Referee Report

Quantum States Prepared by Wormholes:
Long-Wavelength Deviations from Bunch–Davies

Summary of the manuscript

The paper asks which Gaussian state is prepared for linear fluctuations when an expanding baby universe nucleates through a Euclidean "wineglass" wormhole. Section II reviews the $O(4)$-symmetric backgrounds: a scalar interpolates between a non-positive extremum and a positive inflationary region, while either an axionic term proportional to Q_a^2/a^6 or a magnetic term proportional to Q_m^2/a^4 supports the throat. The zero-charge limit is argued, on the basis of the authors' earlier work, to pinch the stem into a disconnected asymptotic space and a no-boundary hemisphere.

Section III constructs the comparison state in exact de Sitter space in the closed slicing. Tensor harmonics with $n \geq 3$ and a massless probe scalar with scalar harmonics are shown to obey the common mode equation (10). Euclidean regularity selects the solution (18), and equations (23)–(30) express the corresponding saddle-point wave function, Riccati variable, kernel, and late-time variance.

Section IV contains the new calculation. A decaying mode is imposed in the asymptotic EAdS or flat region, using equations (32)–(33) or (44), and is evolved inward to the rim. The rim kernel is compared with the exact closed-de Sitter kernel through equation (34). Figures 6, 7, and 9 motivate empirical deviations proportional to $Q_a^{1.8}/(a_0^2 n^2)$ and $Q_m^2/(a_0^2 n^2)$ in equations (35) and (36). Equations (37)–(40) interpret the result as a Bogoliubov admixture, while Figure 8 and equations (41)–(42) describe a subsequent shift and oscillations during an approximately de Sitter phase. The authors conclude that short wavelengths approach the Bunch–Davies state, that long wavelengths retain a charge-dependent signature, and that the state tends smoothly to Bunch–Davies as the throat pinches off.

The question is timely and well motivated. In particular, extracting a state from explicit finite-charge saddles, comparing axionic and magnetic support, and finding weak sensitivity to the asymptotic potential depth could constitute a useful contribution to quantum cosmology. The regular closed-de Sitter solution in equations (13)–(19) is clearly presented, and the asymptotic expansions (32)–(33) and (44) are consistent with the stated Euclidean mode equation. Nevertheless, several issues affect the interpretation and, in some cases, the consistency of the central result.

Recommendation

I recommend **major revision**. The manuscript has a potentially publishable core, but it does not yet establish that the reported effect is a wormhole-prepared, ultraviolet-admissible state on the relevant Lorentzian background. The finite-charge background is replaced by exact de Sitter at precisely the order at which the signal is found; the Euclidean kernel has an unresolved sign convention; tensor and scalar normalizations have been conflated; and the claimed n^{-2} Bogoliubov tail would have problematic ultraviolet energy if it were truly asymptotic. The numerical fits and the observational interpretation also require substantial strengthening. These are not merely presentational matters, but they appear addressable without changing the basic aim of the paper.

Correctness and clarity

The qualitative strategy—choose a Euclidean outer boundary condition, propagate a logarithmic derivative to the nucleation surface, and use it as Lorentzian Gaussian data—is sound. The manuscript also correctly emphasizes that regularity on the limiting de Sitter hemisphere selects the usual closed-slicing Euclidean state. However, the current text does not keep separate three logically different ingredients: the state selected by the Euclidean saddle, the Lorentzian background on which that state evolves, and the exact-de Sitter reference state. At finite charge these ingredients are not identical. Moreover, equations (26), (33), and (34) do not presently define a kernel with a consistent orientation, and equations (8), (11), and (20) do not use a common physical normalization. Until these matters are repaired, the plots cannot be read as a controlled fractional power-spectrum prediction.

Novelty and significance

The background geometries and their no-boundary limit were already developed in the authors' cited earlier papers. The credible novelty here is narrower and more concrete: the finite-charge tensor/test-scalar kernel, its comparison across charge mechanisms and asymptotics, and its small-charge behavior. That result would be interesting if it survives the checks below. By contrast, oscillations from a nonzero Bogoliubov coefficient are generic to excited initial states, so their mere presence is not by itself a distinctive wormhole signature. A characteristic phase, a robust charge–geometry relation, or a correlated scalar/tensor prediction would make the novelty much sharper.

The manuscript should also position the calculation more carefully. The bibliography already contains Gratton and Turok's *Cosmological perturbations from the no boundary Euclidean path integral*, but this directly relevant work is not cited in the discussion of the Euclidean state. Khan et al., *Long Inflation Screens Euclidean-Wormhole Initial States* (arXiv:2605.04166), is included only in a general list even though it bears directly on Bogoliubov data and inflationary screening. A substantive comparison with these works, with the cited closed-universe/no-boundary literature, and with generic non-Bunch–Davies initial states is needed.

Major comments

1. **The finite-charge state is compared and evolved on the wrong reference background without a controlled error estimate.**

   At the time-symmetric rim used for analytic continuation, $a'(0) = \phi'(0) = 0$. Equation (2) then implies

$$\frac{3}{a_0^2} = V(\phi_0) + \frac{Q_a^2}{a_0^6} + \frac{Q_m^2}{a_0^4}.$$

Consequently the Section III relation $L = a_0 = \sqrt{3/V}$ is exact for the limiting de Sitter solution, but not for the finite-charge scalar-plus-charge background. Equation (4) also shows that the continued acceleration differs from that of $a(t) = a_0 \cosh(t/a_0)$ at order Q^2. In particular, with the stated continuation one obtains at the rim

$$\frac{\ddot{a}}{a_0} = \frac{1}{a_0^2} - \frac{Q_a^2}{a_0^6} - \frac{2Q_m^2}{3a_0^4},$$

rather than $1/a_0^2$. The scalar will also roll and the charge density will redshift after nucleation.

This is central because equations (35) and (36) report an approximately Q^2 effect. Replacing the true Lorentzian continuation by exact de Sitter therefore introduces a background error at the same parametric order as the claimed state signal. The authors should evolve the prepared modes on the actual continued background and compare them with a clearly defined adiabatic reference state on that same background. At minimum, they must demonstrate that the exact-de Sitter replacement changes the reported kernel and late-time variance by much less than the plotted per-mille or percent effects. Without this separation, the charge dependence can reflect a background mismatch rather than memory of the Euclidean wormhole.

2. **The Euclidean sign and contour orientation are internally inconsistent.**

   Equation (26) defines the Lorentzian quantity $R_n = -i\dot{h}_n/h_n$ and $\Omega_n = a^3 R_n$. In Section IV.A, however, the same symbol is identified with h_n'/h_n. The decaying EAdS solution (32) has $h_n'/h_n < 0$, explicitly as written in equation (33); the plotted modes likewise decrease as the Section IV coordinate runs outward from the rim. Thus $\Omega_n = a^3 h_n'/h_n$ would be negative. Inserted in equation (28), it would give a non-normalizable Gaussian and could not be compared with the positive de Sitter value used in equation (34).

   For a coordinate increasing from the rim toward the EAdS/flat end, the analytic continuation and outward-normal convention appear instead to require $\Omega_E = -a^3 h_n'/h_n$. The authors should state the complex-time contour, the orientation of the Euclidean normal, and the continuation of the boundary term explicitly, and then use one convention in equations (26), (33), (34), and (44). They should also state which sign was implemented numerically. This point must be resolved before the sign of the variance shift or the Bogoliubov coefficients can be assessed.

3. **The tensor and probe-scalar sectors share a mode equation, but not the normalization claimed in equations (20)–(30).**

   The tensor action (8) carries a coefficient 1/8, whereas the probe-scalar action (11) carries 1/2. Equation (20), with coefficient 1/2, cannot follow from both for the same physical amplitude h_n unless the tensor amplitude is explicitly rescaled by a factor of two. Such a rescaling must then be undone when quoting a tensor variance. The normalization of scalar and tensor harmonics, the two tensor polarizations, and their degeneracies should be given, and separate scalar and tensor spectra should be displayed. Relative logarithmic-derivative shifts may coincide, but the absolute kernels and power normalizations do not.

   There is a second canonical-normalization problem in equation (38). From the displayed solution (14), the $t \to 0^+$ limits are

$$f_2(0) = n e^{-i\pi(n-1)/2}, \qquad \frac{\dot{f}_2(0)}{f_2(0)} = \frac{i(n^2-1)}{nL}.$$

With the prefactor printed in equation (38), these relations give $a^3(u_n\dot{u}_n^* - u_n^*\dot{u}_n) = -i$, not $+i$. An overall phase such as the square root of i cannot reverse a Wronskian sign. The authors must either conjugate the reference mode or change the Wronskian and Bogoliubov conventions, then rederive equations (39)–(40). Furthermore, equations (21) and (28) use harmonics normalized to leave a factor V_{S^3}, whereas the canonical Wronskian in (38) contains no such factor. One RMS or orthonormal convention should be used throughout.

The harmonic ranges must also be treated correctly. Tensor modes begin at $n = 3$, so the $n = 2$ points in Figures 6–9 do not test a tensor prediction. The scalar $n = 1$ harmonic is a homogeneous zero mode: $n^2 - 1 = 0$, the normalization in equation (38) is singular, and there is no ordinary Gaussian Bunch–Davies oscillator for that mode. The analysis should explicitly restrict the probe scalar to $n \geq 2$ and the tensor to $n \geq 3$.

4. **The magnetic tensor calculation requires an underlying perturbation theory.**

A term Q_m^2/a^4 in the background equations is not sufficient to establish the tensor action (8). If the magnetic support is an isotropic $SU(2)$ gauge configuration, its tensor perturbations can mix with gravitational waves and can provide tensor anisotropic stress. An analogous consistency check should be supplied for the axionic realization. The paper should specify the matter actions that realize Q_a and Q_m, derive the quadratic tensor sector, and show under what conditions the matter tensor modes may consistently be set to zero. Otherwise the magnetic results apply only to a frozen phenomenological radiation-like source, not to the advertised magnetic wormholes in general.

Relatedly, the orders quoted in the flat asymptotic expansion (44) are valid for axionic support but not generically for magnetic support. With $r = \tau - \tau_{\min}$, the magnetic constraint gives

$$a(r) = r + \frac{Q_m^2}{6r} + \cdots,$$

and the decaying mode consequently behaves as

$$h_n = r^{-n-1}\left[1 - \frac{Q_m^2(n+2)}{12r^2} + \cdots\right], \qquad \frac{h_n'}{h_n} = -\frac{n+1}{r} + \frac{Q_m^2(n+2)}{6r^3} + \cdots.$$

Thus the displayed relative $O(r^{-4})$ and absolute $O(r^{-5})$ remainders do not cover magnetic flat wormholes. Since Figure 9 presents axionic data, the scope of the flat-space conclusion should be stated explicitly unless magnetic data are added with the corrected boundary expansion.

5. **A true $\beta_n \sim n^{-2}$ ultraviolet tail would not have finite excitation energy.**

Equations (35) and (40) are used to infer $\beta_n \propto n^{-2}$. On S^3 the large-n degeneracy grows as n^2 and the physical frequency as n/a. If this behavior persisted asymptotically, the excitation energy relative to the reference vacuum would scale as

$$\sum_n n^2 \frac{n}{a} |\beta_n|^2 \;\propto\; \sum_n \frac{1}{n},$$

which is logarithmically divergent. The state would not have the ultraviolet behavior needed for a finite renormalized stress tensor and would be in tension with an adiabatic/Hadamard state.

The data only cover $2 \leq n \leq 15$, which is not an ultraviolet test. The authors should perform a high-frequency WKB/adiabatic analysis and extend the numerics to much larger n. A smooth

Euclidean preparation may well cross over to a faster falloff, in which case $1/n^2$ should be described only as an intermediate-range fit. If it does not, the authors must explain the state prescription, renormalized stress tensor, and backreaction. They should also check whether an abrupt replacement of the true continued background by exact de Sitter is itself producing an artificial n^{-2} tail.

6. **The numerical evidence and empirical fits are not documented at the precision required by the claims.**

   The paper reports effects from a few parts per thousand to a few percent, but gives no numerical error budget. It should list, for every background family, the values of a_0, ϕ_0, $\tau_{\min}$ and $\tau_{\max}$, the background shooting accuracy, mode-integration tolerance, retained asymptotic order, and stability under changing the starting surface. Tables of the rim kernels or machine-readable data would make the result reproducible. The authors should also show residuals and parameter uncertainties for equations (35) and (36), rather than only overlaying curves on the data.

   The exponent 1.8 is particularly in need of scrutiny. The axionic background equations depend on Q_a^2, so a regular small-charge expansion would naturally begin with an even power; a non-integer exponent could instead signal the singular topology-changing limit, or simply a finite-range fit that is not statistically distinct from two. Data below $Q_a = 1/20$ and a controlled extrapolation are required before the exponent is assigned physical meaning. Charge units must also be defined. Equations (2)–(4) imply different length dimensions for Q_a and Q_m, and equation (35) is not scale-covariant with a 1.8 exponent unless all quantities have first been nondimensionalized using the fixed V_{top} scale. The formulas should be presented as model-dependent fits in explicit dimensionless variables.

7. **Equation (24) contains a definite algebraic error.**

   Let $x = \sqrt{a_f^2/L^2 - 1}$. Equation (23) can be written as

   $$S_{\text{os}} = \pi^2 a_f^2 (n^2 - 1) \frac{-x + in}{n^2 + x^2} h_f^2.$$

   Its large-a_f real part is therefore

   $$\operatorname{Re} S_{\text{os}} = -\pi^2 (n^2 - 1) L a_f h_f^2 + O(a_f^{-1}),$$

   not $-2\pi^2(n^2-1)La_f h_f^2$ as printed in equation (24). The imaginary term used for the Gaussian weighting is unchanged, so this does not by itself invalidate equation (25), but the derivation should be corrected and the remaining normalizations rechecked.

8. **The topology-changing limit is not yet shown to be smooth.**

   The numerical data show that the rim logarithmic derivative approaches the de Sitter value over a finite charge interval. This is useful evidence, but it does not prove uniform convergence through a pinching throat. In addition, the mode functions in Figures 4, 5, and 9 have arbitrary overall normalization; their large change across the stem cannot by itself diagnose physical amplification or the absence of a buildup. The unresolved physical scalar sector is also precisely the sector in which negative-mode issues are known to arise.

   To support the stronger statement, the paper should study invariantly normalized two-point functions, the quadratic action density, or the renormalized stress tensor in the stem region as $Q \to 0$, and distinguish fixed-n convergence from a limit uniform in n. Otherwise the conclusion should be narrowed to evidence that the *rim kernel for tensor and massless spectator modes* approaches the Euclidean de Sitter kernel. The words "exactly" and "smooth topological transition" should not be inferred solely from equations (35) and (36).

9. **The late-time oscillation admits an analytic derivation.**

   The statement following equation (42) that an exponential solution automatically preserves the original $1/n^2$ scaling is not a sufficient argument, because the coefficient in the exponent is n-dependent. In fact, within the exact-de Sitter approximation, equation (41) can be integrated directly. Using $R_{n,\mathrm{BD}} = -i\dot{u}_n/u_n$ and $\Omega_n = a^3 R_n$ gives

$$\delta\Omega_n(t) = \delta\Omega_n(0) \left[\frac{u_n(0)}{u_n(t)}\right]^2 .$$

   Since the displayed solution has $f_2(\infty) = 1$ and $f_2(0^+) = ne^{-i\pi(n-1)/2}$, this implies, after dividing by the corresponding Bunch–Davies kernels,

$$\left.\frac{\delta\Omega_n}{\Omega_{n,\mathrm{BD}}}\right|_{t\to\infty} \simeq (-1)^{n-1} \left.\frac{\delta\Omega_n}{\Omega_{n,\mathrm{BD}}}\right|_{t=0},$$

   subject to correcting the Wronskian convention in equation (38). This relation supplies the even/odd oscillation and preserves the envelope in the idealized de Sitter evolution. It should replace the qualitative argument after equation (42), and the authors should explain how it changes on the actual finite-charge background. The distinction between the relative kernel difference, the absolute ΔR_n, and β_n must also be maintained.

   Figure 8 plots $a_f^3 \,\mathrm{Re}\, R_n/(n^3 - n)$, which is proportional to an inverse variance, not an angular power spectrum. It should state a_f, verify that all displayed modes are super-horizon, and be supplemented by the physical fractional variance $\Delta_n^2/\Delta_{n,\mathrm{BD}}^2 - 1$. The claimed percent-level effect should be shown for the magnetic and large-charge cases where it is said to be largest, using the actual finite-charge Lorentzian evolution or a quantified approximation.

10. **The manuscript does not calculate a CMB angular spectrum or the observable scalar perturbation.**

   Equations (28)–(30) define a primordial variance for discrete harmonics on S^3; the index n is not a CMB multipole ℓ. No radiation-era transfer functions, projection from closed harmonics, reheating history, inflation duration, curvature constraint, or likelihood calculation is included. More importantly, the scalar calculation concerns a massless spectator. The gauge-invariant adiabatic curvature perturbation couples to the rolling field that supports the wormhole and is explicitly left for future work. Thus statements about an "angular power spectrum," "all perturbative modes," low-multipole anomalies, and a distinctive observable are not established by the present calculation.

   The abstract, introduction, and discussion should consistently say "primordial tensor and massless spectator-mode kernel/spectrum." Either an actual curvature spectrum and C_ℓ prediction should be calculated for a specified inflationary history, or the CMB paragraphs should be presented only as qualitative motivation. Equation (34) is a fractional *kernel* shift at the rim, not directly a fractional late-time power shift; the two should not be interchanged.

11. **The outer boundary condition is a physical state choice and its uniqueness should be discussed.**

    In EAdS a massless mode has both a non-normalizable/source branch and the decaying branch used in equation (32). Choosing the latter is a natural no-source condition, but it is part of the state prescription rather than a consequence of the local wormhole equations alone. The flat-space decaying branch in equation (44) likewise encodes a vacuum choice at infinity. The authors should formulate these choices through the Euclidean action and its boundary terms, state which alternative boundary data are excluded, and explain whether holographic counterterms affect the canonical momentum used for the rim kernel. This would also clarify in what sense the resulting state is uniquely "prepared by the wormhole."

12. **The literature comparison should isolate what is genuinely distinctive.**

    The background construction and zero-charge geometry are results of the authors' previous papers, while Bogoliubov oscillations are generic for excited initial states. The paper should explicitly identify its new result as the kernel extracted from specific finite-charge wineglass saddles and then compare its amplitude, phase, ultraviolet falloff, and inflationary screening with the existing no-boundary and non-Bunch–Davies literature. The already-cited work of Khan et al. deserves more than inclusion in a list, and the uncited Gratton–Turok entry in the bibliography is directly relevant to Section III. If the authors wish to call $1/n^2$ "distinctive," they must show that it survives the ultraviolet and background-matching checks and distinguish it from generic finite-time or finite-adiabatic-order initial states.

Minor comments

1. State the normalization convention for every scalar and tensor harmonic. The factor $V_{S^3} = 2\pi^2$ in equations (21) and (28) otherwise appears inconsistent with the usual orthonormal-harmonic convention, even though a rescaling can reproduce equation (30).

2. Equation (25) displays $|\Psi|$, an amplitude, while the text calls it a probability distribution. The probability is $|\Psi|^2$; this distinction should be kept explicit when deriving the factor of two in equation (28).

3. Equation (34) uses the wormhole kernel rather than the Bunch–Davies kernel in the denominator. The authors should state this convention and note that, on a real matching surface, a positive kernel shift corresponds to a negative fractional variance shift.

4. In equation (38), specify whether the primes are Lorentzian time derivatives; the rest of Section III reserves dots for those derivatives. After correcting the Wronskian sign, writing an explicit phase and a real positive normalization would make the convention easier to verify.

5. Plot $|\beta_n|$ and its phase directly. The phase, not only the envelope, determines the sign and location of the oscillations.

6. The potential (5) should be described as an illustrative model. Calling the ensuing phase approximately realistic on the basis of the curvature at its maximum is not a substitute for a scalar-spectrum and background-evolution analysis.

7. The phrase that inflation assumes Bunch–Davies "often without specific reason" should be balanced by noting the usual adiabatic/Hadamard, short-distance, and Euclidean regularity motivations.

8. The captions of Figures 6, 7, and 9 should report fit ranges and uncertainties, and should distinguish the scalar-only $n = 2$ point from the tensor range.

9. The discussion should distinguish a small kernel shift at nucleation, a Bogoliubov coefficient, a late-time primordial variance, and an observed C_ℓ; these are four different quantities.

10. There are several minor grammatical issues, for example "wineglass wormhole produce" in Section IV.A and "perturbations modes" in Section V. A careful proofreading would improve the presentation.

Conclusion

The manuscript addresses an interesting problem, and the numerical state-preparation calculation could become a valuable result. Publication in a high-energy theory or gravitation journal would be warranted if the authors demonstrate that the signal is defined relative to the actual Lorentzian continuation, repair the sign and normalization inconsistencies, establish acceptable ultraviolet behavior, document numerical convergence, and restrict the phenomenological claims to quantities that have been computed. I would be willing to reassess a substantially revised version that addresses these points.