# Referee Report

On interacting non-Abelian antisymmetric tensor field models

arXiv:2607.12798

## Summary

The manuscript studies the gauge structure produced by a group-manifold reduction of an Abelian higher-form coupled to gravity. Starting with a three-form, the authors retain the Yang–Mills vector associated with internal diffeomorphisms and the tower of components of the higher-dimensional form. A tangent-frame redefinition, equations (2.19)–(2.23), gives lower-dimensional fields that transform covariantly under the Yang–Mills group. The resulting system contains an external Abelian three-form, adjoint two-forms, vectors, and scalars. Section 2 derives their transformations, covariant field strengths, and a reduced action; identifies parameters that generate zero transformations; and interprets particular projections of the vectors and scalars as Stueckelberg fields for massive tensor modes.

Section 3 repeats the gauge-for-gauge analysis for a four-form. The claimed first-stage null parameters in equation (3.53) have the same component structure as the reduced three-form transformations, and equation (3.55) supplies a second stage. The external Abelian four-form retains a third stage. Section 4 then proposes a component formula for the reduction of an arbitrary $(r+1)$-form, equations (4.61)–(4.63), and attempts to construct and iterate the corresponding null parameters in equations (4.64)–(4.68). The headline conclusion is that the non-Abelian part of the reduced tower has the same number of gauge-for-gauge stages as the corresponding charged lower-rank forms, while the complete tower retains the reducibility length of the original Abelian higher-form.

There is a sound and useful idea beneath the component calculations. After the transformations are corrected, the proposed null parameters in the three- and four-form examples are precisely what one expects by decomposing the nilpotent higher-dimensional exterior derivative into a covariant spacetime derivative, an internal Chevalley–Eilenberg differential, and a curvature-insertion term. The cancellations use $\nabla^2$ proportional to $\mathcal{F}$, the Yang–Mills Bianchi identity, and the Jacobi identity. An explicit description of this reduced complex could be useful for a later BV or BRST quantization.

In its present form, however, the manuscript does not reliably establish its main claims. The central transformation table in equation (2.22) differs in three material respects from the transformation actually used in the rest of the paper. As printed, the parameters in equation (2.38) do not generate a zero transformation. The arbitrary-rank discussion contains several rank-inconsistent equations and replaces an all-stage proof by a circular recursion. Completeness is also claimed for a general compact group using assumptions that apply only to a suitably normalized semisimple algebra, with no separation between local off-shell reducibility and global zero modes. Finally, the novelty claim is too broad relative to the tensor-hierarchy and gauge-for-gauge literature.

## Recommendation

**Major revision.** The explicit component gauge-for-gauge maps for this particular group-manifold reduction may constitute a useful technical result, and I see a plausible route to a correct paper. Publication should await a correct master transformation table, a genuine all-rank nilpotency and stage proof, a precise statement of the group and locality assumptions, and a substantially sharper account of novelty. The present version is not yet at the standard required for JHEP or Physical Review D because the displayed formulas do not support the principal three-form result and the general theorem is neither well typed nor proved.

## Major comments

1. **Equation (2.22) is not the transformation law used by the subsequent analysis.**

   With the normalized antisymmetrization convention stated in Section 1, the three-form tower should transform, up to the notation and sign conventions of the manuscript, as

   $$\begin{aligned} \delta B_{\mu\nu\rho} &= 3\partial_{[\mu}\widetilde{\Lambda}_{\nu\rho]} - 3\kappa^{-1}\mathcal{F}_{[\mu\nu|a|}\widetilde{\Lambda}_{\rho]a}, \\ \delta B_{\mu\nu a} &= 2\nabla_{[\mu}\widetilde{\Lambda}_{\nu]a} - \kappa^{-1}\mathcal{F}_{\mu\nu b}\widetilde{\Lambda}_{ab} + f_{amn}\lambda_m B_{\mu\nu n}, \\ \delta B_{\mu ab} &= \nabla_\mu\widetilde{\Lambda}_{ab} + \kappa f_{abc}\widetilde{\Lambda}_{\mu c} + 2f_{[a|mn}\lambda_m B_{\mu n|b]}, \\ \delta B_{abc} &= 3f_{[a|mn}\lambda_m B_{n|bc]} - 3\kappa f_{[ab|d}\widetilde{\Lambda}_{d|c]}. \end{aligned}$$

   Instead, the first line of equation (2.22) contains an additional $\partial_\rho\widetilde{\Lambda}_{\mu\nu}$ even though this derivative is already included in $3\partial_{[\mu}\widetilde{\Lambda}_{\nu\rho]}$. The second line omits the curvature term $-\kappa^{-1}\mathcal{F}_{\mu\nu b}\widetilde{\Lambda}_{ab}$, and the last line has $-\kappa$ rather than $-3\kappa$ multiplying the internal differential. These are not merely cosmetic discrepancies. The missing curvature term is restored in equations (2.26) and (2.35), and all three corrected structures are recovered from the general equation (4.61), showing that the paper itself uses two different versions of its basic symmetry.

   The failure can be seen immediately by inserting equation (2.38) into the printed equation (2.22). The two-form variation contains

   $$[\nabla_\mu, \nabla_\nu]\omega_a = -f_{abc}\mathcal{F}_{\mu\nu b}\omega_c,$$

   with no term available to cancel it. The extra derivative in the first line also leaves a non-antisymmetric residual term, while the missing factor in the scalar line spoils the Jacobi cancellation. Thus the central statement of Section 2 is false as the paper is written, even though it becomes consistent after the evident corrections.

   The authors should replace equation (2.22) by a single verified master table and then explicitly substitute equation (2.38) into every component. They should also recheck the closure parameters in equation (2.29), the homogeneous transformation of all curvatures in equation (2.31), and invariance of the action (2.33). A related normalization error occurs in the component decomposition of $F_{\mu\nu ab}$ immediately before equation (2.30): normalized antisymmetrization requires $2\partial_{[\mu}C_{\nu]ab}$, not $\partial_{[\mu}C_{\nu]ab}$. Given the concentration of missing factors in the foundational formulas, a complete coefficient and index audit is needed.

2. **The arbitrary-rank centerpiece is not rank-consistent and does not prove the asserted reducibility stage.**

   Equation (4.61) has a recognizable bigraded structure and specializes correctly to the intended three- and four-form transformations. The formulas that follow it are much less controlled. For a field component with $n$ external and $r - n + 1$ internal indices, every tensor-shift parameter must have total degree $r$. The commutator table (4.62), however, displays parameters of total degree $r+1$: for example its first parameter has $r$ external indices and one internal index, and its last has $r+1$ internal indices. The generic line has the same one-index excess. These cannot be the parameters appearing in equation (4.61).

Equation (4.68) has further type errors. Its first left-hand side has the right total degree, but the second is printed with $n-2$ external and $r-n+1$ internal indices while its right-hand side carries one more internal index. The third left-hand side is printed with $n$ external and $r-n+1$ internal indices while its right-hand side carries two fewer internal indices. The paragraph following equation (4.68) repeats incompatible ranks. Consequently, the equation that is supposed to encode the general first-stage operator cannot be composed with the field transformation as printed.

Equation (4.67) is also not uniform at the endpoints of the tower. When the field component has only one internal index, the curvature-induced parameter must be purely external, as the $\Lambda_{\mu\nu\rho}$ term in equation (3.53) illustrates. The generic left-hand side of equation (4.67) instead retains a spurious internal index in this case. Endpoint formulas and the ranges on every branch must be written explicitly.

Even after repairing these indices, the prose does not prove an exact stage. Equations (4.64)–(4.67) construct a representative first-stage null combination for one component. The text then declares that the same argument "obviously" applies to other ranks and, after equation (4.68), invokes the already asserted reducibility of equation (4.61) to say that the new parameters can be reduced once again. This is circular: it does not display operators $Z_s$ satisfying

$$RZ_1 = 0, \qquad Z_s Z_{s+1} = 0,$$

establish their nontriviality, or show where the chain terminates. Boundary values of the external degree, and cases in which spacetime or the internal group is too small to support a displayed component, are also absent.

There is also a missing endpoint in the curvature tower. A reduced five-form field strength in the four-form model has, when $\dim \mathcal{G} \geq 5$, a purely internal component

$$H_{abcde} = -10\kappa f_{[ab|m} B_{m|cde]}.$$

Equation (3.48) stops at $H_{\mu abcd}$, and the general equation (4.63) likewise omits the all-internal endpoint. This term contributes to the scalar potential and mass matrix, so it matters for both the claimed reduced Lagrangian and the Stueckelberg discussion. The complete field-strength tower and all of its degree bounds should be supplied.

There is a much cleaner proof available. The authors should assemble the components into a bigraded complex $\Omega^q(M) \otimes \Lambda^s \mathfrak{g}^*$ and define one reduced differential $\mathcal{D}$ containing the covariant derivative, the internal Chevalley–Eilenberg differential, and the curvature-insertion term. They should prove $\mathcal{D}^2 = 0$ from the Bianchi and Jacobi identities, write $\delta B = \mathcal{D}\Lambda$, $\delta\Lambda = \mathcal{D}\omega$, and continue the same construction at each total degree. This would give an actual all-stage proof, make the three- and four-form examples transparent, and force all combinatorial factors and index ranges to be consistent. A short table should then state separately the stage of the external Abelian form, the charged tensor-shift sector, and the irreducible Yang–Mills sector.

3. **The completeness statements require substantially sharper group-theoretic and local/global assumptions.**

The setup repeatedly refers to an arbitrary compact group, but the key inversion uses $f_{abc}f_{abd} = 2\delta_{cd}$. This excludes Abelian factors and fixes a special normalization on the semisimple part. For a product of simple groups, relative normalizations and radii must also be specified. In particular, the conclusion following equation (2.40) that every closed internal two-index parameter is proportional to $f_{abc}\omega_c$ is a semisimple cohomological statement, not a consequence of compactness alone. It should be formulated and proved under the appropriate hypothesis, for example using the relevant Whitehead lemma. A compact group with $U(1)$ factors has additional kernels and no corresponding non-Abelian mass structure in those directions.

The four-form completeness claim after equations (3.56)–(3.57) is even more delicate. The retained invariant internal forms form the Chevalley–Eilenberg complex of $\mathfrak{g}$. Compact simple algebras have nonzero $H^3(\mathfrak{g})$, represented by the invariant Cartan three-form $f_{abc}e^a \wedge e^b \wedge e^c$. The symmetric trace solution noted below equation (3.57) is directly related to this issue. The statement that it is "not compatible" with equation (3.46) is not demonstrated. At minimum, constant coefficients give global zero modes; with horizontal components in the redefined basis, closed invariant forms must be analyzed as a whole. These modes may be excluded from a local BV reducibility complex, but that restriction has to be stated rather than silently assumed, especially because the paper motivates its analysis by future quantization and ghost counting.

The discussion surrounding equation (2.34) likewise conflates an off-shell reducibility identity with field-dependent stabilizers. It is not true that $\nabla_\mu\lambda = 0$ requires a pure-gauge connection: a nonzero curvature can have a nontrivial centralizer, for example for Cartan-valued configurations. Such accidental stabilizers need not count as local reducibility generators, but the paper should say that it seeks local differential identities valid on the full configuration space. The authors should define off-shell versus on-shell reducibility, local versus global zero modes, the allowed parameter space and boundary conditions, and then prove completeness only within the stated category.

4. **The reduced action is not a consistent truncation of the gravitational theory described at the start of Section 2.**

The manuscript explicitly notes that setting the internal metric moduli to $\Phi_{ab} = \delta_{ab}$ while retaining the other fields makes the Einstein equations inconsistent. It then fixes the external metric to Minkowski space, drops the scalar potential and cosmological term, and presents equation (2.33) as the dimensionally reduced action. This may define a useful fixed-background Yang–Mills/tensor model with the inherited gauge symmetry, but it is not an unqualified consistent reduction of equation (2.9). The later appeal to the consistency of equations of motion in the cited Nishino–Rajpoot system does not by itself establish consistency of the scalar- and gravity-truncated action used here.

The authors should either restore the internal metric scalars, their potential, and the appropriate gravitational terms, or state clearly that equation (2.33) is a standalone fixed-background sector used only to study the kinematic gauge complex. Claims about a reduced gravitational theory, physical spectra, and future quantum effective actions must then be limited accordingly. It would also help to give a direct lower-dimensional gauge-invariance and Bianchi check rather than rely on an inconsistent parent truncation.

The role of $\kappa$ needs clarification at the same time. It is naturally an inverse-radius or mass scale, but the displayed Yang–Mills transformation and curvature contain no explicit $\kappa$, while other formulas contain both $\kappa$ and $\kappa^{-1}$. The actual canonically normalized gauge coupling also depends on the gravitational normalization and internal volume. Thus a naive $\kappa \to 0$ "free limit" is singular in the chosen variables. The paper should distinguish the compactification scale, mass parameters, and dimensionless interaction strength and explain the relevant limiting procedure.

5. **The gauge algebra and the relation to no-go theorems are described too strongly.**

   Equations (2.29), (3.47), and the intended version of (4.62) show that two pure tensor-shift transformations commute. Noncommutativity arises when a Yang–Mills transformation acts on a tensor parameter in the appropriate adjoint tensor representation. The algebra is therefore most naturally described as the Yang–Mills algebra acting semidirectly on an Abelian ideal of tensor shifts, together with the external singlet shift. This is certainly a non-Abelian full gauge algebra, but it is not a new non-Abelian bracket among higher-form shifts. The distinction matters when comparing the construction with genuine non-Abelian higher-gauge systems.

   For the same reason, the claim that mass terms allow the model to "bypass" the cited no-go theorems is imprecise. The construction lies outside their premises because it enlarges the field content by Kaluza–Klein Yang–Mills, lower-form, and Stueckelberg fields and uses a coupled hierarchy inherited from a higher-dimensional Abelian form. The mass mixings are part of that enlarged system, not by themselves the mechanism that invalidates a no-go theorem. The manuscript should state exactly which assumptions are evaded and avoid suggesting a non-Abelian deformation of the isolated massless form symmetry.

   The Summary also says that the conclusions should carry over to PST-type self-dual actions because the action played little role. Self-duality and the additional PST gauge symmetries change the constraint and gauge complex, so this extension is not automatic. It should either be demonstrated or presented only as an open question.

6. **The Stueckelberg interpretation needs a mode count, not only formal projections.**

   Equations (2.41)–(2.45) identify plausible projections that shift under tensor parameters, and the projector in equation (2.45) is useful. However, the argument does not establish the physical spectrum claimed in Section 5. The projector property relies on the special normalization of a semisimple algebra; its rank and kernel are not given; and the proposed Stueckelberg variables are projections of the original fields rather than an independently counted set. For a semisimple algebra of dimension $N$, the map generated by $f_{abc}$ has a different complement in $\Lambda^2\mathfrak{g}$ depending on the group (for $SU(2)$ the relevant complement is already trivial), while Abelian factors change the conclusion completely.

   The authors should decompose the vector and scalar spaces into the image and kernel of the internal differential, give the rank of the mass matrices and the number of independent shift symmetries, and verify that gauge fixing leaves the expected degrees of freedom. One worked example, such as $SU(2)$ and one higher-rank simple group, would test the formulas and make the physical content appreciably more interesting. It would also clarify which fields are massive, which are eaten, which remain massless because of internal cohomology, and how the claimed reducibility changes after a Stueckelberg gauge choice.

7. **The novelty claim must be narrowed and supported by a more complete comparison with gauge-for-gauge literature.**

   The reduction itself and the interacting tensor hierarchy are largely inherited from Scherk–Schwarz and from the Nishino–Rajpoot model cited by the authors. The potentially new contribution is the explicit reducibility complex and stage count for this particular ordinary group-manifold reduction. The broad statement in the Introduction that reducibility of interacting $p$-forms has never been studied is not sustainable. The tensor-hierarchy literature already treats trivial gauge parameters and symmetries-for-symmetries. For example, Cagnacci, Codina, and Marques, arXiv:1807.06028, discuss gauging-dependent trivial parameters and gauge-for-gauge structure in tensor hierarchies related to generalized Scherk–Schwarz reduction; Berman et al., arXiv:1208.5884, analyze reducibility and ghost towers for generalized diffeomorphisms. Related structures are also implicit in several of the tensor-hierarchy references already cited in the manuscript.

These works do not make the present calculation redundant, but they change the novelty question. The authors should explain precisely what is special about the present DeWitt reduction, compare its finite complex with the existing tensor-hierarchy and $L_\infty$ formulations, and replace "never studied" by a defensible narrower claim. At present the contribution is classical and kinematic: no new solution, amplitude, quantum calculation, explicit BRST operator, or degree-of-freedom count is supplied. Writing the nilpotent BRST/BV complex or carrying out the mode count proposed above would substantially strengthen the significance for a high-energy theory journal.

8. **The comparison model in equations (4.69)–(4.71) must be corrected before it can support the contrast in reducibility stages.**

   In equation (4.69), the variation of an $(n-1)$-form $C$ is shifted by a $\beta$ written with $n$ spacetime indices. Equation (4.70) confirms that $\beta$ should instead be an $(n-1)$-form parameter. Once the rank is corrected, the first-stage cancellation with $\gamma = -\omega$ appears plausible: the second component makes the gauge-for-gauge map injective and therefore prevents a further stage of the same type. This argument should be shown explicitly with consistent ranks and signs. In a paper whose primary result is a comparison of reducibility lengths, an ill-typed comparison transformation cannot be left as a typographical detail.

## Minor comments

1. Equations (1.7) and (1.8), which define the normalized antisymmetrization convention used throughout the manuscript, are malformed. Equation (1.7) repeats indices in its displayed expansion. Equation (1.8) repeats $\mu_2$ on the left and should involve a two-form $F_{[\mu_1\mu_2}$ multiplied by $A_{\mu_3\cdots\mu_{p+2}]}$. These examples should be replaced by correct, compact formulas.

2. Define the reduced spacetime dimension and the internal group dimension at the start of Section 2. The symbol $n$ first appears as the integration dimension in equation (2.33) and is later reused as an external form degree in Section 4. Distinct notation is needed, together with all allowed index ranges and a convention that terms outside those ranges vanish.

3. The last term of equation (2.27) has a Lie-algebra index appearing more than twice in the same monomial, and the same display switches between tilded and untilded parameters. This line should be rederived, not merely relabeled.

4. The Killing equation in Section 2.1 is written with $+2\Gamma^c_{ab}e_c$. For a covector and the standard definition of the Levi–Civita derivative the sign is negative. Please correct this or state the nonstandard connection convention being used.

5. In the paragraph preceding equation (2.41), the standard Abelian two-form transformation is printed as $\partial_\mu \Lambda_\nu - \partial_\mu \Lambda_\mu$; the second term should be $\partial_\nu \Lambda_\mu$.

6. The text introducing equation (3.53) refers to parameters with ranks that do not match the displayed $\omega_{\mu a}$, $\omega_{ab}$, and $\omega_{\mu\nu}$. Please make the notation uniform and explicitly verify equations (3.53)–(3.55) by composition.

7. The Summary states that all considered gauge transformations are reducible. The Yang–Mills transformations are not locally reducible in the generic off-shell sense used here. The sentence should distinguish the full algebra from its tensor-shift complex.

8. The reduced action should state all overall normalizations that have been suppressed, including the internal volume and gravitational coupling, and should identify which sign and normalization convention is used for the higher-form kinetic term. This is necessary for the subsequent statements about masses. In particular, the $+12\kappa^2 B^2$ quoted in Section 2.6 is the contribution inside the parentheses of equation (2.33); after the overall $-1/24$ it is not itself the Lagrangian mass term.

9. The two bibliography entries for the distinct recent quantization papers currently carry the same arXiv identifier, 2509.01863. The first appears to require arXiv:2507.09312. The bibliography and author/title spellings should be checked systematically.

10. Equation numbering should be reset at the beginning of each section or made globally sequential. The present hybrid notation, in which Section 2 starts at equation (2.9), is unnecessarily difficult to navigate.

11. A careful language edit is needed. There are numerous missing articles, subject–verb disagreements, repeated words, and typographical slips. More important than stylistic polish, the revision should consistently distinguish fields, gauge parameters, gauge-for-gauge parameters, local generators, and global zero modes.