Referee Report

The Infraparticle Edge

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Summary

The manuscript proposes to derive the charged-particle spectral edge from an open-quantum-system description of soft QED. Hard charged data are treated as the system and photons below a scale Λ as the environment. For a hard branch h, the total unresolved energy is assigned the Laplace transform

$$\widetilde{\mathcal{P}}_h(\tau) = \exp\left[\int_0^\Lambda \mathrm{d}N_h(\omega)(e^{-\tau\omega} - 1)\right],$$

as written in equations (2) and (17). For the logarithmic intensity $\mathrm{d}N_h = \eta_h\,\mathrm{d}\omega/\omega$, equations (36)–(40) give $\mathcal{P}_h(E) \sim E^{\eta_h - 1}$. Equations (33)–(35) then relate the total soft energy to the invariant-mass excess $s - m^2$, and equation (41) introduces a hard factor $H(E)$. Assuming the regular nonzero limit (42), the manuscript obtains the central result

$$\rho_{\mathrm{inc}}(s) \sim C\,\theta(s - m^2)(s - m^2)^{-1+\eta_h}$$

in equation (43).

A second theme is the relation between energy counting and decoherence. The sesquilinear soft kernel in equations (18)–(20) gives the diagonal edge exponent κ_{aa} and the difference-current dephasing exponent $\frac{1}{2}(\kappa_{aa} + \kappa_{bb} - 2\,\mathrm{Re}\,\kappa_{ba})$, as displayed in equations (23)–(24). Later sections expand the regulated photon-number sectors, separate a proposed hard threshold exponent, compare the edge with an unparticle power law, classify several infrared intensities, and interpret the no-photon factor $(\mu/\Lambda)^{\eta_h}$ as a finite-resolution residue.

There is a clear and useful formal observation here. The coherent-state overlap in equation (20), checked mode by mode in equations (103)–(104), correctly places the diagonal energy marginal and off-diagonal dephasing in one Gram kernel. The pure logarithmic Laplace calculation in equations (36)–(40) is also correct, subject to the stated soft model. In particular,

$$\int_0^\Lambda \frac{\mathrm{d}\omega}{\omega}(e^{-\tau\omega} - 1) = \mathrm{Ei}(-\tau\Lambda) - \gamma_E - \log(\tau\Lambda),$$

so the displayed power follows for that probability law. The difficulty is not this calculation. It is the unproved identification of a conditional, conserved-current scattering law with the Källén–Lehmann spectral measure of a physical charged operator. The manuscript uses precisely the assumptions needed for that identification in equations (28), (41), and (42), rather than deriving them. A related phase-space inconsistency in equations (18), (19), and (26) also affects the definition of every κ. Consequently the probability-theory result is presently sounder than the advertised QED spectral theorem.

Evaluation of correctness and logical structure

Within a regulated leading-eikonal coherent-state model, the logical chain from the Lévy exponent to the small-energy cumulative law is convincing. Positivity of the Gram kernel gives equation (25), the real part of the overlap gives equation (23), and the endpoint change of variables preserves the power. The separation between edge and dephasing exponents is conceptually important: the former depends on an absolute soft record, while the latter depends only on the distance between two records.

The QED application, however, changes the object under study. Equations (18)–(27) use the eikonal current of a charge-conserving scattering history. Equation (28) instead invokes a vacuum-to-charged-state matrix element of an interpolating field. The manuscript does not specify a gauge-fixed field or a gauge-invariant dressed charged operator, does not include the insertion or dressing current needed by the Ward identity, and does not derive the factorization (41). This omission is decisive: the diagonal coefficient κ_{aa} is sensitive to an absolute coherent displacement and hence to the choice of charged dressing, whereas the difference current in equation (23) is insensitive to a common displacement.

The paper also moves too quickly between four distinct notions: a CP instrument, a conditional scalar probability law for a fixed hard branch, the absolute spectral weight of a two-point function, and a detector-binned distribution. These objects can share an infrared exponent under suitable hypotheses, but they are not identical. The present notation and normalization obscure the matching factors and make several claims about the Källén–Lehmann residue and finite resolution stronger than the derivation supports.

Novelty and significance

The most interesting aspect is the matrix organization in equations (20)–(24): one soft kernel yields an energy-resolved diagonal statistic and a difference-current dephasing statistic, while making clear that their exponents need not agree. This is a potentially useful way to connect infrared quantum information with threshold spectroscopy.

The individual ingredients are otherwise close to established results. Coherent soft-photon exponentiation, the Poisson photon-number expansion, real–virtual cancellation, and the near-mass-shell branch point belong to the Bloch–Nordsieck/Yennie–Frautschi–Suura tradition. Equations (68)–(75) are standard regular-variation and Lévy-process statements. The manuscript should therefore identify precisely what is new relative both to that literature and to the author's three recent precursor papers. At present, the new step seems to be the proposed map from total soft energy to a charged-field spectral density; that is also the step that has not been established.

The discussion of prior work is too narrow for the claim. In addition to classic and modern analyses of the electron propagator near its mass shell, the revision should engage the dressed-state literature of Chung, Kibble, and Kulish–Faddeev; work on physical gauge-invariant charged propagators; and the infrared-decoherence literature. These references bear directly on whether the soft cloud is traced over, included in the asymptotic state, or built into the charged operator. The unparticle comparison should also be set against the literature on deconstructed or massive unparticles, where adding a gap can introduce structure not captured by the formal replacement $P^2 \mapsto s - m^2$.

If the missing charged-operator construction and a benchmark QED calculation were supplied, the open-system packaging could be worthwhile for the high-energy theory and infrared quantum-information communities. Without them, the manuscript mainly relabels a standard compound-soft-emission power and does not yet demonstrate a new physical spectral result.

Recommendation

Reject in the present form, with encouragement to submit a substantially reconstructed version. This recommendation does not reflect a problem with the elementary Laplace asymptotics; those are largely correct. It reflects the fact that the central advertised theorem concerns a charged QED spectral function, while the derivation is presently for a different, conditionally normalized scattering observable. Repair requires choosing and defining the charged operator, restoring the complete conserved current and phase-space normalization, proving the spectral factorization, and matching at least one explicit QED example. Those changes affect the foundation and interpretation of the paper rather than a finite set of local edits.

Major comments

1. **The soft scattering current does not define the charged spectral operator used in equation (28).**

   Equation (26) and equations (105)–(108) use

   $$j_h^\mu(k) = \sum_i \eta_i e_i \frac{p_i^\mu}{p_i \cdot k}.$$

   The claim following equation (108) that current conservation removes gauge-dependent polarizations requires $k \cdot j_h = \sum_i \eta_i e_i = 0$. This holds for a complete charge-conserving scattering process. It does not hold for the literal vacuum-to-one-electron matrix element written in equation (28): the listed charged legs then give $k \cdot j_h = e$. Correspondingly, the amplitude in equation (26) changes under $\varepsilon^\mu \to \varepsilon^\mu + ck^\mu$. In the rest frame of that one outgoing electron, its contraction with a physical transverse polarization even gives $p \cdot \varepsilon = 0$, and equation (108) would yield no edge. These simple checks show that the operator insertion or its dressing carries indispensable soft-current data.

   A local charged field is not a gauge-invariant operator on the physical Hilbert space. One may study a gauge-fixed electron propagator, whose near-shell behavior can depend on the gauge, or a nonlocal gauge-invariant dressed field, whose dressing changes the soft cloud. The manuscript must choose one of these settings and state it explicitly. For a dressed field, the dressing/insertion current must be added to equation (105) and the full conserved current used to recompute κ_{aa}. For a gauge-fixed field, the gauge and the status of positivity must be stated, and the claimed exponent must be compared with the known gauge-dependent propagator result.

   This point also conflicts with the advertised hard-configuration dependence. A genuine multileg scattering current depends on all incoming and outgoing velocities and can vanish when the incoming and outgoing currents coincide. Such a coefficient is not automatically an intrinsic Källén–Lehmann exponent of a fixed electron operator. The authors should provide an operator-specific soft theorem and compute η_h in at least one fully specified example.

2. **The factorization from equation (28) to equation (41) is the main physical assertion, not a derived identity.**

   Equation (28) is an absolute sum of operator matrix elements, while equations (29)–(31) use a soft instrument normalized to unit probability for a chosen hard branch. Equation (41) then inserts the latter into the former and places everything else in $H(E)$. No derivation shows that the virtual factor, operator matching coefficient, recoil, angular correlations, spin sums, and charged dressing factorize in this way. Nor is it shown that the separation-scale dependence $\Lambda^{-\eta_h}$ in equation (40) is canceled by matching-scale dependence in H.

   The assumption

   $$H(E) = H(0) + O(E), \qquad H(0) \neq 0,$$

   in equation (42) and the related assertion $\chi_{\mathrm{e}} = 0$ in equation (56) carry the central physical burden of the paper. They should not be presented as established properties of the one-electron spectral measure without a calculation. A satisfactory revision would derive a soft factorization theorem for the specified charged operator, identify the spin/operator projection entering H, and reproduce the first near-shell logarithm at fixed order. This would also determine whether the exponent and coefficient agree with the established Bloch–Nordsieck or electron-propagator result.

   The proposed generalization in equations (55)–(57) should also be restricted to the specific multiplicative ansatz (41). If a separate hard continuum carries an independent threshold energy, its distribution generally convolves with the soft law rather than multiplying it at the same value of E. Moreover, local finiteness of the displayed density requires $\eta_h > \chi$. Neither point is addressed.

3. **Equation (28) is not a complete Källén–Lehmann definition for an electron field.**

   For a Dirac field the spectral function is matrix valued and is normally decomposed into two scalar functions multiplying $\not{P}$ and the identity, with associated positivity constraints. The unqualified absolute square in equation (28) suppresses the spinor indices and does not state which scalar projection is claimed to have the edge (43). The phase-space notation is also ambiguous: $\mathrm{d}\Phi_{e+n\gamma}(P)$ usually fixes the total momentum P, while the additional $\delta(s - P^2)$ requires one to say whether and how P is integrated.

   There is a further infrared circularity. Equation (28) uses a sharp on-shell electron state $p^2 = m^2$ as a constituent of an exact spectral sum, although the purpose of the paper is to explain why the exact charged sector has no Wigner one-particle state at that mass. A regulated Fock-space factorization may be a useful intermediate construction, but the regulator removal and the resulting representation of the charged state must be specified. The rigorous infraparticle results cited in the Introduction establish the absence of an isolated mass shell; they do not by themselves supply the positive scalar measure or the particular power asserted here.

   The paper should define the charged two-point function, its gauge or dressing, its Dirac projection, the regulated state space, and the precise spectral sum before stating the Soft-instrument edge theorem after equation (43).

4. **Equations (18), (19), and (26) have an inconsistent one-photon normalization.**

   The eikonal amplitude in equation (26) scales as $s_h \sim \omega^{-1}$. Substituting that expression literally into equation (18), which integrates $s_b^* s_a \,\mathrm{d}\Omega\,\mathrm{d}\omega$, gives $\mathrm{d}\omega/\omega^2$, not the logarithmic measure $\kappa_{ba}\,\mathrm{d}\omega/\omega$ asserted in equation (19). It also gives a dimensionful exponent.

   Equation (106) contains the missing radial phase-space factor:

   $$\mathrm{d}N_h(\omega) = \sum_\lambda \frac{\omega\,\mathrm{d}\omega\,\mathrm{d}\Omega}{2(2\pi)^3}\,|\varepsilon_\lambda \cdot j_h|^2.$$

   The authors must either put this factor into equation (18), or define s_h there as a phase-space-weighted amplitude and then modify equation (26) accordingly. The normalization and dimensions of s_h, $\mathrm{d}N_{ba}$, and every κ_{ba} should be made uniform throughout. Until this is done, the numerical edge and dephasing exponents have not been consistently defined.

5. **The scope of the quantum instrument and the infrared limit must be made precise.**

   Equations (7), (13), and (29) alternate between an instrument, an effect, and a scalar probability measure. For CP maps, normalization is the completeness relation $\int K_X^\dagger K_X = 1$; the integral of the operations is a channel, not the number one. For a fixed hard outcome, summing only soft outcomes generally gives the probability of that hard outcome unless the distribution has first been conditioned. The manuscript should define $\mathbb{P}_h(\mathrm{d}X) = \mathrm{Tr}\,\mathcal{I}[\mathrm{d}X](\rho_h)$, explain the conditioning on h, and keep that scalar law distinct from the CP map and from an absolute spectral weight.

   The approximation is also not stated sharply enough. The compound-Poisson form is a leading-soft, eikonal, coherent-displacement result for a regulated hard history. The text should state the perturbative/all-orders sense in which it is used and explain why subleading soft terms, recoil, and hard–soft correlations are integrable additions that cannot change the claimed exponent. Statements about the "exact charged sector" currently exceed this scope.

   Finally, the outcome space in equation (6) consists of finite photon configurations. For $\mathrm{d}N = \eta\,\mathrm{d}\omega/\omega$, however, the total activity is infinite as $\mu \to 0$. At finite μ, equations (47)–(49) define an ordinary finite-photon Poisson law. In the limit, every fixed-n weight vanishes, the typical configuration contains infinitely many photons accumulating at zero, and the coherent displacement is not a vector in the original photon Fock representation. The total energy does have a well-defined infinite-activity Lévy limit. The authors should construct the instrument at finite μ, prove the weak limit of the energy law, and state whether a limiting CP instrument exists in an enlarged representation. The sentence following equation (54) should be changed: the unregulated edge is carried by n growing without bound, not by finite-n sectors that retain nonzero weight.

6. **The endpoint theorem and stability claim need stronger Tauberian hypotheses.**

   For the pure logarithmic measure, the density in equation (40) is correct. More generally, positivity and Karamata's theorem give the cumulative asymptotic

   $$W(E) \sim \frac{e^{-\eta_h \gamma_E}}{\Gamma(1+\eta_h)}\left(\frac{E}{\Lambda}\right)^{\eta_h},$$

which the manuscript correctly records in equations (61) and (66). A pointwise density asymptotic does not follow from a large-τ Laplace power without an additional local Tauberian or monotone-density hypothesis. The pure logarithmic subordinator has the required density, but an arbitrary merely integrable remainder in equations (74)–(75) can contain atoms or irregular structure accumulating near zero.

The main theorem should therefore either be stated for cumulative spectral weight, or assume that the finite remainder is absolutely continuous and sufficiently regular. If the remainder is allowed to be signed after subtracting the logarithmic part, finite total variation is needed, not only the displayed integral. The theorem should also state $\eta_h > 0$; at $\eta_h = 0$ the limiting object is an atom rather than the density (43).

The Endpoint classes theorem after equation (73) is likewise too broad. Equations (72)–(73) establish the stated result for one power-singular family. They do not classify every intensity stronger than $\mathrm{d}\omega/\omega$; slowly varying modifications generate intermediate logarithmic behavior. The result should be presented as representative classes or reformulated in terms of regular variation of the integrated Lévy measure.

7. **Equation (44) is not the required fixed-order distributional expansion.**

   The equation expands a bare, dimensionful power pointwise. The normalized endpoint kernel also contains $1/\Gamma(\eta_h)$, a reference scale, a delta function, and plus distributions. Schematically, with $x = s - m^2$ and $Q = 2m\Lambda$,

$$\frac{e^{-\eta_h \gamma_E}}{\Gamma(\eta_h)} Q^{-\eta_h} \theta(x) x^{-1+\eta_h} = \delta(x) + \eta_h \left[\frac{\theta(x)\theta(Q-x)}{x}\right]_+ + O(\eta_h^2),$$

   up to matching-dependent delta terms beyond the local approximation. Equation (44) instead begins with an unregulated $1/x$, omits the delta and plus prescription, and writes the logarithm of a dimensionful variable. The correct distributional expansion is important because it is where fixed-order real–virtual cancellation should match the resummed edge. Appendix A's delta-sequence statement does not repair equation (44) as written.

   The discussion after equation (45) should also note that positive integer threshold powers generally produce logarithmic nonanalyticities in the dispersive integral. Noninteger η_h is not the only case in which the propagator lacks an analytic Taylor expansion at threshold.

8. **The finite-resolution residue conflates an infrared regulator with experimental binning.**

   Equation (77) is the no-photon probability for the pure logarithmic model after a genuine mode floor is imposed. With the regular addition in equation (74), the factor is instead

$$P_0(\mu) = \left(\frac{\mu}{\Lambda}\right)^{\eta_h} \exp\left[-\int_\mu^\Lambda \mathrm{d}N_{h,\mathrm{reg}}\right],$$

   and an actual spectral atom also contains the hard/operator normalization represented by $H(0)$. Thus $(\mu/\Lambda)^{\eta_h}$ is at most the pure-log soft part of a regulated residue, not the complete Källén–Lehmann residue.

   A detector threshold or invariant-mass resolution does not create a literal delta function in the exact spectral measure. It bins continuum weight. The corresponding operational quantity is the cumulative $W(\Delta E)$ in equation (61). Even in the pure logarithmic model it differs from equation (77) by the factor $e^{-\eta_h \gamma_E}/\Gamma(1+\eta_h)$ at leading order. A photon mass or finite box, a finite observation time, a per-photon threshold, and a total-energy bin are different regulators or measurements. Proposition 5 and the prose following equation (79) should distinguish them rather than setting all of them equal to one μ.

9. **The unparticle comparison is formal and should not carry the novelty claim.**

   Equation (58) defines $d_{\text{eff}} = 1 + \eta_h$ so that the two displayed powers agree. This is a relabeling of the endpoint exponent, not a derivation of an operator dimension, a conformal sector, or unparticle dynamics. Moreover, equation (4) is the spectral law for a scalar unparticle, while the claimed QED observable is an electron correlator with Dirac spectral structures. A fermionic unparticle comparison would have a different relation between operator dimension and spectral power.

   The analogy may remain as a short descriptive observation if it is qualified in these terms. It should not be described as a "gapped unparticle spectrum" without engaging the literature on deconstructed and massive unparticles, where a mass gap can be accompanied by additional poles or resonances. Likewise, the final claim that the classification is unique to soft QED is incorrect: logarithmic bosonic baths, vanishing overlaps, and power-law edges also occur in independent-boson and orthogonality-catastrophe problems. QED-specific content resides in the conserved soft current and its angular coefficient, which is exactly the part that still needs to be computed for the chosen charged operator.

Minor comments

1. Equation (35) should be written using the smooth inverse of the endpoint map and its Jacobian. Adding an $O(s-m^2)$ term directly to a delta distribution is not a precise distributional statement.

2. In equation (47), the scope of the ω_i integrations and the final exponential is typographically ambiguous. The product form used in equation (101) is clearer and should be adopted.

3. Equation (67) must require $a > 0$ and $a \neq 1$. At $a = 1$ the displayed ratio is $0/0$.

4. The soft cutoff Λ and the condition $\omega < \Lambda$ are frame dependent. For a claimed Lorentz-invariant spectral exponent, the text should explain why changing the slicing changes only finite matching terms. This statement is plausible for a complete conserved current but not for the one-leg current currently used in the spectral application.

5. The soft model allows arbitrarily large total energy through sufficiently many photons even though each photon is below Λ. Its interpretation should be restricted explicitly to endpoint asymptotics; it is not a global model of the full QED spectral density.

6. Appendix A should specify a compactly supported or sufficiently decaying test function in the delta-sequence limit and retain the reference scale needed on dimensional grounds.

7. Section 8 repeats the sentence that QED lies in the logarithmic class. The repetition can be removed. More generally, substantial material in the appendices duplicates the main text; that space would be better used for the missing operator matching and explicit QED benchmark.

8. The bibliography is insufficient for the scope of the claims. In particular, the paper needs direct references on the electron propagator branch point and its gauge dependence, physical/dressed charged propagators, dressed asymptotic states, infrared decoherence, Lévy–Khintchine and Karamata theory, and massive or deconstructed unparticles.

9. The arXiv strings in references [7]–[9] are malformed ("arXiv://" or "arXiv:/"). They should use the standard `arXiv:YYMM.NNNNN [category]` form. The quotation marks in references [8]–[9] and the bibliographic details of the 1987 Buchholz contribution should also be corrected.

10. The final sentence promising immediate applications to diverse multiscale phenomena is unsupported. It should be replaced by one or two concrete observables for which both the energy marginal and the dephasing kernel can actually be evaluated.