Referee Report

Conformal line defects from matter-coupled 5D $N = 4$ gauged supergravity

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Summary and overall assessment

The paper studies supersymmetric $AdS_2 \times S^2$-sliced solutions of five-dimensional half-maximal gauged supergravity coupled to vector multiplets. The slices have the bosonic symmetry expected of a line defect in a four-dimensional conformal field theory. Their curvature is supported by two-form potentials, while the remaining fields depend only on the transverse radial coordinate. Three gaugings are considered: $SO(2)_D \times SO(3)$, $SO(2) \times ISO(3)$, and $SO(2) \times SO(3) \times SO(3)$.

The common structural result is that, within the invariant scalar truncations and spinor ansatz used in the paper, supersymmetry and the two-form equations impose

$$f = h, \qquad B_1 = b_1 \,\mathrm{vol}_{AdS_2}, \qquad B_2 = b_1 \,\mathrm{vol}_{S^2}, \qquad b_1 \propto \frac{e^f}{\Sigma}.$$

The scalar and warp-factor equations then reduce to equations familiar from holographic RG flows without the two-forms. In the $SO(2)_D \times SO(3)$ model, compatibility with the two-form equation leads to the obstruction in equation (64): a nonzero matter scalar ϕ_3 requires $g_2 = 0$, thereby removing the $N = 2$ AdS_5 vacuum. The paper instead gives an analytic flow from the remaining $N = 4$ vacuum to a singular interior and applies the Gubser bounded-potential criterion. In the $SO(2) \times ISO(3)$ model, two analytic scalar flows previously obtained in the author's work are dressed by the two-forms. One of these is uplifted explicitly to eleven dimensions and is interpreted as a compactification/RG-flow geometry involving M2- and M5-brane charges. Finally, in the $SO(2) \times SO(3) \times SO(3)$ model, a known scalar domain wall between two AdS_5 critical points receives the same two-form dressing, and a one-ended singular solution is also presented.

There is worthwhile material here. The restricted obstruction in the first gauging, the systematic matter-coupled extension of the charged-domain-wall mechanism, and especially a correct explicit M-theory uplift would be useful additions to the holographic-defect literature. The manuscript also makes clear that matter multiplets constrain these solutions much more strongly than one might have expected from analogous constructions in six and seven dimensions.

The present draft is not yet reliable enough for publication, however. Several equations central to the argument are mutually inconsistent. In particular, the cancellation used to claim that the known scalar RG flows remain solutions does not occur in equation (65) as printed; the main $SO(2) \times SO(3)$ BPS system is inconsistent with its own analytic solution; multiple endpoint expansions cannot approach the stated AdS_5 vacua; and the displayed eleven-dimensional metric (122) does not follow from the reduction ansatz. The appendix also disagrees with the main BPS equations in powers of Σ and in spinor factors. These are not isolated cosmetic errors: they affect the proof of the solution-generating mechanism, the source/vev assignments, the claimed no-go result, and the strongest new output of the paper.

The physical interpretation also needs to be made more precise. The $AdS_2 \times S^2$ isometry is necessary for a conformal line defect, but the manuscript does not exhibit the flat-space defect sources, identify the preserved one-dimensional superalgebra, or establish the holographic source/vev map. This is especially important for the $\Delta = 2$ scalars at the AdS_5 Breitenlohner–Freedman bound and for the $SO(2) \times ISO(3)$ solutions, where the AdS_5 region is an IR fixed point and the UV is asymptotically locally AdS_7. I therefore view the work as potentially publishable only after a substantial technical and conceptual revision.

Recommendation

Major revision. The central strategy is plausible and the new matter-coupled solutions may be of interest to readers of JHEP or PRD. Before publication, the author should provide a corrected, internally consistent BPS and field-equation system; rederive every endpoint expansion and operator assignment; repair and verify the eleven-dimensional uplift; and delimit the scope of the no-go statements. The defect interpretation should also be supported by an explicit boundary dictionary. If the numerous contradictions are transcription errors, these tasks should be feasible without changing the basic construction, but they must be completed before the correctness and significance of the claimed results can be assessed definitively.

Major comments

1. **The scope and completeness of the BPS analysis and the no-go result must be established.**

   The obstruction in equation (64) is potentially one of the most useful results of the paper, but the derivation establishes it only in a restricted branch. All vector fields are set to zero, only selected singlet scalars are retained, the two-form orientations are gauge-fixed, $\theta = 0$ is chosen, and an additional internal projector is imposed in the appendix "to avoid too many algebraic constraints." The derivation then sets coefficients of several gamma-matrix structures to zero separately. After the radial and internal projectors have been imposed, the manuscript does not demonstrate that these structures remain linearly independent on the allowed symplectic-Majorana spinors. Setting all apparent coefficients separately certainly produces a sufficient BPS branch, but without a rank or basis argument it does not prove that the branch is exhaustive.

   There is a related issue between equations (30) and (31). The constraint leaves two constants, k and $\widetilde{k}$, describing orientations in the $(1, 2)$ and $(6, 7)$ tensor doublets. The text invokes an $SO(2)_D$ transformation to set both to zero. A single diagonal rotation does not obviously eliminate two independent relative phases. If independent rotations are available as global automorphisms that leave the embedding tensor and scalar ansatz invariant, this should be shown explicitly. Otherwise a physical relative orientation may have been discarded, and the subsequent no-go statement would not apply to it.

   The author should formulate the projected spinor problem as an explicit finite matrix system, give the dimension of its solution space in each branch, and show which gamma structures are independent. The $\theta = 0$ and $\theta = \pi/2$ branches, the cases $\phi_3 = 0$ and $\phi_3 \neq 0$, and all allowed tensor orientations should be accounted for. A table listing the active fields, projectors, preserved supercharges, bosonic symmetry, and endpoint geometry for every final solution would also be valuable. Unless completeness is proved, statements such as "there is no line defect solution" should be replaced by "there is no solution within the present vector-free invariant ansatz."

2. **The five-dimensional equations require a systematic algebraic audit and an explicit check of the bosonic field equations.**

   There are too many contradictions in central formulas to treat them as harmless typographical slips. The following examples should all be corrected and traced through the subsequent results.

   - Equation (32) assigns M_{66} both the value 1 and a nontrivial scalar-dependent value. It lists M_{17}, whereas the contraction in equation (65) uses M_{16}. The complete scalar matrix entries relevant to the tensor kinetic term should be given consistently.
   - The dressed embedding tensors fail simple origin checks. At the scalar origin, equation (20) gives $\widehat{\xi}^{12} = \xi^{12} = -g_1$, whereas equation (42) evaluates to $-g_2$. The coefficient of $\cosh 2\phi_1$ in (42) therefore appears to have the wrong relative sign. Similarly, the printed $\widetilde{\xi}^{14}$ in equation (146) remains nonzero at $\phi_1 = 0$, even though the ϕ_3 transformation then leaves the original $(1, 2)$ plane supporting ξ^{12} untouched; the final reduced equations do not contain the corresponding term. All dressed components should be recomputed from explicit coset matrices before the BPS systems are regenerated.
   - Equation (59) contains the undefined component $\widehat{\xi}^{34}$, although equations (42), (48), and (57) involve $\widehat{\xi}^{45}$. This component is precisely relevant to the g_2-dependent compatibility condition.
   - The first term in equation (65) is printed as

     $$g_1^2 M_{11}(\widetilde{b}_2^2 e^{+4h} - b_1^2 e^{-4f}).$$

     It does not vanish when $f = h$ and $\widetilde{b}_2 = b_1$, contrary to the paragraph immediately following the equation. Presumably the first exponential should be e^{-4h}, as expected from contraction with the inverse metric. The cancellation is the logical step that permits old scalar RG-flow solutions to be dressed by the tensors, so it must be corrected and derived rather than merely asserted.
   - At the fixed point of equations (67)–(69), one finds $f' = h_1/(2\Sigma_*)$. Thus $L = 2\Sigma_*/h_1$, not $L^2 = 2\Sigma_*/h_1$ as written in equation (71). Equation (162) uses the former convention.
   - In the $SO(2) \times ISO(3)$ model, equations (97), (99), and (101) imply

     $$\frac{b_1'}{b_1} = f' - \frac{\Sigma'}{\Sigma} = -\frac{g_1}{\sqrt{2}}\Sigma^2.$$

     The compatible tensor equation is therefore $b_1' + (g_1/\sqrt{2})\Sigma^2 b_1 = 0$, while equation (102) has the opposite sign. The claim immediately following (102) that compatibility is automatic is false for the printed equations.
   - The first equation in (104) is missing a factor Σ^{-1}. This follows directly from equation (94) after setting $\phi_1 = 0$ and $\phi_5 = \phi_3$, and the factor is also required for the radial relation in equation (105) when $d\widetilde{r}/dr = \Sigma^{-1}$.
   - The claimed consistent truncation $\phi_5 = \phi_3$ in Section 4.2 does not make equations (134) and (135) coincide. The latter contains $1 + 2e^{2\phi_3} + e^{2\phi_5}$, whereas consistency requires the middle sign to be negative. This sign must be corrected before the reduced system and equation (140) are assessed.

- Equation (155) lacks the derivative on ϕ_3. Equation (161) lacks the derivative on b_1 and has a sign incompatible with equations (157)–(159). Indeed those equations give

$$\frac{b_1'}{b_1} = f' - \frac{\Sigma'}{\Sigma} = \frac{g_1}{\sqrt{2}}\Sigma^2,$$

so the compatible tensor equation is $b_1' - (g_1/\sqrt{2})\Sigma^2 b_1 = 0$.

The appendix compounds the problem. Equations (180)–(182) disagree with the main equations (45)–(46) in factors Σ versus Σ^{-1} and in the placement of $\sin\theta$ and $\cos\theta$; equation (181) even prints $\cos\theta$ in both of its last two lines. Equation (192) omits the factor of Σ multiplying one tensor contribution that is present in the main gravitino variation. The author should regenerate the main BPS systems from one fixed set of supersymmetry conventions and make the appendix exactly reproduce them.

Because tensor fields obey first-order duality equations and the vector truncation imposes additional constraints, it is not enough to state that the remaining second-order equations are "readily verified." For each gauging, the revised paper should substitute the proposed solution into the scalar, Einstein, tensor, and truncated-vector equations and display either the resulting identities or a concise general integrability proof. This is particularly important for equation (65), on which the claimed universal dressing rests.

3. **The endpoint expansions and holographic mode assignments must be recomputed.**

Several asymptotic formulas contradict the exact solutions from which they are said to follow. For the $SO(2) \times SO(3)$ solution in Section 4.1, equation (105) gives

$$\Sigma^3 = -\frac{g}{\sqrt{2}g_1 \cosh\phi_3}.$$

Consequently, near $\phi_3 = 0$,

$$\Sigma - \Sigma_* = -\frac{\Sigma_*}{6}\phi_3^2 + O(\phi_3^4).$$

If $\phi_3 \sim e^{2r/L}$ as stated, the leading correction to Σ is of order $e^{4r/L}$, not $e^{-2r/L}$ as in equation (106). Moreover, the two exponentials printed in (106) cannot both tend to zero in the same radial limit. The conclusion that there is no independent source for the $\Delta = 2$ scalar may still be correct, but it does not follow from the displayed expansion.

The same problem is more severe in equations (141)–(142). The terms $e^{+2r/L}$, $e^{-2r/L}$, and $e^{-4r/L}$ cannot all describe deviations approaching one AdS_5 vacuum in either radial direction. The exact expressions in (140) should be series-expanded on a stated real branch, and the relation between r, $\widetilde{r}$, the signs of the gauge couplings, and the positive physical radius L should be fixed before assigning dimensions. At the second AdS_5 vacuum of Section 5.1, equation (167) should expand $\phi_3 - \phi_{3,*}$, since $\phi_{3,*}$ in equation (163) is nonzero. The printed correction to Σ grows as $r \to -\infty$ and therefore cannot represent approach to that vacuum.

At every critical point, the author should linearize the full coupled scalar system, diagonalize the mass matrix, and identify source and response modes in the mass-eigenstate basis. Raw fields such as ϕ_3 and Σ generally mix at the second vacuum, so assigning distinct operator dimensions to them without diagonalization is not justified. For the $m^2L^2 = -4$ modes with $\Delta = 2$, the Fefferman–Graham expansion permits a logarithmic branch. An $e^{-2r/L}$ term cannot automatically be called a vacuum expectation value; the conclusion depends on the coefficient of the logarithmic source, the boundary conditions, mixing, and counterterms. The revised source/vev discussion should use this full expansion and should treat the massive two-form modes in the same convention.

4. **The explicit eleven-dimensional uplift in equations (122)–(133) is incorrect as written and needs an independent verification.**

   This uplift is arguably the strongest new result in the paper, so the error in equation (122) is consequential. Equations (108), (119), and (120) imply

$$T_{ab}^{-1} D\mu^a D\mu^b = \big(e^{6\lambda}\sin^2\xi + e^{-4\lambda}\cos^2\xi\big)d\xi^2 \\ + e^{6\lambda}\cos^2\xi\,(d\theta - \omega_H)^2 + e^{-4\lambda}\sin^2\xi\, d\widehat{\mu}^{\hat{a}} d\widehat{\mu}^{\hat{a}}.$$

   Equation (122) instead repeats $\sin^2\xi$ in both terms multiplying $d\xi^2$ and omits the factors $e^{6\lambda}$ and $e^{-4\lambda}$ from the final two terms. Its later UV limit (132) uses the expected structure and therefore directly contradicts (122).

   There is also a flux-level omission. The term in equation (114) that is discarded because T_{ab} is diagonal is not generally zero when λ runs: a pair of indices in the two eigenspaces contributes

$$e^{6\lambda}d(e^{-4\lambda}) - e^{-4\lambda}d(e^{6\lambda}) = -10e^{2\lambda}d\lambda.$$

   Thus equations (115) and (123) omit a scalar-gradient contribution to the four-form. The full uplift should be rederived from the stated consistent-truncation formulas, not patched only at the metric level. In particular, equation (123) for $G_{(4)}$, its limiting forms (129) and (133), and the rotated H^2 vielbeins should be checked against $dG_{(4)} = 0$ and the eleven-dimensional field equation. The relation between the seven-dimensional coupling m and the five-dimensional parameters g, g_1 should be stated. For a global M-theory solution, the author should also specify whether H^2 is replaced by a compact quotient H^2/Γ, identify the relevant cycles, and discuss flux/Page-charge quantization. Without these checks, the claim of an M2–M5 interpretation remains suggestive rather than established.

5. **The manuscript should demonstrate that the solutions are holographic line defects, not merely $AdS_2 \times S^2$-invariant source backgrounds.**

   The preserved $SO(2,1) \times SO(3)$ symmetry is the correct bosonic symmetry for a line in four dimensions, and the equal warp factors are consistent with the standard conformal frame. Nevertheless, a complete holographic identification requires more than this kinematic observation. The paper should give the Weyl map between the boundary $AdS_2 \times S^2$ metric and flat four-dimensional spacetime with a straight or circular line removed. The leading scalar and two-form data should then be transformed to flat coordinates so that the reader can see their profile and singular support at the line. The dual dimension-three tensor operator, its $N = 2$ multiplet, and the preserved one-dimensional superconformal algebra should be identified. This would also clarify whether the construction corresponds to a familiar Wilson/'t Hooft-type line or to a distinct defect defined by a tensor source.

   For the $SO(2) \times ISO(3)$ solutions, the interpretation needs additional care. The AdS_5 region is an IR fixed point while the UV is asymptotically locally AdS_7. The full geometry is therefore a defect-preserving compactification/RG trajectory, not an asymptotically AdS_5 defect state of an isolated four-dimensional SCFT. From the six-dimensional viewpoint the object should wrap the compactification surface, so the manuscript should state precisely how it reduces to a one-dimensional defect after compactification. Likewise, the second AdS_5 region in Section 5.1 is an IR throat of an RG flow, not a second conformal boundary.

The source/vev discussion should be connected to holographic renormalization. References [17] and [48] develop tools directly relevant to these $AdS_2 \times S^2$ slicings. At minimum, the paper should identify the renormalized boundary data and one-point-function structures permitted by the defect symmetry. Computing one quantitative datum—for example a renormalized defect free energy, a one-point coefficient, or a Page charge in the uplift—would substantially strengthen the physical content and help distinguish the new solutions from their known RG-flow seeds.

6. **Parameter domains, global branches, and singular interiors need a complete treatment.**

   The analytic formulas contain cube roots, square roots, logarithms, inverse trigonometric functions, and inverse hyperbolic functions whose real branches are not specified. For example, the second vacuum (163) requires conditions on h_1, h_2, g_1 such that $h_2^2 > h_1^2$, the logarithm is real, Σ^3 has the intended sign, and L_2 is a positive physical radius. The interpolation (165) further requires a choice of scalar interval and branch of $\tanh^{-1}$. Similar sign choices are needed for g and g_1 in (85), (105), and (140), and for C_0 in the one-ended solutions. There is an immediate reality problem in (140): for $x = e^{\varphi_2} \in (0, 1)$ and the coupling signs that make the AdS_5 value of Σ positive, the displayed logarithm has a negative argument and the radicand of b_1 is negative. A sign or absolute value is missing, or a different real branch is intended.

   For each family, the revised manuscript should state the allowed coupling and integration-constant ranges, the radial interval, the endpoint values of every scalar, and whether any denominator or warp factor vanishes in the open interval. Curvature invariants and tensor norms should be checked for the purportedly smooth interpolation. This is necessary both for reproducibility and for deciding which formal expressions are genuine Lorentzian supergravity solutions.

   The conclusions about singularities should also be moderated. The condition that the scalar potential be bounded above along a flow is a useful necessary Gubser criterion, but $V \to -\infty$ alone does not prove that a singularity is "physical." In the absence of an uplift or a finite-temperature cloak, the paper should say that the singularity passes this necessary criterion, not that it has been resolved or shown to be acceptable in string/M-theory. This qualification applies to the tuned solution in Section 3.2 and to both branches discussed after equations (171)–(172).

7. **The novelty and publication case should be delineated more sharply.**

   The paper is candid that many scalar and warp-factor profiles are known, but the division between previous and new results should be made explicit. Section 2 is standard formalism. In Section 4 the scalar solutions and their AdS_5/AdS_7 limits come from Ref. [43]; the new data are the tensor dressing and the tensor-dependent uplift flux. In Section 5 the scalar interpolation is inherited from Refs. [20] and [22], while the new ingredient is again $b_1 \propto e^f/\Sigma$. The closest pure-supergravity line-defect construction, Ref. [17], already contains the same broad charged-domain-wall mechanism and goes further in holographic renormalization.

   A concise comparison table—gauging, known seed, newly activated fields, preserved supersymmetry, endpoints, singularity status, higher-dimensional origin, and new observable—would make the contribution transparent. More substantively, the recurring relations $f = h$, $B_1 = B_2$, and $b_1 \propto e^f/\Sigma$ suggest a general solution-generating statement: under clearly stated algebraic conditions, a scalar domain wall can be promoted to a charged $AdS_2 \times S^2$-sliced solution. Formulating and proving such a statement would turn the paper from a catalogue of dressed examples into a reusable result. If the author prefers to retain the example-by-example format, then at least one quantitative defect or brane datum would be important to support the significance of the extension.

Minor comments

1. The Introduction says that all three gaugings can be constructed with three vector multiplets, whereas Section 3 uses $SO(5,2)$ and explicitly states that there are two vector multiplets. The relation between these descriptions should be clarified.

2. Equation (36) appears to mean $\beta_1 = \sigma^2$, not $\beta_2 = \sigma^2$, since the AdS_2 tangent indices are $0, 1$ and equation (37) uses $\beta_0\beta_1$.

3. In equation (149), the third coset vielbein is again labeled P^1_m; it should presumably be P^3_m. Similar index and tilde-placement errors occur in equations (29)–(32).

4. The unit-radius and orientation conventions for AdS_2, S^2, and H^2 should be stated once, including the signs of the volume forms and the normalization of $\kappa_i, \widetilde{\kappa}_i$. These signs enter both the tensor duality equations and the uplift.

5. The notation $\sin^{-1} x$ in equation (140) is ambiguous between $\arcsin x$ and $1/\sin x$. The former should be written as $\arcsin x$. All logarithm and inverse-function branches should be fixed explicitly.

6. The phrase "wrap factor" before equation (113) should read "warp factor." The manuscript also contains many easily corrected language errors, including "decates," "seection," "two-fom," "opeartors," and several subject–verb disagreements. A careful copy-edit is needed.

7. The bibliography contains corrupted quotation marks and apostrophes in several entries. The Gubser–Klebanov–Polyakov entry has an empty title and a malformed arXiv identifier; other early AdS/CFT identifiers should also be standardized.

8. M5-brane theories are conventionally denoted six-dimensional $N = (2,0)$ theories. If the use of $N = (0,2)$ reflects an opposite chirality convention, the convention should be stated.

9. The abstract and conclusion should distinguish an asymptotic AdS_5 region from an IR fixed point and from a second conformal boundary. These notions are not interchangeable in a domain-wall geometry.

10. Once the equations are corrected, the author should perform a global notation pass so that L always denotes a positive radius and signs are carried by the couplings or the chosen radial orientation, not implicitly by fractional powers.