Referee Report on “A de Sitter Anti-Scrambling Algebra”

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Summary and overall assessment

The paper studies an algebra of observables available to a semiclassical observer in de Sitter space when insertions at very early and very late times are separated by a scrambling time. The starting point is a four-dimensional Schwarzschild–de Sitter construction with a timelike observer boundary. In the double-scaling limit (2.16), gravitational interactions between early and late excitations are encoded by the eikonal phase (2.28). A near-Nariai limit isolates the s-wave sector and gives the two-dimensional scattering operator

$$S = \exp(ixP_A P_B), \qquad x < 0,$$

with the sign opposite to the familiar AdS JT-gravity problem.

The main body then analyzes this negative-phase model using two chiral CFTs. The authors derive exact position- and frequency-space four-point functions, equations (3.8)–(3.12), and identify a monodromy which violates the KMS and detailed-balance conditions for $x < 0$, while both conditions hold for $x > 0$. They further argue that the canonical Hartle–Hawking functional on a crossed-product algebra fails to be cyclic, prove that the vacuum is cyclic but nonseparating for the finite-$x < 0$ algebra, give restricted constraints on its commutant, and conjecture that the commutant is trivial. At parametrically later times, $x \to -\infty$, the mixed moments approach those of a free product. A de Sitter JT construction supplies a geometric interpretation, and Appendix C presents a three-dimensional free-scalar check.

The negative-phase KMS monodromy is a novel and potentially important observation. The exact two-dimensional calculation, its AQFT interpretation, and the connection to a nontracial crossed-product functional would be of clear interest to readers working on de Sitter holography and gravitational operator algebras. The paper is ambitious, technically substantial, and generally well organized. In particular, the agreement between the direct special-function computation and the commutator/contact-term derivation is valuable evidence that the central physical effect is real.

I do not, however, think the manuscript is ready for publication in its present form. The crossed-product discussion treats a nonnormalizable weight as a Hilbert-space vector, the central noncyclicity formula contains an algebraic error, and the evidence offered for a trivial commutant relies on ill-defined generalized vectors and an OPE assumption that is not valid for a general chiral CFT. There are also gaps in the claimed test-function estimates and in the treatment of the $x \to 0^-$ asymptotics. These issues affect claims highlighted in the abstract and conclusion, rather than only peripheral details.

Recommendation: major revision. I would be open to publication after a revision that places the crossed product on a correct operator-algebraic footing, repairs the explicit formulas, substantially narrows or strengthens the commutant claims, and states precisely the scope in which KMS violation has been established.

Correctness and logical structure

The most convincing part of the paper is the finite-x two-dimensional four-point calculation. Starting from the momentum integrals in Appendix D, the hypergeometric-U expressions (3.8) and (3.9) have the expected perturbative ${}_2F_0$ asymptotics. The distinction between upper, principal, and lower sheets gives a natural sign-sensitive mechanism: the $x > 0$ continuation implements the KMS relation, whereas the $x < 0$ continuation acquires the monodromy displayed in (3.55) and (3.60). The frequency-space result (3.63) is consistent with the same picture. The half-sided-translation argument surrounding (3.36)–(3.41) also gives a useful physical explanation of why a time advance can bring an originally commuting antipodal operator into causal contact.

The later conclusions require more care. Equation (3.36) is a diagnostic involving the undeformed modular data; it is not by itself a theorem that every negative eikonal phase violates KMS. The explicit nonvanishing result is established for the special two-dimensional model and is supported, more formally, by the free-scalar example in Appendix C. Similarly, Proposition 4 establishes nonseparation of the vacuum only after accepting domain manipulations with unsmeared frequency fields. Propositions 5 and 6 exclude narrow ansatz classes, but they do not control a general square-integrable two-particle component of $O|\Omega\rangle$. The conjecture $\mathcal{A}' = \mathbb{C}$ therefore remains far from a classification theorem.

Novelty and significance

The paper appropriately builds on the observer algebra of Ref. [24] and the positive-phase scrambling algebra of Ref. [40]. The new content is not the modularly twisted product itself, nor the free-product argument, but the negative sign appropriate to the de Sitter time advance and the resulting KMS/crossed-product consequences. Gravitational time advances and causal contact between opposite static patches already occur in Ref. [47], while Ref. [44] found related nonfactor/type-I behavior from observer recoil. The exact negative-phase monodromy and its proposed interpretation as a failure of Hartle–Hawking cyclicity appear to be the genuinely new core.

That core is significant if formulated with the right scope. It challenges an observer-centric version of static-patch holography only under the additional premise that the microscopic theory must reproduce the enlarged algebra of selected early/late OTOCs with the semiclassical Hartle–Hawking functional. Section 5 itself observes that ordinary time-ordered detector evolution is insensitive to these OTOCs. The abstract and conclusion should make this premise explicit. The revision should also explain precisely how the conjectured factor result differs from the type-I nonfactor or sector structure of Ref. [44].

Major comments

1. **The Hartle–Hawking object in (3.100) is a weight, not a vector state.**

   For the "perfect clock" the auxiliary Hilbert space is declared to be $L^2(\mathbb{R}, dq)$, but $e^{-\pi q} \notin L^2(\mathbb{R}, dq)$. Correspondingly, the displayed relation $p|\Psi_{\mathrm{HH}}\rangle = i\pi|\Psi_{\mathrm{HH}}\rangle$ cannot hold for an eigenvector of a self-adjoint p; it is a convenient formal identity for a nonnormalizable function. This distinction is central because the paper repeatedly calls the resulting object a state, evaluates bra-ket correlators with it, and then infers whether it is a trace.

   The crossed product should instead be formulated using the canonical normal semifinite weight (or another precisely specified weight), with its domain and normalization stated. The authors should identify a dense algebra of bounded analytic elements on which all cyclicity manipulations are valid. The insertions $\delta(q - E_i)$ in (3.104) and (3.107) are also distributions rather than bounded projections, so noncyclicity should be demonstrated first with smooth bounded clock functions and only then recovered as a controlled limit. If a lower-bounded imperfect clock or the CLPW compression is intended to produce a normalizable state or a type II_1 trace, that construction must be carried through rather than mentioned informally. Without this reformulation, the statement that the Hartle–Hawking state is not a trace on $\mathcal{A}_{\rm cr}$ is not mathematically well posed.

2. **The central crossed-product formulas (3.122) and (3.123) need to be rederived.**

   Let $k = E_1 - E_2 + E_3 - E_4$. For $x < 0$, subtracting (3.120) from (3.119) gives

$$C_{E_1E_2E_3E_4}\, e^{-\pi(E_2+E_4)}(1 - e^{-2\pi k})F_x(k),$$

   whereas (3.122) contains $e^{-\pi(E_2-E_4)}$. This appears to be a definite algebraic error. Equation (3.123) then omits the common factor $F_x(k)$ altogether and appears to inherit the same exponential mismatch. Although these corrections may leave the qualitative nonvanishing conclusion intact, they propagate into the Gaussian smearing beginning at (3.124), and likely into (3.125), and therefore require a complete check of that subsection. The revised paper should show the subtraction explicitly, state which factors can be divided out and on what set, and repeat the no-product-form-weight argument using legitimate smoothed elements.

3. **The finite-x commutant evidence contains formal and explicit errors.**

   The proof of Proposition 4 is physically plausible, but it uses delta-normalized frequency fields as though they were bounded elements in the Tomita domain. More basically, equation (4.7) identifies $\Delta = SS^\dagger$, whereas the polar convention stated in Appendix A gives $\Delta = S^\dagger S$ and $SS^\dagger = \Delta^{-1}$. Thus (4.7)–(4.9) do not follow as written. Equality of unbounded Tomita operators on a merely dense set is also insufficient without showing that this set is a common core. A rigorous version should correct the modular identity, smear the fields, and exhibit a nonzero detailed-balance defect for bounded analytic elements.

   The subsequent commutant calculation has further concrete problems. In the proof of Lemma 3, equation (4.16) commutes $O \in \mathcal{A}'$ past the undressed fields φ, although only the dressed generators $\widetilde{\varphi}$ are known to belong to $\mathcal{A}$; this step needs to be redone. Equations (4.33) and (4.35) display $2\pi(\lambda_1 + \cdots)$ where the preceding calculation and the next integration require $2\pi\delta(\lambda_1 + \cdots)$; equation (4.35) also contains frequency variables that do not occur in its bra and ket. Appendix D repeats an integration variable in (D.46), and the Appell parameter in (D.57) contains $h_{k'} - i(\lambda_1 - \lambda_A)$. The claimed reduction of the Appell factor to one would instead require the sum $h_{k'} - i(\lambda_1 + \lambda_A)$, given $h_{k'} = h_i + h_k$. These are not cosmetic errors because they are used to establish Propositions 5 and 6.

   The revision should provide corrected, smeared operator kernels, specify convergence contours and $i\epsilon$ prescriptions, and verify that the two inversions leading to (4.17) and (4.18) hold on the stated domain. Until that is done, the restricted commutant exclusions should not be described as established propositions.

4. **The proof of Proposition 6 assumes a non-generic OPE spectrum and its support condition is essentially distributional.**

   The proof states that, for arbitrary primaries i, k, a primary k' with $h_{k'} = h_i + h_k$ and $C_{k'ik} \neq 0$ is guaranteed to appear in the OPE. This is false for a general interacting chiral CFT. An OPE contains primaries and descendants allowed by the theory; it need not contain a primary of exactly additive weight. The simplification used in (D.47)–(D.59) therefore establishes a conditional result only for theories and channels with such an operator.

   There is a second issue. Proposition 6 restricts $O|\Omega\rangle$ to sets with one frequency fixed exactly, such as $|\lambda_A^*, \lambda_B\rangle$. For a continuous spectral measure, a nonzero Hilbert vector supported at one exact frequency is not square-integrable; it is a generalized vector. The proposition is consequently vacuous for ordinary bounded O, unless a distributional completion is explicitly intended. A meaningful result should use wave packets or spectral support in sets of positive measure and should control a general L^2 coefficient $c^{AB}_{\lambda_A,\lambda_B;k,l}$. At minimum, Proposition 6 must be made conditional and the type-I conclusion sharply downgraded. The opening sentence of Section 4.2 should say that triviality of the finite-x commutant is *conjectured*, not that it has been argued as a result.

5. **The $x \to 0^-$ distributional statements need a theorem with a precise topology and remainder.**

   Proposition 1 relies on the vertical-contour condition in Lemma 1. Appendix D.1.3 verifies a decay estimate for one specially centered bump function and then asserts that finite sums of shifted bumps establish the result for generic C_c^∞ test functions. That approximation does not by itself give the uniform complex-frequency bounds required to move the contour. The authors should either prove an appropriate Paley–Wiener/Mellin estimate for a clearly stated test-function space or restrict Proposition 1 to a class for which the estimate is actually shown. In addition, equation (3.25) has $2h_A + n$ in the V_- denominator where the derivation requires $2h_B + n$.

   The same precision is needed for Proposition 3. Equation (3.86) is an asymptotic stationary-phase expansion, not an exact "identity valid to all orders"; its derivatives must be evaluated at $U_- = V_- = 0$, and a remainder estimate is required before it is inserted under the U_+, V_+ integrals. The resulting series in (3.84) should be marked asymptotic unless convergence is proved. Most importantly, Proposition 1 gives convergence of smeared real-time correlators to the undeformed QFT, while Proposition 3 permits an order-x^0 defect after analytic continuation. This is potentially one of the paper's most interesting results, but the order of smearing, analytic continuation, boundary-value limit, and $x \to 0^-$ limit must be stated in one place, together with the topology in which each operation is continuous or discontinuous.

6. **The generality of the KMS conclusion is overstated.**

   The four-dimensional construction gives a general angular kernel in (2.28), but the nonzero KMS defect is calculated only after the near-Nariai and null-hypersurface assumptions reduce the problem to $S = e^{ixP_AP_B}$. Conditions (3.38) and (3.39) are useful sufficient conditions for preserving KMS; their failure does not logically imply that a particular correlator has a nonzero defect. A negative Shapiro shift likewise shows the possibility of causal contact, not a universal theorem about every matter algebra.

   The abstract, Introduction, and Discussion should therefore distinguish: (i) the general eikonal diagnostic (3.36); (ii) the exact proof in the two-dimensional chiral model; and (iii) the formal three-dimensional example. Claims such as "time advance implies breakdown of KMS" should be qualified by "in the models studied" unless the authors prove nonvanishing for a substantially broader class. The role of the assumed null-hypersurface formalism and of the UV chiral fixed point should be stated as part of the result, not only as setup.

7. **The geometric conventions used to establish the sign of the recoil should be reconciled.**

   The sign of the de Sitter time advance is physically central, yet Appendix B contains mutually inconsistent Kruskal assignments. Substituting its claimed north-pole trajectory $U = 2e^t$, $V = -2e^{-t}$ into the embedding map (B.2) gives $X_4 = -\cosh t$, opposite to (B.5); the latter instead corresponds to $U = -2e^{-t}$, $V = 2e^t$. The same appendix places conformal infinity at $UV = -4$, whereas (B.2) makes it $UV = +4$, and the tangent displayed in (B.7) is ∂_U at $U = 0$, not ∂_V at $V = 0$. The horizon assignment and the recoil translation (B.18)–(B.19) should be rederived in one convention and checked against the main-text convention.

   There is an independent scaling error in the simplified model. Substituting (2.76) into (2.74) gives the correction $-ce^{T+2t}h_Ah_B$ in (2.77), not $-ce^{T}h_Ah_B$; the omitted t dependence is essential to the stated $t \to -\infty$ limit. These corrections need not reverse the physical sign, but the paper should not rely on that expectation: it should show explicitly that the corrected conventions reproduce the phase in (2.28).

8. **Appendix C does not presently provide a controlled higher-dimensional check.**

   Equation (C.52) is written as a limit $x \to 0$, but its right-hand side contains $|x|^{-1}e^{-iU_{13}V_{24}/x}$ and therefore has no pointwise limit. Moreover, the remainder is written $1 + \mathcal{O}(x^{-1})$, which grows rather than being subleading. This must be replaced by a correctly stated asymptotic formula, with the power of the remainder checked. The preceding extraction of an "arc at infinity" as a delta function is distributional and should be tested after smearing.

   The normalization of the three-dimensional shock is also unclear. Equations (C.23)–(C.25) assign P_U, P_V eigenvalues already proportional to $e^{T/2}k$, while (C.32) introduces another factor e^T in the scattering operator, even though (C.31) contains only the single boost factor e^T. The authors should define whether P_U, P_V are boosted or unboosted momenta and remove the apparent double counting before using this appendix as an independent check.

   There are also scope mismatches: the main two-dimensional model assumes integer-weight bosonic primaries, whereas the comparison after (C.52) uses $h_A = h_B = 1/2$; and the conformally coupled value $s = i/2$ lies outside the initial assumption $s > 0$. These analytic continuations may be useful, but they need to be declared and justified. Since Appendix C is the principal evidence beyond the chiral toy model, these points deserve a careful revision and a concise summary in the main text.

9. **The free-product result and the finite-x algebra must be kept conceptually separate.**

   The moment limit (4.49) is an interesting de Sitter application of the sign-insensitive argument of Ref. [40], and the JT diagrams give it useful physical content. It is not a new proof of a general free-product theorem. The manuscript should state the needed clustering and integrability hypotheses rather than assuming that decay of connected correlators alone licenses dropping an oscillatory exponential under all integrals. It should also state the hypotheses under which the resulting free product is type III_1.

   Section 4.3 correctly observes that the $x \to -\infty$ limit defines a new emergent algebra made from a selected set of very early and very late operators, with intermediate-time operators omitted. This caveat should appear already at the start of Section 4.2 and in the abstract. More precisely, the calculation establishes convergence of moments followed by a new GNS construction, not operator-topology convergence on the original tensor-product Hilbert space. There is no demonstrated phase transition from a finite-x type-I observer algebra to a type-III_1 algebra; indeed, the finite-x type-I statement is itself conjectural.

10. **The novelty claims should be sharpened against Refs. [40], [44], and [47].**

    The paper is strongest when it presents the negative-phase KMS monodromy and crossed-product obstruction as its contribution. The modularly twisted construction and free-product limit are adapted from Ref. [40], while time advances were studied in Ref. [47]. Ref. [44] already found a recoil-driven observer algebra with type-I/nonfactor behavior and a nontracial Hartle–Hawking functional. The revision should add a focused comparison explaining the respective Hilbert spaces, centers or superselection sectors, gravitational approximations, and meanings of "trivial commutant." This would make clear why the present exact backreaction calculation is an advance while avoiding the impression that all algebraic conclusions are new.

11. **The physical interpretation of the holographic tension should be stated conditionally.**

The manuscript argues that anti-scrambling challenges observer-centric static-patch holography, but Section 5 also explains that a conventional observer coupled through a time-ordered interaction cannot operationally access the relevant OTOCs. The tension is therefore with a microscopic proposal required to reproduce the enlarged early/late algebra and its canonical functional, not automatically with every finite-dimensional model of static-patch observables. This conditional statement would be both more accurate and more informative. It would also help separate the robust scattering result from the stronger, presently conjectural claim that the complete observer algebra is $\mathcal{B}(\mathcal{H})$.

Minor comments

1. The normalization in (2.43) is inconsistent with the resolution of the identity (2.44), the matrix element (2.46), and the four-point normalization in (3.1). With $\overline{dP} = dP/(2\pi)$, equations (2.44) and (2.46) produce an extra $1/(2\pi)$ in the two-point function. The later formulas appear to use the normalized version; correspondingly, the left-hand side of (3.49) is missing $1/(2\pi)$ relative to its printed right-hand side. Please make the convention uniform.

2. The KMS-strip statements below (3.27) write $\operatorname{Im} t_1 \in (-2\pi i, 0]$, and the Kruskal version similarly writes an argument interval containing i. Imaginary parts and arguments are real numbers; the intervals should be $(-2\pi, 0]$ and $(-2\pi, 0)$, respectively.

3. In Appendix B, (B.5) has $X_{\text{Obs}} = (\sinh M\tau, \ldots, \cosh M\tau)$ while calling τ proper time. Its tangent has norm $-M^2$, not -1. Either remove M from the trajectory or explain that τ is a rescaled parameter and that the derivative is the momentum rather than the unit velocity.

4. The static-patch metric (C.2) is missing the square on dt, and the same display contains an undefined η. The mode normalization in (C.8), (C.12), and (C.13) also appears inconsistent with the global convention $\overline{dk} = dk/(2\pi)$. The statement below (C.31) about a "zero-mode" is confusing: the constant mode of $\partial_\varphi^2 + 1$ has eigenvalue 1; the degeneracy at the pure-de Sitter circumference concerns the $n = \pm 1$ harmonics.

5. The analytic-continuation formula (D.30) omits the factor e^{-s} from the Laplace transform. Appendix D.1.3 twice writes $\lim_{\Lambda \to 0}$ while defining a $\Lambda \to \infty$ limit, and (D.36) uses $|x| \geq R$ where $|z| \geq R$ is intended.

6. The AQFT review defines a separating vector with a misplaced universal quantifier. The intended statement is: for each $a \in \mathfrak{A}$, $a|\Omega\rangle = 0$ implies $a = 0$. Its characterization of the Bunch–Davies state as a maximally entangled vector in a left/right tensor factorization also conflicts with the type-III framework just reviewed; this should instead be phrased as algebraic restriction to a static-patch KMS state. The same appendix has a missing pair of parentheses in the second dressed generator in its final paragraph.

7. In the free-product commutant discussion, the operator denoted B' should belong to $\mathcal{A}'_{0,B}$, not $\mathcal{A}_{0,B}$, and it acts on $\mathcal{H}_F$, not on an algebra denoted $\mathcal{A}_F$. The initially defined right action is only densely defined; bounded extension should be justified before calling it an operator on all of $\mathcal{H}_F$.

8. A thorough copy edit is warranted. Examples include "Thee" below (2.22), "discrepency," "nontrvial," "worrysome," repeated "with with," and the repeated integration measure in (D.46).

Recommendation

The paper contains a strong and potentially publishable central idea: a negative eikonal phase produces a calculable KMS monodromy and obstructs the canonical Hartle–Hawking cyclicity for an enlarged scrambling-time algebra. This is novel, technically interesting, and relevant to current work on de Sitter holography. Publication should nevertheless await a major revision. The indispensable items are a correct weight-based crossed-product formulation, repair of (3.122)–(3.123), a mathematically meaningful reformulation of the commutant evidence, and precise distributional control of the $x \to 0^-$ and Appendix C limits. If those points are addressed and the claims are scoped accordingly, the manuscript could become a valuable contribution.