Referee Report

Enhanced Correlations in Hawking Radiation from Near-Extremal Collapse

arXiv:2607.12989

Summary

The manuscript studies quantum-gravitational corrections to Hawking radiation from a near-extremal Reissner–Nordström black hole formed by charged-shell collapse. After restricting four-dimensional Einstein–Maxwell theory coupled to a scalar to the spherically symmetric sector, the authors impose the gauge $R = r$, $\pi_L = 0$ and solve the gravitational constraints. This produces the scalar-only phase-space action (3.18), which is local in Schwarzschild time and nonlocal in the radial coordinate. Its near-extremal, near-horizon limit is equation (3.25), controlled by

$$g = \frac{G}{\pi r_0^3 T_H},$$

as stated in equation (3.26). The intended perturbative regime is the window (1.4), $G/r_0^3 \ll T_H \ll 1/r_0$.

The authors then use the near-horizon action to compute thermal AdS_2 boundary correlators. After integrating out the scalar momentum to first order in g, they obtain the stress-tensor interaction (4.10). The connected mixed four-point function is written as Schwarzian exchange in equations (4.24)–(4.32) and gives the position-space answer (4.33). A one-loop scalar two-point function is obtained in equations (4.36)–(4.41), after adopting a prescription in which induced quadratic operators from equal-time momentum contractions are set to zero. These answers agree with the known Schwarzian dressed-bilocal correlators.

Section 5 gives a useful formal explanation of this agreement. The authors construct metric functions N and L from the Hamiltonian variables, show that the resulting on-shell two-dimensional metric has $R = -2$, recover the Schwarzian from the boundary extrinsic curvature, and argue that the Hamiltonian and Schwarzian-plus-bilocal formulations obey the same source-functional equations. The action (5.42) is thereby claimed to generate the same separated-point tree-level boundary correlators in both descriptions.

Finally, Section 6 applies the AdS_2 results to late-time radiation. The collapse coordinate map (2.20), with asymptotic form (2.22), reproduces the free Hawking two-point function. Approximate reflection at the outer edge of the throat is then used to replace the late-time near-horizon state by a thermal one. The resulting connected four-point function (6.21) is interpreted as a correlated temperature fluctuation through equation (6.23), while the continued one-loop two-point function (6.24) gives the flux correction (6.25). Under an additional assumption about the flux–energy relation, the latter is converted into the mass and partition-function corrections (6.30)–(6.31).

The work has a strong and potentially publishable core. In particular, the constraint derivation provides an illuminating Lorentzian starting point, the all-tree-level argument of Section 5 is conceptually valuable, and the infrared statement that correlations become enhanced at separations of order T_H^{-1} is physically interesting. Several steps central to the advertised asymptotic-radiation and one-loop conclusions are not yet under adequate control, however. The most important concerns are the omission of the parametrically small greybody transmission from the displayed null-infinity observables, the replacement of the collapse state by an equilibrium correlator, and the regulator and measure dependence of the one-loop calculation.

Evaluation of correctness and logical structure

The classical and tree-level parts form a coherent chain. The near-extremal scaling in Section 2 is standard, the constraint solution leading to (3.18) is plausible within the stated spherical truncation, and the appearance of the coupling g in (3.25)–(3.26) has the expected thermodynamic scaling. The explicit agreement between (4.33) and the Schwarzian dressed-bilocal answer is a nontrivial check. I also find the derivation in Sections 5.1–5.3, especially the passage from (5.10) to (5.20) and the recovery of the boundary Schwarzian in (5.28)–(5.29), to be one of the clearest and most interesting parts of the manuscript.

The logical bridge to the principal physical claim is less complete. Section 6.2 first writes the correct type of Schwinger–Keldysh evolution for the collapse problem, but the displayed answers are obtained only after restricting all interactions to the throat, replacing the state by a thermal density matrix, evaluating a Euclidean AdS_2 correlator, analytically continuing it, and propagating it through the exterior with the transmission coefficient temporarily set to one. Each step is physically motivated, but their combined error is not estimated. Moreover, Appendix C shows that the actual low-frequency transmission is small, not close to unity. Consequently, equations (6.21), (6.24), and (6.25) are not yet complete correlation functions or fluxes at future null infinity.

The loop-level claims require still more care. Section 4.1 explicitly encounters induced quadratic operators, denotes them collectively by $\mathcal{O}_2$, and removes them by a dimensional-regularization prescription. The subsequent coincidence limit used for the stress tensor is a composite-operator limit and can be affected by precisely such local terms. The manuscript is commendably candid in Section 7.1 that its Hamiltonian theory is not UV complete, but the current text nevertheless presents the coefficient in (6.25) and the thermodynamic inference (6.30)–(6.31) too definitively. A complete renormalization prescription and an independent justification of the flux–energy step are needed before those conclusions can be treated as derived results.

Novelty and significance

The paper addresses a timely question and should be of interest to the near-AdS_2, black-hole evaporation, and quantum-gravity communities. Its most important original contributions appear to be the source-functional equivalence established in Section 5 and the attempt to express enhanced multi-time correlations of the outgoing radiation directly in a collapse framework. These are more distinctive than the coupling g, the Schwarzian thermal correlators, the Lyapunov exponent, or the $\frac{3}{2}\log T_H$ correction, all of which have substantial antecedents.

The novelty should be delineated much more precisely. The scalar Hamiltonian and the initial in-in program already appear in the cited precursor arXiv:2509.04293. The present manuscript seems to complete that program by supplying the four-point function, a revised one-loop treatment, the all-tree-level equivalence, and the late-time map toward null-infinity observables. A direct comparison is necessary, including an explanation of how the approximately reflected thermal ingoing sector changes the earlier outgoing-only treatment. The paper should also distinguish its connected multi-time observables from recent Schwarzian evaporation studies, notably arXiv:2411.03447 and the other works cited in the cluster [14–19], which already incorporate metric fluctuations and greybody factors in inclusive emission spectra.

If the central qualifications below are addressed, I believe the Hamiltonian perspective and the explicit correlation observables would provide a useful journal contribution. As written, however, the strongest claims exceed what has actually been demonstrated.

Recommendation

Major revision. I would be willing to reconsider a substantially revised version. The tree-level framework is promising and the main physical effect is plausible, but the authors should complete or sharply qualify the null-infinity map, establish the status of the loop renormalization, and separate conditional consistency checks from independently derived results.

Major comments

1. **The displayed results are not yet the physical correlators at future null infinity.** Section 6.1 explains that one transmission factor must multiply each outgoing external leg, but Section 6.3 sets that factor to one. Appendix C then finds

   $$\mathcal{T}(\omega) = -2i\omega r_0 + \mathcal{O}((\omega r_0)^2)$$

   in equation (C.8). Thus, for the thermally relevant frequencies $\omega \sim T_H$, the transmission is parametrically small in the very regime under study. It is not a harmless order-one normalization for an absolute asymptotic correlator: every external leg changes the scaling and the time profile. The statement that the relative order-g enhancement survives may be correct, but it is not equivalent to having computed the observable advertised in the title.

   The authors should write the frequency-space version of the connected four-point and one-loop two-point functions with the appropriate product of transmission amplitudes, then give at least their leading low-frequency form at $\mathcal{I}^+$. The same exercise is required for the flux. In particular, the temperature dependence of $\mathcal{T}(\omega)$ modifies the derivative relation (6.23), as the text already briefly acknowledges. Equations (6.21), (6.24), and (6.25) should either be replaced by the greybody-filtered answers or be labeled explicitly as transparent-boundary/throat correlators rather than null-infinity results.

2. **The reduction from the collapse in-in problem to a thermal Euclidean correlator needs controlled justification.** Equation (6.19) is a genuine collapse Schwinger–Keldysh expression, but it is not evaluated. Instead, the interactions are restricted to the near-horizon region, the late-time state is taken to be thermal for both ingoing and outgoing modes, and the answer is imported from Section 4 by Euclidean continuation. Approximate reflection explains the free ingoing two-point function, but by itself does not establish that the interacting state or every required contour-ordered correlator is thermal.

   There is also an ensemble issue at exactly the retained order. The collapse problem starts from a pure state with specified shell data and apparently fixed ADM mass and charge, whereas the Euclidean calculation is canonical at fixed T_H. Equation (6.23) identifies the displayed connected four-point function with canonical temperature fluctuations proportional to $1/C_Q$, which is itself order g. Conditioning on a sharp energy modifies or removes precisely such a covariance. The authors must specify the energy distribution of the shell/black-hole state and either justify the canonical ensemble for the collapse observable or derive the corresponding fixed-energy Schwinger–Keldysh correlator and its relation to (6.21).

Please state a hierarchy of time and frequency scales and estimate the errors in powers of $T_H r_0$, g, and $e^{-2\pi T_H u}$, together with the adiabatic error from the slow evaporation. The contour ordering and $i\epsilon$ prescription of (6.21) and (6.24) should be explicit; a time-ordered (VVWW) correlator is not interchangeable with every Wightman or measurement correlator. It would also be valuable to explain directly how the reflected thermal component alters or cancels the nonequilibrium one-loop behavior found in the precursor arXiv:2509.04293. Unless a direct real-time derivation is supplied, claims that the explicit $\frac{3}{2}$ result is obtained by a Lorentzian calculation should be revised: the framework is Lorentzian, while the displayed late-time thermal functions are evaluated in Euclidean signature and analytically continued.

3. **The one-loop answer depends on an incompletely specified measure and renormalization prescription.** The reduction to (3.18) is presented as formally exact at the quantum-mechanical level, and (4.3) uses a flat phase-space measure. The paper should show how the Faddeev–Popov determinant and the Jacobian from solving the constraints reduce to this measure, including any horizon or radial-boundary contribution. This is especially important because the Schwarzian comparison attributes physical information to its symplectic measure, whereas Section 7.1 emphasizes that no analogous loop-level definition has yet been derived here.

   Separately, setting the induced $\mathcal{O}_2$ operators to zero needs more than the statement that an equal-time $\delta(0)$ is scale-free. The thermal circle, AdS boundary, and nonlocal radial kernel introduce physical structure, and local boundary or bulk counterterms can affect a two-point function and its coincidence limit. The authors should list the allowed order-g quadratic counterterms, specify renormalization conditions, and show which part of (4.41) is scheme independent at separated points. This is visible directly in (4.36): after summing over the internal positive mode, the Fourier coefficient approaches a nonzero constant at large external mode, so the formal series contains a contact distribution. The noncoincident continuation (4.41) does not by itself fix that contact term. For (6.25), the renormalized stress tensor and its subtraction should be defined as explicitly as in equations (6.6)–(6.7). If the coefficient is prescription dependent, the claim should be restricted accordingly.

4. **Equations (6.30)–(6.31) are a conditional consistency check, not an independent derivation of one-loop thermodynamics.** The inference starts by assuming that the tree-level relation (6.26) between flux and energy remains valid after the quantum mass shift. The manuscript itself notes after (6.31) that this assumption needs justification. At the same order, however, quantum corrections can change the relation between mass, temperature, near-horizon flux, and the flux transmitted to infinity. The greybody issue in the first comment makes this particularly concrete.

   To claim a derivation of $\delta E = \frac{3}{2} T_H$ and $\delta \log Z = \frac{3}{2} \log T_H$, the authors need an independent Ward identity, first-law argument, or source-functional calculation showing why (6.26) retains the required form at one loop. Otherwise, these equations should be presented as a nontrivial match obtained after imposing the known Schwarzian thermodynamic relation, and the abstract, equation (1.3), and surrounding text should be softened. The comparison should also include the near-extremal four-dimensional thermodynamics literature rather than citing only the formal Schwarzian result.

5. **The moving shell and radial boundary terms need to be included in the Hamiltonian derivation.** In the collapse geometry the shell position is $r_s(t)$, yet equations (3.15), (3.17), (3.18), and (3.20) display a fixed lower integration limit and no moving-boundary or corner term. Treating independent shell fluctuations as WKB suppressed does not by itself show that the classical motion of this boundary, its junction conditions, and its canonical contribution can be omitted after the exterior constraints are solved.

   The authors should derive the reduced action with a time-dependent lower limit and show which surface terms cancel, or explain precisely why a late-time coordinate choice makes the displayed action sufficient for the observables considered. Relatedly, the gauge is adapted to Schwarzschild time and does not cross the future horizon. Figure 5 and the discussion around (6.16)–(6.19) should make clear that the chosen slices and their shell segments provide the needed evolution domain. This issue bears directly on the claim that the Hamiltonian gives a uniform description of the collapse spacetime, even if the stationary AdS_2 calculation itself is unaffected.

6. **The scope and uniqueness of the all-tree-level equivalence should be stated more precisely.** Section 5 treats a free scalar with $m^2 = \Delta(\Delta - 1)$, separated boundary points, a particular asymptotic branch, and a fixed AdS_2 saddle. Equality of the differential relations (5.39)–(5.41) implies equality of generating functionals only after boundary/initial conditions, integration constants, the $SL(2, \mathbb{R})$ redundancy, and local contact terms have been matched.

   There is also a concrete counterterm problem in Appendix B. From (B.7), the leading term in $F\phi'\delta\phi$ is $(\Delta - 1)r_b^{2\Delta-1}\chi\delta\chi$, not the $(\Delta - 1)^2$ coefficient printed in (B.8). Moreover, (B.9) is not the full cutoff-local counterterm and does not cancel this power divergence. A regulated term such as $-(\Delta - 1)r_b\phi(r_b)^2/2$, supplemented as required in special or logarithmic cases, cancels both the divergent variation and the unwanted $\chi\delta\phi_\Delta$ term before the finite response (B.10) is extracted. This calculation should be corrected because Appendix B supplies an input to the arbitrary mass discussion in Section 5.

   Please state those data and explain why they select the same solution in the two formulations. The conclusion at (5.42) should be phrased as equality of noncoincident tree-level correlators within this free-scalar sector, rather than as a general quantum equivalence. It would also help to identify explicitly which parts of the proof continue to hold in Lorentzian signature and for the collapse state, since Section 5 itself is an equilibrium AdS_2 argument.

7. **The approximations require a unified power-counting discussion.** The window (1.4) is nonempty for a parametrically large extremal black hole, $r_0^2/G \gg 1$. Within that window, the authors neglect higher partial waves, independent shell fluctuations, backreaction inside the shell, departures from quasi-equilibrium, and loop effects associated with degrees of freedom removed by the spherical reduction. The statement that the enhanced mode lies in the s-wave is plausible, but it does not by itself establish that every omitted contribution is smaller than the order-g effects retained, especially for the one-loop thermodynamic claim.

   A compact table or subsection should give the expansion parameters and the suppression of each omitted effect for external frequencies $\omega \sim T_H$. This should distinguish exact solution of the constraints *within the s-wave truncation* from exactness in the four-dimensional theory. It should also state the range of retarded times over which the black-hole parameters may be treated as fixed and explain why nonspherical modes do not modify the claimed $\frac{3}{2}$ coefficient at the same order.

8. **The novelty relative to prior Hamiltonian and Schwarzian evaporation work must be made explicit.** The reduced action and the low-temperature coupling build directly on arXiv:2509.04293; the thermal four-point and one-loop functions are known Schwarzian results; and recent work such as arXiv:2411.03447 already treats corrected emission spectra with metric fluctuations and greybody factors. The present paper's strongest advances seem instead to be the completed Hamiltonian derivation of the correlators, the source-functional equivalence of Section 5, and connected multi-time radiation correlations rather than inclusive spectra.

   The Introduction should contain a precise paragraph enumerating what is new beyond the precursor and how the present observable differs from the spectrum calculations in Refs. [14–19]. The known $\frac{3}{2}\log T_H$ result and Appendix D's standard Lyapunov exponent should be described as checks, not new physical predictions. This sharper positioning would strengthen the paper: the independent derivation and multi-time observables are interesting on their own and do not require broader novelty claims.

Minor comments

1. When Gauss' law is solved below equation (3.5), state explicitly that the variational principle is at fixed charge and display the electromagnetic boundary term, or explain why none is required in the chosen ensemble.

2. The notation (M) is used both for the mass parameter of the background shell and inside the quasilocal/ADM construction. Section 3 should distinguish the fixed background parameter from the total Hamiltonian once scalar excitations are present.

3. Section 4 restricts to a massless scalar in equation (4.2), whereas some surrounding statements sound general. Mark clearly which correlator and equivalence results have actually been evaluated for $\Delta = 1$ and which follow for arbitrary Δ.

4. Equation (6.21) is written for two distinguishable fields (V) and (W). Give the channel sum and combinatorial factors needed to recover the connected four-point function of the single physical scalar used in the collapse problem.

5. The use of the Weyl anomaly in equations (4.37)–(4.39) should specify the regulator, the boundary conditions, and whether boundary-anomaly terms can contribute. This would also clarify why diagram 4(b) vanishes beyond the formal Fourier argument.

6. Section 5.2 reuses (u) for the boundary proper-time coordinate even though (u) already denotes the asymptotic retarded coordinate central to Section 6. A different symbol would prevent confusion.

7. Since the normalization of the main answer is important, show explicitly between equations (6.20) and (6.21) how the four factors $\partial_{\tilde{u}} = \frac{1}{2}\partial_z$ and the coordinate rescaling $\tilde{u} = 2\pi T_H u$ combine.

8. Appendix D determines only the exponential scaling (D.19), not its coefficient. The text should consistently call this a derivation of the Lyapunov exponent rather than of the complete OTOC. In addition, equation (D.4) defines H_I with an integral over dt; a Hamiltonian should contain only the spatial integrals, with the time integral appearing in the Dyson expansion. The placement of $W_I(t_2)$ and the contour limits in (D.6) should also be corrected. After removing the time integral, the sign inherited from (4.10) appears inconsistent with the overall sign in (D.9); this does not change the reported exponent, but the contour and sign bookkeeping should be made internally consistent.

9. In Appendix E, equations (E.7)–(E.8) determine the cutoff only parametrically. Accordingly, the definite equality and coefficient in (E.10) should be replaced by a parametric relation unless a matched asymptotic calculation fixes z_c. There is also an apparent factor-of-two error: the term $Q^2/(2r^2)$ in (E.2) gives a near-horizon kernel $1/(2\sinh^2 z)$, whereas (E.5) uses $1/\sinh^2 z$. The enhanced interaction and the normalized four-point functions (E.9)–(E.17) should be rechecked accordingly.

10. The claimed normalization match in Appendix F also appears inconsistent. Using the near-horizon h stated below (F.4), the variable changes (F.5), and the $8G_3h$ exponent in (F.3) gives $-2g \int dz\, \sinh^2 z(\tilde{\pi}_\phi^2 + \tilde{\phi}'^2)$ with $g = 2G_3/r_h$, not the $-g/2$ coefficient in (F.6). The factor-of-four discrepancy among (F.3), (F.6), and (F.7) should be resolved before Appendix F is cited as an exact check of the four-dimensional normalization.

11. The statement that (6.23) is a direct temperature-fluctuation relation should specify that C_Q is the fixed-charge heat capacity and state the accuracy with which its near-extremal approximation is used.

12. The phrase “first controlled breakdown of the free Hawking state” in Section 7 should be narrowed or compared with the older self-interaction and microcanonical radiation literature. The present effect is specifically a low-frequency, long-time near-AdS$_2$ correlation, distinct from high-energy recoil and energy-conservation corrections.

13. The logarithms in equations (2.17) and (2.20) have dimensionful arguments. Insert reference scales, or state explicitly that they are absorbed into the arbitrary origin of the tortoise coordinate and hence into the constant C in (2.22).

14. Correct the phrase near the end of Section 6.3.2 referring to the “assumption stated below (6.27)”; the relevant assumption is made after (6.27). Throughout Section 6, label correlators consistently as connected, time ordered, Wightman, or contour ordered.