Referee Report

Thermodynamic scaling and string dynamics in rotating holographic QCD

arXiv:2607.12906

Summary

The manuscript studies thermodynamics, a string-based confinement criterion, and holographic Schwinger pair production in a bottom-up Einstein–Maxwell–dilaton (EMD) model. Starting from the static reconstructed background of Section 2.1, the authors compactify one boundary direction as $x_3 = l\theta$ and apply the boost in equation (11). The resulting stationary metric, equations (14)–(18), depends on the local velocity $v = \omega l$, and the temperature and chemical potential are related to the static parameters by $T_0 = \gamma T$ and $\mu_0 = \gamma\mu$.

Section 3 analyzes the small- and large-black-hole branches using the Hawking temperature, entropy, an integrated grand potential, and the response functions C_μ, χ, and c_s^2. The central thermodynamic claim is equation (38),

$$\Omega(T, \mu, \omega) = \Omega_0(\gamma T, \gamma\mu),$$

from which the authors infer the scaling of all first-order and crossover lines and of the critical end point in equations (39)–(40). In this sector, rotation is therefore claimed to have only the kinematical effect dictated by the boost.

Section 4 studies an open string whose endpoints lie on a finite-radius probe brane and whose separation is along x_1. Equations (41)–(48) give the static embedding, the connected-string energy and separation, and the effective string tension. A positive minimum of that tension is identified with a dynamical infrared wall and with string confinement. For the fixed-θ embedding, the factor entering the string action vanishes at $G(z_*) = \omega^2 l^2$, equation (53). The manuscript calls this a worldsheet horizon and uses it to delimit the radial range of connected strings. Numerically, the infrared-wall phase line does not obey the simple thermodynamic scaling. The authors report that rotation enlarges the region between the thermodynamic transition and the string-wall criterion, lowers the two quoted critical electric fields E_s and E_c, lowers the potential barrier, and reduces both $L_{\max}$ and the largest turning point of the preferred connected branch.

The proposed comparison of several probes in one background is potentially useful. In particular, a controlled study of the separation between a low-temperature thermodynamic branch and a fundamental-string wall could be of interest to the holographic-QCD community. The current manuscript, however, contains mutually inconsistent background equations, an incorrect near-boundary charge formula, an incomplete rotating thermodynamic ensemble, and an apparent error in the DBI critical field. More fundamentally, the string embedding is not co-rotating as claimed: it is the standard plasma-wind or moving-dipole configuration. This distinction changes the interpretation of the worldsheet surface and of the advertised violation of the thermodynamic scaling. Consequently, the present formulas do not yet support the quantitative phase diagrams or the main physical conclusion.

Evaluation of novelty and significance

The manuscript combines three ingredients: an established boost-generated stationary black brane, the authors' earlier static EMD analysis with a dynamical string wall and supercritical region, and the established potential method for the holographic Schwinger effect. The most distinctive new content is the simultaneous numerical tracking of the wall, E_s, E_c, the connected-string branches, and the mismatch between thermodynamic and string criteria as the boost velocity is varied. If made internally consistent, that synthesis could be suitable for a specialist journal such as JHEP or Physical Review D.

The novelty should nevertheless be stated more narrowly. The exact thermodynamic rescaling is expected from the boost and is not an independent dynamical prediction. Likewise, the surface $G = v^2$ is the familiar critical surface encountered for moving probes and "hot wind" Wilson loops, a literature that the manuscript cites only after introducing it as a new rotational effect. The setup has no rotational backreaction, depends only on the product ωl, and predicts a transition-temperature trend opposite to current rotating-lattice results. The publishable contribution would therefore be a model-specific application and comparison of probes, not a new universal mechanism of rotating QCD. A revised paper should compare its configuration explicitly with the earlier moving-quarkonium and rotating-Schwinger calculations cited in the Introduction and identify which numerical observations are genuinely new relative to the authors' static study.

Recommendation

Major revision. The project is potentially publishable, but the background and probe calculations must first be made mutually consistent. The sign errors in equations (6) and (8), the charge-density problem in equations (32)–(33), and the DBI determinant underlying equation (56) can change the numerical results. The authors must also decide whether the observable is defined for a laboratory-stationary dipole or a co-rotating dipole, since only the former has the critical surface in equation (53). Publication should await a rederived background, a complete thermodynamic ensemble and boundary stress tensor, a frame-consistent probe analysis, recomputed plots with documented numerical inputs, and a substantially more limited interpretation of the results.

Major comments

1. **The displayed EMD background does not solve one consistent set of field equations.**

   For the action (1) and metric (2), the Maxwell equation reduces to

$$\partial_z \left(\frac{e^{A_e(z)} f(z)}{z} A_t'(z) \right) = 0.$$

   The gauge kinetic function printed in equation (6), $f = e^{-cz^2 - A_e}$, would therefore require $A_t' \propto z e^{+cz^2}$. It is incompatible with equation (7), for which $A_t' \propto z e^{-cz^2}$. Equation (7) instead requires $f = e^{+cz^2 - A_e}$. The same positive sign in the exponent of f is also the one consistent with the e^{-cx^2} weight used later. The authors must correct the model definition and verify the dilaton solution and reconstructed potential with the corrected convention.

There is a second, independent sign problem in equation (8). Define

$$I(z) = \int_0^z dx\, x^3 e^{-3A_e(x)}, \qquad J(z) = \int_0^z dx\, x^3 e^{-3A_e(x)-cx^2},$$

and $K = 2c\mu_0^2/(1-e^{-cz_h^2})^2$. With equation (7) and the corrected kinetic function, the charged solution of the Einstein equation has the structure

$$G(z) = 1 - \frac{I(z)}{I(z_h)} + K\left[\frac{J(z_h)I(z)}{I(z_h)} - J(z)\right].$$

The charge term printed in equation (8) is the negative of this expression. Moreover, differentiating the corrected expression at the horizon gives the charge-dependent sign displayed in the Hawking temperature (24), whereas differentiating equation (8) as printed gives the opposite sign. Thus equations (8) and (24) cannot both describe the same metric. Since $G(z)$ enters every thermodynamic curve, every infrared-wall calculation, and the condition (53), the manuscript must state which expression was used in the numerics and regenerate all affected figures from a verified solution. A direct residual check of the Maxwell, Einstein, and scalar equations should be included.

2. **The string is not co-rotating, and the main "new rotational" effect is a relative-velocity effect.**

The manuscript correctly identifies the horizon generator as $\chi = \partial_t - \omega\partial_\theta$ in equation (19). A co-rotating endpoint must therefore follow $\theta = -\omega t + \text{const.}$. The embedding (41), by contrast, holds θ fixed and has tangent ∂_t. Transforming back with equation (11) shows that this is a transverse dipole moving at speed $v = \omega l$ through the static plasma. It is not the co-rotating pair asserted in Section 4.1.

This is not a semantic distinction. For the fixed-θ embedding,

$$g_{\tau\tau}^{\text{ws}} = g_{tt} = -H_s\gamma^2(G - v^2),$$

which yields equation (53). For the actual co-rotating embedding, the temporal tangent is χ and

$$g(\chi,\chi) = -\frac{H_s G}{\gamma^2}.$$

The additional zero at $G = v^2$ then disappears; the only horizon is at $G = 0$, and the existence condition for the string-frame wall is the static one up to an overall factor. Thus the non-scaling wall line in Figures 13–15 and the worldsheet mechanism in Figures 20–21 arise from a laboratory-stationary probe moving relative to the fluid, not from a co-rotating equilibrium observable.

The authors should define the intended frame at the start of Section 4. If the laboratory-stationary probe is intended, the paper should be reformulated as a plasma-wind calculation and compared quantitatively with the cited moving-dipole literature. If a co-rotating equilibrium pair is intended, Sections 4.1–4.3 must be recomputed with a $\theta(\tau)$ embedding. It would be especially informative to present both configurations. The claim in Section 2.2 that ∂_t remains timelike everywhere outside the horizon is also directly false: equation (52) shows that it becomes null at z_* and spacelike between z_* and z_h. This surface is first a bulk stationary-limit surface for ∂_t; calling it a worldsheet horizon requires specifying an embedding that actually reaches it and demonstrating the induced causal structure.

3. **The gauge source and baryon density in equations (32)–(33) are algebraically and thermodynamically inconsistent.**

   Even before the boost, direct expansion of equation (7) gives

   $$A_t(z) = \mu_0 - \frac{c\mu_0}{1 - e^{-cz_h^2}} z^2 + O(z^4) = \mu_0 - \frac{c\mu_0 e^{cz_h^2}}{e^{cz_h^2} - 1} z^2 + O(z^4).$$

   Equation (33) is missing the factor $e^{cz_h^2}$. After the boost there is a further problem: the one-form is

   $$A = \gamma A_t(z)(dt + \omega l^2 d\theta).$$

   The text below equation (20) omits the factor l^2 in A_θ, even though that factor is required to obtain $\mu = \mu_0/\gamma$ in equation (21). The temporal component at the boundary is $\gamma\mu_0$, not the physical chemical potential μ, which is the potential along χ. It is therefore incorrect simply to write the boosted temporal component as $A_t = \mu - \rho z^2 + \cdots$ in equation (32).

   The charge density should be derived from the renormalized canonical Maxwell flux, including its normalization and the transformation of the boundary current. As a basic consistency check, equation (38) implies $\rho = \gamma\rho_0(\gamma T, \gamma\mu)$ for a density per fixed physical volume. Equation (33) has neither this factor nor the correct static coefficient. This affects the susceptibility in equation (35), the energy in equation (29), and all finite-density response plots. The revised paper should demonstrate explicitly that the bulk current equals $-\partial\Omega/\partial\mu$ and that the mixed T, μ derivatives of the proposed grand potential agree.

4. **The rotating thermodynamic ensemble and stress tensor are incomplete.**

   A boosted compact black brane carries momentum, interpreted here as angular momentum. Its grand potential obeys

   $$d\Omega = -s\, dT - \rho\, d\mu - \mathcal{J}\, d\omega,$$

   where $\mathcal{J}$ is the charge conjugate to ω. Restricting this identity to fixed ω gives equation (26), but treating ω as an external parameter does not remove $\omega\mathcal{J}$ from the Legendre relation for the energy. Equation (29) is therefore not the laboratory-frame energy unless a nonstandard definition is being used. The boosted state also has an off-diagonal momentum density and unequal longitudinal and transverse stress components. Consequently, a single pressure $p = -\Omega$ and the trace formula $I = \epsilon - 3p$ in equations (28)–(30) require a precise comoving convention; they do not follow for the coordinate-frame stress tensor used elsewhere.

   The correct way to settle these questions is to compute the renormalized Brown–York stress tensor, Maxwell current, Euclidean on-shell action, and angular-momentum density in one convention, and verify the quantum statistical relation and first law. The entropy density (25) contains an explicit factor of l, while the Stefan–Boltzmann normalization and equation (38) do not state whether densities are per unit $d\theta$ or per unit physical length $l\, d\theta$. This volume convention must be fixed before comparing thermodynamic densities.

   Equation (38) is currently introduced by saying that the numerical results “indicate” it, but it is then called exact and universal. Integrating entropy as in equation (27) determines Ω only up to a function of μ and ω, not a universal constant. The reference $z_h \to \infty$ is explained only at $\mu = 0$. A first-principles derivation must address this integration function and the compact volume. At minimum, the paper should provide collapse plots and residuals for the claimed scaling. The locations of branch crossings may still inherit a simple boost mapping, but that fact alone does not establish the full thermodynamics claimed in Sections 3.1–3.3.

5. **The DBI threshold in equation (56) does not include the off-diagonal metric correctly for the standard probe D3-brane.**

   A D3-brane at fixed z_0 in this boundary geometry normally spans (t, x_1, x_2, θ). With an electric field along x_1, the determinant of $g + 2\pi\alpha' F$ contains the $g_{t\theta}$ term. Its reality bound is

$$\frac{E_c^2}{T_F^2} = -g^s_{x_1x_1}\left(g^s_{tt} - \frac{(g^s_{t\theta})^2}{g^s_{\theta\theta}}\right) = H_s(z_0)^2 N(z_0),$$

   not $H_s^2(N - RP^2) = k$, as used in equation (56). The printed result would follow from discarding the compact worldvolume direction, but then the object would not be the standard space-filling probe D3-brane and its missing fourth worldvolume direction would have to be specified.

   The manuscript should write the full induced brane metric and DBI determinant, state whether the brane and electric field are stationary or co-rotating, and derive the critical field in that frame. A transverse electric field also mixes with a magnetic field under the inverse boost, so comparisons at fixed E need a frame convention. Figure 17 and all statements about the discontinuities and observational implications of E_c must be recomputed after this issue is resolved. In particular, when the manuscript's z_* reaches z_0, one has $G(z_0) = v^2$ and hence $N(z_0) = v^2/(1 + v^2)$, which is finite. The vanishing of the plotted E_c at that point is therefore an artifact of using $k(z_0)$, not the standard filling-D3 DBI threshold. The barrier calculation should then use the same field and brane conventions as the DBI bound.

6. **The connected-string analysis does not establish the physical screening configuration or its free energy.**

   The manuscript labels one of the two connected branches in Figures 18–19 as energetically favored, but it never displays a regulated disconnected-string free energy or a connected/disconnected crossing. At finite temperature, the connected solution generally ceases to be globally preferred at a screening length smaller than the geometric $L_{\max}$. Thus $L_{\max}$ is not, without an energy comparison, the largest separation at which a connected pair is stable. Comparing only the two connected roots of equation (45) is insufficient.

   For the fixed-θ probe there is an additional complication. A straight static string cannot be continued through z_*, because its Nambu–Goto determinant changes sign there. The isolated-quark solution in a plasma wind is normally a trailing string with a nonzero profile in the boost direction and momentum flux. The assertion after Figure 18 that the physical configuration beyond $L_{\max}$ is simply two straight strings extending into the bulk is therefore incompatible with the same reality condition used to define equation (53). The authors should construct the appropriate disconnected or trailing solutions, renormalize both actions in a common scheme, find their crossing, and distinguish geometric existence, worldsheet stability, and global thermodynamic preference. This is also needed to support equation (58), which is presently asserted from a schematic plot without a quantitative limiting analysis.

7. **The physical domain and all numerical inputs must be specified before the figures are reproducible.**

   Every rotational result depends on $v = \omega l$, but the manuscript never assigns a value or unit to l. The phase curve in Figure 10 reaches $T = 0$ at $\omega = 1\,\mathrm{GeV}$, strongly suggesting an unstated choice $l = 1\,\mathrm{GeV}^{-1}$. That endpoint is $v = 1$, where γ diverges and the construction violates its own strict condition $v < 1$; it may only be approached as a limit. Results should primarily be shown versus the dimensionless velocity v, with the conversion to ω given for several physically motivated radii. The use of $\omega = 0.3$–$0.9\,\mathrm{GeV}$ also needs to be reconciled with the tens-of-MeV scale invoked in the Introduction.

The string results additionally require T_F (or α'), the probe position z_0, and the map between z_0 and the quark mass. None is specified for Figures 16–20; $z_0 = 0.5\,\mathrm{GeV}^{-1}$ first appears in the caption of Figure 21, and no numerical value of T_F is given. The dilaton profile $\phi(z)$, which is essential to $A_s = A_e + \phi/\sqrt{6}$ and hence to every string result, is also absent.

A revised numerical section should state the integration grids, precision, root-finding and near-turning-point regulators, branch-selection rules, grand-potential integration paths, and criteria used for the CEP, response extrema, loss of the infrared wall, $z_c^{\max}$, and energetic crossings. Representative tables of phase coordinates, scaling residuals, critical fields, and $L_{\max}$ should accompany the plots. Without these inputs, the many smooth curves cannot be independently checked.

8. **Several physical identifications require qualification.**

   Both thermodynamic phases discussed in Section 3 are black-hole branches with entropy of order N_c^2. A swallow-tail transition between them is a small-/large-black-hole thermodynamic transition; it is not, by itself, a confinement order parameter. The string wall is the actual Wilson-loop-like criterion studied here. The phrase "thermodynamically confined but string deconfined" should therefore be replaced by a neutral description such as "the low-temperature thermodynamic branch without a string infrared wall", unless an independent confinement order parameter is supplied. This is especially important because Section 2.1 states that the reconstructed potential $V(\phi)$ inherits thermodynamic-parameter dependence. The authors must explain how states at different (T, μ) nevertheless solve one fixed bulk theory, or quantify the limitation of comparing their free energies.

   At finite density, equation (36), $(\partial p/\partial \epsilon)_{\mu,\omega}$, is not generally the adiabatic hydrodynamic sound speed. The physical sound channel involves the full susceptibility matrix and an appropriate fixed entropy-per-charge condition. Its use as a mechanical-stability criterion and as a pseudocritical sound line should be rederived or renamed as an isochemical thermodynamic slope. Local grand-canonical stability likewise requires the appropriate Hessian, not only the signs of individual plotted derivatives.

   Finally, Section 4.3 says that E_s vanishes simultaneously with the infrared wall, whereas Section 4.2 says the wall disappears abruptly at a finite z_{IR}. In the latter situation $T_F \sigma_{\mathrm{eff}}(z_{\mathrm{IR}})$ remains finite up to the boundary of its domain and then ceases to be defined; it does not continuously tend to zero. The authors should show the limiting behavior and use precise language. Potential analysis establishes a change in a static barrier, not a pair-production rate, so the claims about improving experimental prospects in heavy-ion collisions should also be reduced unless an actual rate and a phenomenologically calibrated parameter set are provided.

9. **The global and geometrical meaning of the boost construction needs a sharper statement.**

   Boosting a noncompact planar direction is locally a coordinate transformation. After compactification, choosing the new θ to be periodic defines a different global quotient and a rotating grand-canonical ensemble. The paper should state the resulting identifications, the boundary metric, the thermal identification, and which time generator defines energy and the ensemble.

This would also clarify the normalization of χ, the gauge potential in equation (20), and the angular-momentum term discussed above.

The claimed fixed-radius interpretation is heuristic. The boundary is $\mathbb{R}^2 \times S^1$, not a full cylindrical geometry with a radial coordinate, and the two noncompact directions remain equivalent in the metric. Identifying x_1 uniquely as the physical rotation axis while discarding a radial direction but retaining x_2 should be explained. At present, the construction is more precisely a boosted compact plasma than a spatially resolved rotating QGP. This limitation should be reflected in the title, abstract, and phenomenological conclusions.

Minor comments

1. The Introduction contains a near-duplicate paragraph beginning "This qualitative discrepancy"; the second occurrence also uses literal "[33, 34]" rather than citation commands. It should be removed or merged.

2. At $\mu = 0$, the locations in z_h of the extrema of $T(z_h)$ are exactly, not merely "nearly", independent of ω, provided equation (23) is the only rotational dependence. This is another useful analytic check of the implementation.

3. The normalization of the coefficient called ρ in equation (32) should include the factors from the Maxwell action, the compact volume, and the chosen normalization by N_c^2. A coefficient in a near-boundary series is not automatically the physical baryon density.

4. The orientation discussion in Section 4.1 alternates among directions "perpendicular to the compactified circle", "parallel to the rotation axis", and a fixed-radius slice. A diagram and a coordinate table would make the intended boundary geometry and probe orientation unambiguous.

5. Figure 15 contains one enlarged plot, although its caption refers to "views". The text describes shaded mismatch regions, but no shading or legend mapping the three curves to angular velocities is visible. Figure 12 has the typo "obtaind". Several captions should also state the missing fixed parameters rather than requiring the reader to infer them from surrounding text.

6. The discussion of Figure 19 says that $E = 5\,\mathrm{GeV}^2$ lies in (E_s, E_c) for all displayed backgrounds, but the same paragraph and caption identify the $\mu = 0.4$, $\omega = 0.6\,\mathrm{GeV}$ curve as string-deconfined, where E_s is undefined. The legend of Figure 21 also labels the red curve by $\mu = 0.28$, whereas its caption and text specify $T = 0.28$, $\mu = 0.4\,\mathrm{GeV}$.

7. The placement of deferred floats substantially interrupts the logical flow. In the compiled manuscript, Figures 10–12 from Section 3 occur after Section 4 has begun and are interleaved with equations (41)–(50). A float barrier or revised placement is needed at the section boundary.

8. The bibliography contains corrupted typography in the title of the reference labeled `Sch8`. The rotating-Schwinger references should also be discussed individually in the text rather than cited as a group when the paper's novelty is defined.