134)–(138) then conclude that the product of this assumed unitary operator with the already assumed unitary $S_{\rm sq}$ is unitary. Even in a finite regulator where both factors exist, this is an algebraic tautology, not a proof of unitarity for an interacting theory on a dynamical black-hole spacetime. There is also an explicit ordering error in equation (137):

$$(SS_{\rm sq})^\dagger(SS_{\rm sq}) = S_{\rm sq}^\dagger S^\dagger S S_{\rm sq},$$

   not the ordering printed in the manuscript.

   The operator order is inconsistent elsewhere as well: equation (139) contains $SS_{\rm sq}$, while the in-representation manipulation in equation (140) yields $S_{\rm sq}S$. These products are not interchangeable unless a commutation or conjugation relation is established. Nor is a factorization of curved-spacetime interacting evolution into a free geometric squeeze followed by a flat/out-space scattering matrix derived. In a time-dependent background, the interaction-picture Hamiltonian is time ordered and expressed in time-dependent modes; the required Møller operators and their asymptotic domains would have to be constructed.

   More importantly, equation (131) retains exterior and interior factors, after which the interior operators are simply suppressed. The completeness relation in equations (133)–(136) is taken only over $\mathcal{H}_{\rm out}$ even though the state constructed in equation (131) belongs, at best, to $\mathcal{H}_{\rm out} \otimes \mathcal{H}_{\rm int}$. Purity of this joint intermediate state is the standard Hawking squeezed-state statement. After tracing the interior, the exterior evolution is a quantum channel and is not unitary. The information problem asks for a dynamical map from the initial asymptotic data to a complete late-time asymptotic state after the interior/horizon has disappeared. No such map is constructed here.

   The ad hoc replacement of the Hawking temperature in equations (147)–(148) does not solve this problem. The fact that an instantaneous squeezing parameter tends to zero when an imposed endpoint temperature tends to zero does not undo correlations emitted earlier or transport interior information to future null infinity. A valid claim would require a well-defined asymptotic Hilbert space, backreaction, and a dynamical evolution of all degrees of freedom without assuming the desired unitary S at the outset.

3. **The claimed scope is incompatible with the hypotheses, and the Rindler/Hawking material is not new.**

   Section II assumes a globally hyperbolic spacetime with Cauchy surfaces, and the subsequent discussion further assumes stationary in- and out-regions admitting preferred positive-frequency splittings. A generic curved spacetime has neither preferred particle states nor an S-matrix. A singular evaporating black-hole spacetime also does not automatically possess the final Cauchy structure used in the opening construction. The phrase "a general spacetime" therefore substantially overstates the domain even before the implementability problem is considered.

   The diagonal relation in equation (71) is a special mode construction, not a general relation between arbitrary Minkowski plane waves and localized Rindler modes. The necessary wave-packet/Unruh-mode choice and the usual single-mode qualification must be stated. In the black-hole application, greybody scattering and the distinction among Unruh, Hartle–Hawking, and Boulware states must also be kept explicit rather than treating the Rindler and collapse geometries as interchangeable.

   There are corresponding black-hole mode errors. Schwarzschild modes are labeled by frequency, angular quantum numbers, and radial channel, not by a conserved spatial three-momentum with $\omega(\mathbf{k}) = \sqrt{\mathbf{k}^2 + m^2}$ as asserted after equation (111). In the expansion from equations (110) to (111), the coefficient of an N-pair state should contain $e^{-4\pi M\omega N}$; the displayed middle line omits the power N. These omissions matter when the result is promoted from a near-horizon, modewise identity to a claimed complete evaporation state.

   The squeezed-state representation of the Unruh and Hawking vacua, their reduced thermal density matrices, and their occupation numbers are established results. Ref. [15] already develops the fermionic Rindler construction used here. Moreover, the public arXiv record flags text overlap with that work, and Sections IV.A–IV.B follow it closely in sequence, notation, the Alice/Rob presentation, and the vacuum/partial-trace calculation. A citation alone does not clearly disclose the extent of adaptation. This material should be drastically condensed and rewritten with transparent provenance, and the overlap should be resolved editorially. The genuinely new claims must then be separated from textbook review.

4. **A two-mode squeezed state is not, by itself, a superconductor or a superfluid.**

   Equations (175)–(177) display a formal equality between the coherence factors of a regulated two-mode fermionic state and those of a BCS ansatz. The physical identification made from this equality is not justified. The proposed pair is an exterior electron and an interior positron. It is electrically neutral, nonlocal across the horizon, and inaccessible as a local pair to either exterior or interior observers. The comparison table on page 31 explicitly states that the relevant $U(1)$ symmetry is unbroken, which undercuts the claim of superconductivity. No charged order parameter, phase stiffness, Meissner response, conductivity, or gauge-invariant long-range diagnostic is computed. Tracing over the inaccessible partner gives a thermal state, not a locally observable superconducting condensate.

   The bosonic statement has the analogous problem. A two-mode squeezed vacuum does not establish Bose–Einstein condensation or superfluidity. The paper supplies neither macroscopic occupation of a one-particle mode, off-diagonal long-range order, nor a superfluid response. Similarity between Gaussian wavefunctions is not equivalence of phases of matter. The title, abstract, and conclusions should therefore use "BCS-like" or "BEC-like" only as an algebraic analogy unless the author derives the relevant local and gauge-invariant diagnostics.

5. **The reverse-engineered Hamiltonian and "mass gap" are non-unique and internally inconsistent.**

A Gaussian state admits infinitely many quadratic parent Hamiltonians. Equations (173)–(184) choose a BCS parametrization and solve a coherence-factor identity for a quantity called $\Delta_{\mathbf{k}}$; they do not derive an interaction Hamiltonian from QFT in the given geometry. This distinction is decisive. The manuscript sets $\xi_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + m^2}$ rather than an energy measured relative to a chemical potential. For a massive field it never reaches the asserted Fermi surface $\xi_{\mathbf{k}} = 0$. The paragraph following equation (184), which says that the Hamiltonian and gap remain correct regardless of the self-consistency condition, is therefore untenable. Without a microscopic action, a saddle-point equation, stability analysis, and a comparison of free energies, $\Delta_{\mathbf{k}}$ is only a reparametrization of the Hawking/Unruh mixing angle.

There are also direct errors. From equation (180),

$$\xi \ \mathrm{csch}\left(\frac{\xi}{2T}\right) = 2T - \frac{\xi^2}{12T} + O\left(\frac{\xi^4}{T^3}\right),$$

whereas equation (182) contains $-\xi/(12T)$, which is dimensionally and algebraically wrong. At fixed T the same function decays exponentially for large ξ; the statement in the caption of Figure 4 that it approaches $2T$ for large $\xi_{\mathbf{k}}$ reverses the limit.

Equation (196) is inconsistent with the state in equation (175). Direct evaluation in the pure two-mode state gives, up to its phase,

$$2\left|\left\langle c_{\mathbf{k}}^{(\mathrm{out})\dagger} d_{-\mathbf{k}}^{(\mathrm{int})\dagger}\right\rangle\right| = \mathrm{sech}(\pi\Omega_{\mathbf{k}}),$$

without the additional $\tanh[(\pi\Omega_{\mathbf{k}})\coth(\pi\Omega_{\mathbf{k}})]$ in equation (196). The extra factor arises from applying a finite-temperature quasiparticle gap equation to a state that was already fixed as a pure Hawking/BCS vacuum, thereby mixing two distinct constructions.

Finally, equations (185)–(194) assume a Newtonian-shaped kernel, set $N = \mathrm{Vol}/\ell_{\mathrm{P}}^3$, and fit its coefficient to selected curves. This is not an independent derivation of a graviton-mediated interaction or Planck-mass "atoms of space." The normalization, locality, spin structure, and relation to a gravitational scattering amplitude are not established. Such material can at most be presented as a speculative toy ansatz, not evidence for the claimed phase.

6. **The entropy and area-law calculations contain multiple independent errors.**

The momentum integrals in equations (204) and (208) are too large by a factor $8\pi^3$ when $\kappa_+ = 2\pi T$. For the mode count used in the manuscript, direct integration gives

$$\frac{S_{\mathrm{B}}}{V} = \frac{2\pi^2}{45} T^3 = \frac{\kappa_+^3}{180\pi}, \qquad \frac{S_{\mathrm{F}}}{V} = \frac{7\pi^2}{180} T^3 = \frac{7\kappa_+^3}{1440\pi}.$$

Equation (206) also changes the required minus sign multiplying the $\sinh^2 r \log(\sinh^2 r)$ term to a plus sign, although the next line returns to the standard expression.

The subsequent near-horizon calculation compounds this normalization problem:

- The lower-limit expansion in equation (219) has the wrong signs. From the antiderivative in equation (218), the lower-limit contribution gives $1/\delta - 4\log\delta - 13/3$, plus the upper-limit terms.
- With $\delta = \delta r/(2M)$, the proper distance is $\alpha \simeq 4M\sqrt{\delta}$, not $4M\sqrt{2\delta}$ as in equation (220). Thus $\delta = \alpha^2/(16M^2)$, not $\alpha^2/(32M^2)$.

- Even using the manuscript's formulas, the numerical identification in equation (221) is false:

$$\frac{1}{45\pi}\frac{2A}{\pi\alpha^2}\bigg|_{\alpha^2=\ell_{\rm P}^2/720} = \frac{32}{\pi^2}\frac{A}{\ell_{\rm P}^2},$$

  not $A/(4\ell_{\rm P}^2)$.
- The terms denoted $O(X)$ in equation (233) grow as X^3 and are not finite when the "entire exterior space" is included. A cavity, subtraction, or a state-dependent infrared prescription is required.

Conceptually, this is a thermal-atmosphere/brick-wall calculation. Choosing a proper-distance cutoff so that its leading divergence equals $A/(4\ell_{\rm P}^2)$ does not derive the Bekenstein–Hawking coefficient and leaves the usual cutoff and species dependence. The manuscript should distinguish this construction from vacuum entanglement entropy, cite the brick-wall literature, specify the quantum state, and avoid the claim in Figure 10 that the two entropies "perfectly overlap." Equation (222) is also said to coincide with equation (156) for $M_{\rm p} = 1/2$, but the displayed denominators do not agree for that value.

7. **The positive-energy evaporation and endpoint analysis are not derived and contain further errors.**

The negative Killing-energy flux used in the standard semiclassical description of Hawking evaporation is not an on-shell ghost and does not represent a negative-norm state. Inside a horizon the stationary Killing vector changes causal character; statements about energy measured at infinity cannot be replaced by assigning positive oscillator energies to both members of a pair. Refuting the standard bookkeeping would require a calculation of the renormalized stress tensor and its horizon flux, which is absent.

Equations (242)–(246) again omit the $(2\pi)^{-3}$ density-of-states factor. For one massless bosonic degree of freedom, the standard energy density is $\rho = \pi^2 T^4/30$, and an isotropic outward flux introduces an additional angular factor; it is not obtained by dividing an energy density by a black-hole volume. The paper also moves from an energy expectation value to a power without introducing a time interval, and omits greybody factors, species, and backreaction. Consequently, equations (244)–(246) do not derive the Stefan–Boltzmann law or an evaporation rate.

The endpoint bound is likewise unsupported. Equation (249) uses an incorrect geometric volume: for a sphere of radius r_s and area $A = 4\pi r_s^2$,

$$V = \frac{4\pi r_s^3}{3} = \frac{A^{3/2}}{6\sqrt{\pi}},$$

not $(4\pi/3)A^{3/2}$. The mass estimate in equation (250) therefore does not follow even within the toy model.

Section IX also calls the Schwarzschild $r = 0$ singularity timelike; it is spacelike. The claim that Einstein gravity is "secretly conformally invariant" and that singular and regular metrics are gauge equivalent requires a separate compensator theory and careful treatment of singular Weyl transformations and matter clocks. It cannot be used as an established ingredient in a proof of predictability here. This entire section should be clearly separated as conjectural unless a concrete, globally defined dynamical model is supplied.

Additional technical and presentation comments

1. Equation (12) should contain $\alpha^\dagger\alpha - \beta^T\beta^* = \mathbf{1}$ in its first line, as follows directly from equations (10)–(11); the printed $\alpha\alpha^\dagger$ is not the same condition for a general noncommuting matrix.

2. Equation (30) should end in the occupation states $|n_a, n_b\rangle$, not a sum of scalar coefficients multiplying $|0\rangle$.

3. The disentangling formulas in equations (22) and (26) contain phase/sign errors in the final ab exponential. They should be rederived from the stated squeeze generator before being used as operator identities. The middle factor in the fermionic formula (37) is also inconsistent with its action on singly occupied states; the exact generator leaves those states invariant.

4. Equation (36) does not define a fermionic vacuum: $(a^2)^\dagger$ and $(b^2)^\dagger$ vanish identically by the canonical anticommutation relations. The required conditions are $a|0\rangle = b|0\rangle = 0$.

5. In the fermionic check, equation (45) incorrectly writes $\cosh^2|\xi| + \sinh^2|\xi| = 1$; it should use $\cos^2|z| + \sin^2|z| = 1$. Equation (47) contains the same stray hyperbolic cosine.

6. Equation (53) assigns a negative norm to negative-frequency Dirac modes, contradicting the positive-definite Dirac inner product in equation (54). The negative sign is appropriate to the Klein–Gordon product, not to the Dirac product used there.

7. Equation (76) does not preserve the canonical anticommutators for a general phase as printed: if the upper-right entry is $e^{-i\phi}\sin r$, the lower-left entry requires the conjugate phase $-e^{+i\phi}\sin r$.

8. The sharp temperature in equation (148) drops the factor of G from the Hawking prefactor while retaining it in the threshold. Units and Planck conventions should be made consistent throughout equations (147), (148), and (156).

9. Figure 7 refers to the two sides of "(7)," because its figure label is used where the consistency equation should be cited. This and other cross-references should be checked against the rendered manuscript.

10. Sections II–IV are disproportionately long for review material, while the central infinite-dimensional and dynamical issues are omitted. The repeated finite-matrix checks should be condensed, and the manuscript needs a precise statement of assumptions, state choices, domains of operators, regulators, and limits.

11. The prose requires careful editing for grammar, terminology, and consistency (for example, "Bogoliubons," "condense," and repeated assertions that a result is rigorous or obvious before its hypotheses are established). Such editing would improve readability but would not resolve the scientific objections above.