Referee Report

Out-of-time-ordered Correlators in de Sitter Revisited

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Summary

The manuscript studies scalar four-point out-of-time-ordered correlators in d-dimensional de Sitter space by adapting the gravitational eikonal construction to the static-patch horizons. The first central observation is that the transverse shock equation contains the operator $\Delta_{S^{d-2}} + (d-2)$. Its kernel is the $L = 1$ harmonic sector, so the localized source in equation (2.14) has no inverse unless its $L = 1$ projection is removed. The authors then derive the generalized-ladder exponentiation (3.14) and express the eikonal phase as the integrated Feynman graviton propagator in equation (3.23).

The proposed interpretation of the propagator is as follows. A gauge-dependent vector contribution has the logarithmically divergent large-distance behavior shown in equation (4.14), proportional to $\alpha \cos\theta/(u\bar{v})$. After the $L = 1$ sector is omitted, the remaining phase is the transverse Green function in equation (4.15). Its $L = 0$ eigenvalue has the sign associated with a de Sitter time advance. The manuscript further attributes this $L = 0$ term to the non-decaying four-derivative contribution (4.21), and hence to a large diffeomorphism rather than to the local physical tensor structure alone.

To lift the $L = 1$ zero mode, the authors add the mass-like term in equation (4.31). Equations (4.32)–(4.33) then isolate an $L = 1$ phase proportional to

$$\frac{G_N}{m_g^2}\, p^u p^v\, y \cdot \bar{y}.$$

The horizon wavefunctions are evaluated for principal- and complementary-series scalar masses, with the compact Bessel-function result given in equation (5.11). For insertions at a pole, angular integration kills odd powers of $y \cdot \bar{y}$, so the series (5.27)–(5.28) begins at order

$$\left(\frac{G_N}{m_g^2} e^{t/\ell}\right)^2.$$

This is interpreted as a Lyapunov exponent $2/\ell = 4\pi/\beta_{\mathrm{dS}}$. The general-position formula (5.45) recasts the same exchange as $\exp[\mathfrak{a}\, \partial_{\vec{q}_{13}} \cdot \partial_{\vec{q}_{24}}]$ acting on two de Sitter two-point functions; equation (5.47) makes the pole selection rule explicit.

Finally, the authors return to strictly massless gravity and retain only the tree-level $L \neq 1$ phase. Uniform pole wavefunctions select its $L = 0$ component. Equations (5.66)–(5.67) give a positive correction on the quarter-thermal-circle contour, proportional to $G_N e^{t/\ell}$. This has the usual maximal boost exponent $1/\ell = 2\pi/\beta_{\mathrm{dS}}$, but the sign is opposite to the familiar decay of a regularized thermal OTOC.

There is substantial and potentially useful work here. The transverse $L = 1$ obstruction is correctly identified, the pole selection rule is correct, the dimension-shifting identity used in equation (5.52) has the stated normalization, and the angular integral in equation (5.62) correctly selects $L = 0$. Given the contour adopted in the paper, the sign of the tree-level term (5.66) also follows: the

two momentum integrals become positive Euclidean integrals while $ie^{t_{41}/\ell}$ is positive. The compact general-position result (5.45) and the extension of the horizon wavefunctions to general d and positive scalar mass are technically interesting.

The principal difficulty is that the most striking conclusions do not yet follow for a physical, gauge-invariant de Sitter observable. The claimed loop divergence is explicitly gauge-parameter dependent, the doubled exponent is obtained after a gauge zero mode is lifted by a non-Fierz–Pauli deformation, and the observer dressing invoked to define the observable is not included in the calculation. In addition, the massive extension of the invariant propagator in Appendix F does not follow from the displayed tensor equations. These are foundational issues rather than matters of presentation.

Evaluation of correctness and logical structure

The massless tree-level chain is the strongest part of the paper. Projecting the compact transverse equation away from $L = 1$, selecting the uniform $L = 0$ component with pole wavefunctions, and carrying out the momentum integrals gives a coherent derivation of (5.66). This result is worth developing. Its physical status is nevertheless conditional because the term is said to arise from a large diffeomorphism and because the calculated external operators are not the dressed relational operators described in the Introduction.

The argument for a genuine loop obstruction is substantially less complete. Equation (4.14) is proportional to the gauge weight α, and the text allows a correlated $0 \times \infty$ limit that assigns its coefficient an arbitrary finite value. Exponentiating this term within generalized ladders shows that this particular truncated set of diagrams is ill-defined. It does not show that a BRST-invariant four-point observable is divergent. At the same order, Ward identities can involve the second-order scalar-gravity vertex, nonlinear gravitational diagrams, ghosts, trajectory corrections, and the observer/dressing sector. The manuscript neither includes these terms nor gives a power-counting or Ward-identity argument that excludes their participation.

The mass deformation does produce the advertised inverse of the transverse operator for the special traceless null shock ansatz in Appendix A. However, equation (4.31) is not a consistent massive spin-2 action, and it explicitly breaks the symmetry whose zero mode is being studied. The residue assigned to that mode therefore requires a regulator-independence argument. Moreover, the Appendix F claim that the deformation changes only one coefficient equation is not compatible with the paper's own bitensor expansion. Thus the double-scaled series is presently a result in a chosen gauge-breaking regulator, not a demonstrated property of massless de Sitter gravity.

The logical status of the chaos statements should consequently be narrowed. Every exchanged eikonal graviton carries the ordinary boost factor $e^{t/\ell}$. Pole symmetry removes the odd moments, leaving the square of that factor as the first nonzero term. This is an angular selection rule in a power series, not by itself a new Regge intercept or a physical Lyapunov mode. The general-position result restores the linear term. A claim of a violation of the quantum chaos bound additionally requires a precise thermal Hilbert-space observable satisfying the analyticity, boundedness, and factorization assumptions of the bound. Those hypotheses are not established for the undressed bulk fields, and they certainly are not automatic in the gauge-breaking massive regulator.

Novelty and significance

The manuscript goes materially beyond a short reprise of the de Sitter shockwave calculation. Its useful contributions include the general horizon-wavefunction formula (5.11), the treatment of both principal and complementary series, the displaced-insertion representation (5.45), and the attempt to connect the compact shockwave obstruction directly to sectors of a covariant graviton propagator. If the $L = 0$ time-advance term can be shown to survive in an explicitly dressed observable, the massless result would be of clear interest to the de Sitter quantum-gravity and quantum-chaos communities.

The novelty of the loop conclusion is harder to assess because the paper does not yet separate a physical linearization-stability obstruction from a gauge zero mode in a truncated perturbative expansion. Likewise, the doubled exponent is novel only if it is regulator independent and has an operational meaning beyond the first allowed even power of the ordinary eikonal parameter. At present these stronger interpretations run ahead of the derivation.

The literature discussion also needs updating. The comparison with the three-dimensional conformally coupled calculation of Aalsma and Shiu is important, but the assertion that an arcsinh contribution was dropped is confined to a footnote. The bibliography contains coordinated *work in progress* entries by Chen et al., Harlow–Zhao, and Cui–Kolchmeyer, while the note added does not cite or compare their now relevant conclusions. A journal version should give a substantive comparison and should also relate the $L = 1$ condition to the established linearization-stability literature for gravity on compact de Sitter slices.

Recommendation

Major revision. The massless tree-level result is technically promising and could become publishable in a journal such as JHEP or PRD. Before publication, however, the authors should either establish the loop and doubled-exponent claims for a gauge-invariant dressed observable or clearly reframe them as regulator-dependent diagnostics. The propagator and mass-deformation derivations also require substantive correction and enough intermediate detail to make the central claims reproducible.

Major comments

1. **Construct the observable that is claimed to be gauge invariant.**

   The Abstract and Introduction state that the scalar insertions can be dressed to an observer worldline. The actual master formula (5.3), however, uses free bulk two-point functions on a rigid de Sitter background and local scalar insertions. The observer stress tensor, recoil, clock variables, and gravitational dressing are absent. Section 6 explicitly acknowledges that the dressing has not been treated.

   This omission is decisive here because the finite $L = 0$ answer is attributed entirely to the non-decaying gradient term (4.21). A local bulk field and a relationally dressed field need not transform in the same way under precisely such a large diffeomorphism. Dressing contributions and observer recoil are also of order G_N, the order at which (5.66) is obtained. It is therefore not enough to say that dressing changes only external lines: the definition of the external observable fixes the boundary terms by which the large diffeomorphism enters the exchange amplitude.

The authors should select an explicit one- or two-observer relational construction, derive the dressed four-point function through order G_N, and show that the coefficient and sign in (5.66) are independent of the covariant gauge parameters and of allowed changes of dressing. They should also state which transformations are redundancies and which are physical large symmetries under the chosen boundary conditions. Without this step, the most robust calculation in the paper remains a gauge-fixed bulk correlator rather than the advertised de Sitter observable.

2. **Distinguish the global constraint from an ordinary IR divergence.**

   Equation (2.14) is an equation on a compact transverse sphere. For a localized source with a nonzero $L = 1$ projection it has no solution; after the source is projected, the solution is nonunique up to an $L = 1$ homogeneous mode. This is naturally tied to the global gravitational constraints and to linearization stability on compact de Sitter slices. It is not merely the usual statement that a propagator has a soft momentum divergence.

   The physical source should be the total source, including the observer, dressing, and any compensating recoil. The manuscript instead projects the matter source at tree level and later permits the forbidden mode to propagate inside loops. The authors need to formulate the relevant constraint on the physical state or total stress tensor, identify the associated de Sitter charge, and show whether a pair of dressed insertions satisfies it. They should then explain why the same constraint does or does not remove an internal $L = 1$ line. A mass term that explicitly relaxes the constraint cannot answer this question by itself.

3. **A gauge-dependent ladder divergence is not yet a physical loop divergence.**

   The large-distance term in equation (4.14) is proportional to α. The discussion immediately below it permits taking $\alpha \to 0$ first, which removes the term, or correlating α with the IR cutoff, which assigns an arbitrary finite coefficient. A physical divergence cannot be established by a gauge-dependent $0 \times \infty$ prescription.

   At order G_N^2, exponentiating the linear $h_{\mu\nu}T^{\mu\nu}$ vertex does not automatically give the complete gauge-fixed amplitude. The $h^2\phi^2$ seagull vertex, nonlinear graviton diagrams, ghost contributions, corrections to the eikonal trajectories, and the observer/dressing sector can be related by Ward or BRST identities. Some may be suppressed in standard fixed-impact-parameter eikonal power counting, but that must be shown in the singular $L = 1$ channel rather than assumed.

   A publishable loop claim requires either an explicit gauge-invariant calculation through the first nontrivial loop order or a derivation of an eikonal Ward identity demonstrating that all omitted terms are subleading and that the final divergence is independent of (α, β). The argument in Section 6 that an internal propagator cannot be affected by external dressing is not a substitute for such a calculation.

4. **The mass regulator does not presently define a universal massless limit.**

   Equation (4.31) adds $m_g^2 h_{\mu\nu}h^{\mu\nu}$, not the Fierz–Pauli combination, to an already gauge-fixed quadratic action. It breaks diffeomorphism and BRST symmetry and generally propagates additional components. If it were interpreted as a physical massive spin-2 theory, the limit toward $m_g = 0$ would also pass through the familiar nonunitary region below the de Sitter massive-spin-2 bound. If, as seems intended, it is only an analytic regulator, the residue of the lifted pure-gauge mode must be shown to be independent of the regulator.

There is also a concrete problem in Appendix F. Equations (F.28)–(F.35) show that the gradient functions X, Y, W contribute to all five bitensor coefficients. Once the mass term breaks gauge invariance these terms can no longer simply be absorbed into the Λ ambiguity. Multiplication of the full propagator by m_g^2 adds $m_g^2 f_i T^{(i)}$ to every relevant coefficient equation, not only $m_g^2 H_1 T^{(3)}$. Consequently the assertion (F.61), and hence the isolated massive-scalar equation (F.62), does not follow from the displayed ansatz. The special shock ansatz of Appendix A may still solve the chosen operator for a traceless null source, but it does not establish the claimed full propagator.

The authors should invert the complete regulated quadratic operator, including all tensor sectors, and demonstrate that the same $L = 1$ residue controls the dressed observable. A calculation with a second regulator, such as a constraint-preserving projection, a finite global-time regulator, or a well-defined Stueckelberg/Fierz–Pauli completion, would be especially valuable. Absent such universality, equations (4.33) and (5.27) should be presented only as properties of the chosen gauge-breaking deformation.

5. **The doubled exponential should not yet be called a Lyapunov exponent or a violation of the chaos bound.**

   The expansion parameter in equations (5.27)–(5.28) is $\chi = (G_N/m_g^2)\ell^{-d}e^{t/\ell}$. Pole symmetry implies $\int (y \cdot \bar{y})^{2k+1} = 0$, so the first allowed term is χ^2. This establishes an even-harmonic selection rule. It does not, without additional analysis, establish a new Lyapunov mode with exponent $2/\ell$. Higher powers $e^{nt/\ell}$ are expected in an eikonal series, while the per-exchange boost and the generic displaced-insertion term remain $e^{t/\ell}$. Indeed, equation (G.13) restores the linear term when neither pair is at the pole.

   At fixed nonzero m_g, the complete phase (4.32) also retains the regular $L = 0$ contribution at order $G_N e^{t/\ell}$. The special double scaling keeps χ fixed while sending this regular term to zero. The $e^{2t/\ell}$ contribution is therefore not the leading early-time correction of the regulated theory at fixed physical couplings. The authors should specify the operational definition of the claimed Lyapunov exponent, the parametric time window in which it is measured, and why a selected quadratic term rather than the underlying eikonal intercept is the relevant quantity.

   Application of the Maldacena–Shenker–Stanford bound requires an explicitly normalized thermal trace (or an equivalent positive-state construction), suitably bounded or smeared operators, factorization, and control of the analytic strip. These assumptions are not established for the undressed bulk fields. The massive regulator is not a unitary gravitational theory, so its double-scaled answer cannot itself diagnose a conflict with unitary quantum mechanics. For the massless result (5.67), the authors should show that the observer-dressed correlator is precisely the regularized OTOC to which the bound applies and identify which hypothesis fails if the positive growth survives. Until then, *conditional tension* is more accurate than *violation*.

6. **The central full-propagator result must be made reproducible.**

   The main text reports calculations in $d = 3, 4, 5$, but equation (4.14) contains an unspecified dimension-dependent coefficient and the cancellation leading to (4.15) is not displayed. The allowed range of β is described only as *not too negative*. Equation (4.21) gives a schematic contribution without the coefficient needed to verify the sign of the $L = 0$ phase. Appendices B and F determine a universal H_0/H_1 coefficient modulo gradient and metric pieces, but they do not solve the non-decaying gradient functions whose boundary flux is responsible for the claimed cancellation and time advance.

Appendix F also has a structural problem before the massive extension. The operator in (F.36) acts at x only and therefore does not preserve endpoint-exchange symmetry. The two terms combined into $T^{(4)}$ in the five-tensor basis must in general be treated as separate structures. The text nevertheless expands the result in only five tensors in (F.37), then later presents three further equations (F.63)–(F.66) in addition to the three equations used for H_1. This is not a closed five-coefficient derivation. There is also a sign inconsistency: substituting the displayed solution (F.57) into (F.55) gives a $+2uc_C$ term on the right-hand side of (F.58), not the printed $-2uc_C$. The sign disappears only after c_C is set to zero, but it signals that the reduction needs to be redone.

Finally, equations (F.2)–(F.6) convolve the propagator with the trace-reversed source $\widetilde{T}_{\mu\nu}$ while treating primed gradient terms as harmless against a conserved source. Even when $T_{\mu\nu}$ is conserved, $\widetilde{T}_{\mu\nu}$ is not conserved for a varying trace. The leading null eikonal source is tracefree, but the appendix advertises a general graviton inverse and the underlying massive scalars are not tracefree away from the strict high-energy limit. The trace-reversal projector and all gradient terms must be handled consistently.

At least one full $d = 3$ or $d = 4$ propagator contraction should be shown from start to finish, including the (α, β) dependence, the large-$u\bar{v}$ asymptotics, the boundary terms, and the normalization of (4.15). A general-d mode argument is needed before the conclusions are stated for arbitrary dimension.

7. **Specify and justify the contour and branch prescriptions.**

The eikonal reduction uses the rotation (3.24), and the OTOC formulas use complex rescalings of positive null momenta, complex powers such as $(-e^{t_{ij}/\ell})^{(d-1)/2}$, and $K_{i\mu}$ on negative or imaginary arguments. The relevant cuts and endpoint arcs are not tracked. Appendix G states that the transform is first defined in a Euclidean region and then continued, but it does not give a domain or prove that no singularity is crossed.

This is not a peripheral technicality. Section 4.4 explicitly says that the large-diffeomorphism term is contour sensitive and that Euclidean and Lorentzian calculations may differ. The authors should specify the Bunch–Davies $i\epsilon$ prescription for all four insertions, the branches of the Bessel and power functions, the allowed deformations of the p^u, p^v contours, and the boundary conditions at past and future infinity. They should then verify (5.11), (5.45), and the sign of (5.67) from one common prescription.

There is a related orientation inconsistency. Appendix G defines $\rho = -e^{t_{BA}/\ell}\sqrt{a_A/a_B}$ and prescribes $(A, B) = (4, 2)$ for the 24 sector. With the manuscript convention $t_{ij} = t_i - t_j$, this gives $\rho = -e^{t_{24}/\ell}\sigma_{42}$, whereas equations (5.40)–(5.42) use $\rho_{24} = -e^{t_{42}/\ell}\sigma_{42}$. The disconnected invariant contains a cosh and can hide this reversal, but $\vec{q}/\rho$ and the linear term (G.13) cannot. The orientation and sheet must be corrected and the general-position formula rechecked.

The intermediate discontinuity formula also needs care in even dimension: equation (5.9) contains $1/\Gamma(2 - d/2)$, while the accompanying integral develops the compensating singularity. A distributional or analytic continuation argument should be supplied, with an explicit check in $d = 4$. The claim of arbitrary scalar mass should exclude or separately treat the minimally coupled massless endpoint, for which the de Sitter-invariant two-point function in (4.4) and the small-momentum integral are singular.

8. **Qualify the assertion that the ladder diagrams have no UV divergences.**

   The transverse Green function in (4.15) has the standard coincident behavior

$$\mathcal{G}(\theta) \sim \begin{cases} \log\theta, & d = 4, \\ \theta^{4-d}, & d > 4. \end{cases}$$

   At n exchanges its n-th power is integrated with relative angular measure $\theta^{d-3}\,d\theta$. Thus the two-exchange integral is logarithmically divergent already in $d = 6$ and power divergent for $d \geq 7$; in $d = 5$, the three-exchange integral is logarithmically divergent. This contradicts the unqualified statement in Section 3 that the generalized ladders generate no UV divergences.

   The short-impact-parameter region is also where the eikonal approximation and the neglect of nonlinear gravitational dynamics require the most care. The authors should restrict the claim to separated impact parameter and the dimensions/orders for which the integral converges, or supply the required renormalization and matching. They should also justify interchanging the $m_g \to 0$ limit, the $L = 1$ truncation, angular integration, and the perturbative expansion.

9. **Clarify novelty relative to prior and coordinated work.**

   The claimed correction to the Aalsma–Shiu calculation affects the order-G_N result and should be demonstrated explicitly, with a side-by-side comparison of conventions and the allegedly omitted arcsinh term. The discussion should also distinguish the present compact-source obstruction from classic de Sitter linearization-stability constraints and from the known gauge dependence of covariant de Sitter graviton propagators.

   The note added should replace anonymous references to *several other groups* with full citations and a technical comparison to the coordinated papers. In particular, the authors should state which conclusions about the observer algebra, negative shocks, positivity, and the regularized OTOC agree or disagree, and which depend on a different dressing or contour. This is needed to establish what is new beyond the existing dS_3 shockwave result and the contemporaneous static-patch analyses.

Minor comments

1. Equation (4.2) conflates two statements. The exact zero-mode values are

$$m^2\ell^2 = d - 2 - L(L + d - 3), \qquad L = 0, 1, 2, \ldots,$$

   whereas the direct large-distance integral can diverge throughout a continuous range $m^2\ell^2 \leq d-2$. These should be stated separately.

2. Equation (4.20) is missing a factor of L. The harmonic denominator following from $\Delta f - \mu^2 f$ should be $-L(L+d-3) - \mu^2$, not $-(L+d-3) - \mu^2$. With the printed expression, the subsequent comparison of signs is not valid.

3. The paper alternates between saying that the localized shock equation has no solution and that it has no unique solution. A source with nonzero $L = 1$ projection gives no solution; after projection, the inverse is nonunique by addition of an $L = 1$ homogeneous mode.

4. The terminology for the gauge choice should be made precise. The $\beta = -2$ condition has de Donder form, while α controls the weight with which it is imposed. Section 4 associates de Donder gauge with $\alpha = 0$, whereas Appendix A uses $\alpha = 1, \beta = -2$; these are not the same propagator.

5. The claim *any dimension* should be restricted at least to $d \geq 3$. The formulas containing $1/(d-2)$ and the transverse-sphere construction do not cover two-dimensional Einstein gravity.

6. The domain called *any bulk masses* should be stated explicitly. Positive complementary- and principal-series masses are covered by the displayed convergence condition. The minimally coupled massless endpoint, tachyonic values, and discrete zero modes require separate treatment.

7. In equation (5.52), $G_{d+2k,\mu}$ is an auxiliary dimension-shifted function at fixed μ; it is not the propagator of a field with the same physical mass in $d + 2k$ dimensions. This should be said when the identity is introduced.

8. The sign convention for the plotted coefficient f is unnecessarily indirect: equation (5.30) writes $1 - f\chi^2$, while the cases of interest have $f < 0$. The figures should have labeled axes, state the precise ϵ and branch conventions, and preferably be accompanied by the numerical data or code.

9. The notation ℓ and ℓ_{dS} should be used consistently, and the dimensionless combination $m_g^2\ell^2$ should be retained in all mode equations.

10. In Appendix B, the displayed action of the wave operator on the bitensor structure is missing a factor of H_0 in the curvature term $2H_0/\ell^2$. In Appendix F, equations (F.45) and (F.47) use an undefined capital D, apparently in place of d, and equation (F.57) writes c_c instead of c_C.

11. Appendix C calls the geometry BTZ but uses the nonperiodic Green function $e^{-R|x|}/(2R)$ and a single AdS propagator. For genuine BTZ, x is periodic and both objects require an image sum. The calculation should either be completed on the quotient or relabeled as the AdS-Rindler/universal-cover check.

12. Correct *pode–antipode* near equation (5.59), and revise the empty subfigure captions in Figures 5 and 6 so that each panel can be identified without decoding the prose in the common caption.

13. The phrase *initial growth* should be tied to a precise regime: large boost time compared with ℓ, small eikonal parameter, and earlier than the relevant crossover or scrambling scale.