Referee Report

on "Negative shocks versus static patch holography"
arXiv:2607.14042

Summary of the paper

The manuscript studies a concrete version of static-patch de Sitter holography in which Euclidean gravitational amplitudes with an observer worldline and operator insertions are proposed to equal matrix elements of a positive cyclic trace, as summarized in equation (1.1). The authors first review two encouraging tests. The sphere amplitude with no insertions has the expected positive semiclassical density, and the observer two-point function can be transformed between fixed-average-mass and tracial descriptions without an apparent conflict with positivity or cyclicity.

The main analysis concerns out-of-time-order four-point functions. In dS_3, the observer produces a conical defect with angular parameter $\alpha = 2\pi(1 - 4Gm)$. Solving the shock equation gives the profile in equation (2.12). Its small-Gm expansion, equation (2.13), contains an $O(1/m)$ $\ell = 1$ piece describing observer recoil together with the $O(G)$ gravitational time-advance contribution. This is an elegant way to put recoil and gravitational backreaction into the same shock-wave calculation. The profile is then inserted into an eikonal representation of the OTOC. Because the profile is negative over part of the transverse circle, the perturbative series has two lateral completions, denoted $\mathcal{F}_{12}$ and $\mathcal{F}_{14}$. Their distinction is particularly transparent in equations (2.48)–(2.51), where it appears as the prescription $h \pm \mathrm{i}0$, while equation (2.35) shows that a cyclic rotation exchanges the two functions.

For the antipodally regularized configuration, equation (2.42) gives a positive leading correction proportional to $G(e^t + e^{-t})$, followed by positive recoil and backreaction terms. This is the opposite of the usual black-hole OTOC sign. The paper also studies the pure-recoil, large-mass limit through geodesic graphs on a sphere. A dual flat tetrahedron solves the saddle equations for the opposite-side configuration, while a second saddle describes particle trajectories on the same side. Equations (2.68) and (2.69) relate the different analytic sectors of the tetrahedron action to the four eikonal expressions. The heavy-light model in Section 2.6 then makes the associated Stokes phenomenon and contour choice especially concrete.

Section 3 presents the conceptual conclusion. Assuming the trace representation, the authors define the Gaussian-regulated function in equation (3.1), compare it with the fixed-average-mass OTOC, and apply Cauchy–Schwarz using two inequivalent cuts. If the regulated function is analytic and suitably bounded in the relevant half-strip, the maximum-modulus argument would forbid the increase implied by equation (2.42). The authors therefore interpret the positive eikonal correction as an obstruction to realizing the observer amplitudes as a positive trace. Appendix A supplies a useful wrong-sign de Sitter JT model, and the remaining appendices develop the tetrahedron geometry, higher-dimensional shock waves, recoil wave functions, and a tracial OTOC.

Evaluation

The manuscript contains substantial and interesting physics. Several nontrivial parts of the calculation are internally consistent. In particular, the operator in equation (2.10) has the advertised negative $\ell = 0$ mode and pure-gauge $\ell = 1$ zero modes at $\alpha = 2\pi$; the observer lifts the latter. Direct expansion of equation (2.12) reproduces equation (2.13), including the coefficient of the recoil term. The first two transverse moments quoted before equation (2.42) also agree with the stated profile. More generally, in the antipodal configuration $\Upsilon_\nu^n(\pi)$ is minus a positive Bessel-function moment, while $(-1)^n \int h^n > 0$ for the physical range $0 < \alpha < 2\pi$. This supplies a short explanation of the all-positive perturbative series. The energy-domain formulas also correctly show why the two lateral prescriptions agree at every perturbative residue even though they differ after resummation.

The flat-tetrahedron construction is geometrically appealing, and the matching of its analytic sectors to the eikonal answers is a valuable account of the Stokes phenomenon. The algebraic Cauchy–Schwarz step in Section 3 is likewise sound once a positive cyclic trace is assumed. I therefore find the central physical mechanism plausible: de Sitter time advance gives the regularized OTOC the sign that is incompatible with the expected positivity structure under additional semiclassical assumptions.

The main reservation is that the paper currently presents this conclusion more strongly than the argument establishes. The maximum-modulus step uses approximate, not exact, boundary estimates; the finite-time part of the half-strip boundary is not treated; and the errors in replacing the fixed-average-mass amplitude by the Gaussian-regulated trace are only estimated schematically. These gaps look repairable, but they concern the central claim of falsification. There are also unresolved contour and saddle-selection issues in the all-orders eikonal and tetrahedron discussions, as well as one explicit inconsistency in Appendix E.

The novelty is sufficient for a journal such as JHEP or PRD, provided the claims are sharpened. The initial growth of de Sitter OTOCs and the unusual Lyapunov behavior have antecedents in Aalsma–Shiu, Kolchmeyer–Liu, and Narovlansky. The distinctive contributions here are the unification of observer recoil and gravitational backreaction through the lifted shock modes, the explicit $h \pm \mathrm{i}0$ energy-space formulas, the tetrahedron interpretation of the Stokes data, and the formulation of the issue directly as a trace-positivity obstruction. These are worthwhile results.

Recommendation

I recommend **major revision**. I would be favorable toward publication after the authors close, or clearly delimit, the logical gap in the positivity argument; correct the gravitational and higher-dimensional formulas discussed below; and state more precisely what is and is not defined by the lateral resummations. I do not see evidence that the shock profile or the sign of the leading OTOC correction is wrong. The required revision is substantial because the strongest advertised conclusion rests on assumptions that are presently not quantified.

Major comments

1. **The half-strip maximum-modulus argument needs a complete boundary estimate.**

   Write the analytic variable as $z = \tau + \mathrm{i}s$. The domain used in Section 3 is the half-strip $-\pi/2 < \tau < \pi/2$, $s > t_0$. Its boundary contains not only the two sides $\tau = \pm\pi/2$, but also the segment $s = t_0$. Equations (3.8) and (3.10) address the two sides, after factorization, but no bound is stated on the finite-time segment. A Phragmén–Lindelöf argument also needs a clear growth condition or a finite-rectangle limiting argument. Finite-dimensionality makes the regulated trace entire and keeps real-time evolution bounded, but it does not by itself normalize every boundary value to one.

   The proof could be repaired in more than one way. The authors could work on a finite rectangle, bound the lower and remote boundaries, and then show that the unwanted boundary has parametrically small harmonic measure in the time range of interest. Alternatively, they could formulate a precise uniform-bound hypothesis. In either case, the normalization constant c should be defined explicitly, and the notation $f(\mathrm{i}t)$ versus $f(\tau + \mathrm{i}s)$ should be replaced by a single complex variable. As written, the conclusion should be stated as a conditional conflict under analyticity, positivity, and suitable boundary factorization, rather than as a consequence of positivity and cyclicity alone.

2. **The factorization and smearing errors must be controlled in the same parametric window as the growing term.**

   The exact Cauchy–Schwarz expressions preceding equations (3.8) and (3.10) are replaced by products of two-point functions. This is an additional dynamical assumption. The OPE argument in the footnote is reasonable, but the central no-go requires a bound on all connected time-ordered contributions, including their dependence on s. It is enough to show that they remain non-growing and parametrically smaller than the positive OTOC correction, but this should be demonstrated rather than asserted.

   Similarly, equation (3.5) uses

   $$\sum_i (m - m_i)^2 = 4(m - \bar{m})^2 + \sum_i (m_i - \bar{m})^2,$$

   but leaves the second term as $O(\delta^2\omega^2)$ and assumes typical energy transfers are of order one. The authors should give the exact quadratic form in independent frequency variables, include the normalization and the lower spectral endpoint, and state whether ν, ν' are fixed as $m \to \infty$. In the heavy-particle regime, ω can scale with ν, changing the estimate to $O(\nu^2/m)$ for $\delta \sim m^{-1/2}$.

   Most importantly, the compatible time window should be displayed. For the gravitational term one needs, schematically,

   $$\frac{1}{m} \ll G e^t \ll 1,$$

   whereas in the pure-recoil regime one needs

   $$\frac{1}{m} \ll \frac{e^{2t}}{m^2} \ll 1.$$

   Such windows can exist, but writing the full hierarchy would show that the positive term dominates the regulator and boundary errors while the eikonal expansion remains controlled. The order of the $G \to 0$, $m \to \infty$, and late-time limits should be included in the statement of the result.

3. **Equations (2.6)–(2.7) omit the cosmological term and misidentify the shock equation.**

   The distribution written in equation (2.6) cannot literally be the Ricci component R_{--}. For pure de Sitter and an $\ell = 1$ gluing, the displayed operator vanishes, as it must for a pure-gauge shock, while the metric has $g_{--} = A(0)X^+\delta(x^-) \neq 0$. The vacuum equation is instead

   $$R_{--} - (d-1)g_{--} = 0$$

   for unit de Sitter radius. Thus equation (2.6) is the singular Einstein residual (or a correspondingly subtracted curvature component), and equation (2.7) should include the cosmological contribution or define the subtraction explicitly. The final operator in equation (2.10) and the solution (2.12) appear correct, so this is a correctable but conceptually important error. The same terminology should be fixed in Appendix C.

   The subsequent functional shock action in equation (2.21) is also asserted without a derivation from the Einstein–Hilbert action about the conical-defect saddle. Since its normalization, sign, gauge treatment, and integration contour are precisely what control equations (2.22)–(2.23) and the two i0 prescriptions, the paper should either supply that derivation or give an explicit matching argument sufficient to fix all of these ingredients. At minimum, the status of this action as an eikonal effective description should be stated carefully.

4. **The all-orders eikonal completion and the cyclicity claim need sharper scope.**

   Equation (2.25) defines an all-time approximation by adding the $t > 0$ and $t < 0$ eikonal pieces and subtracting the disconnected answer. This is plausible for collecting separately enhanced powers of e^t and e^{-t}, but it is not shown that mixed terms, non-growing terms, or additional saddles are absent at the accuracy later used. The resulting expression is then Fourier transformed over all time to infer detailed energy-space analyticity. The authors should state the error of equation (2.25) and explain why omitted terms cannot modify the Stokes conclusions in equations (2.48)–(2.51).

   The contour statement also deserves more precision. The profile $h(\phi)$ changes sign for part of the physical parameter range, and rotations of the $p_\pm$ contours encounter Bessel-function cuts and the zeros of h. The footnote following equation (2.35) says that opposite rotations cover every OTOC configuration, but the homotopy and prescriptions are not demonstrated. A contour inherited from a specified Euclidean integration cycle would be ideal; otherwise $\mathcal{F}_{12}$ and $\mathcal{F}_{14}$ should consistently be called lateral completions of the same asymptotic eikonal series.

   Relatedly, the existence of two lateral functions exchanged by equation (2.35) is a contour ambiguity, not by itself an unavoidable violation of cyclicity. The manuscript itself observes that an average can restore cyclicity. The robust conclusion is stronger and cleaner: subject to the assumptions in Section 3, a cyclic completion cannot also satisfy both positivity bounds. The abstract and Discussion should emphasize this formulation.

5. **The unequal-mass energy-domain formulas attach the field masses to the wrong frequencies.**

   This can be seen directly by Fourier transforming equation (2.32). The red V insertions at θ_1, θ_3 carry the parameter ν', while the blue W insertions at θ_2, θ_4 carry ν. Consequently, the first line of Gamma functions in equation (2.47) should have the structure

   $$\Gamma\left(\frac{1 + \mathrm{i}\omega_{1,3} \pm \mathrm{i}\nu'}{2}\right), \qquad \Gamma\left(\frac{1 - \mathrm{i}\omega_{2,4} \pm \mathrm{i}\nu}{2}\right),$$

   with the product over the two indicated insertion frequencies and the independent signs. The printed expression instead assigns ν to ω_1, ω_3 and ν' to ω_2, ω_4. The same interchange is present in equations (2.48)–(2.51), with their displayed frequency signs. Later comparisons do not expose the error because they specialize to $\nu = \nu'$, but the general formulas and any unequal-mass claims must be corrected.

   There is also a scale zero mode in the passage from equation (2.47) to (2.48). With energy conservation imposed, the transformation $p_+ \to \lambda p_+$, $p_- \to p_-/\lambda$ leaves the product p_+p_- invariant and produces an integral $\int \mathrm{d}\lambda/\lambda$. Distributionally, this is tied to the energy-conserving delta function already displayed on the left of equation (2.47). The derivation should introduce, for example, $u = (-p_+)(-p_-)$ and the logarithmic ratio variable, regulate or quotient the latter, and track the resulting $2\pi\delta(\sum_i \omega_i)$ and normalization. As written, the delta function is inserted while the redundant double integral is still present.

6. **The tetrahedron result is presently an exponent-level saddle diagnostic, not a defined path integral.**

   Equation (2.55) writes the recoil amplitude as $e^{-I_{\text{tetra}}} + e^{-I_{\text{same}}}$ with unit coefficients. No integration cycle, fluctuation determinant, Maslov phase, or Stokes multiplier is derived, and other winding or caustic saddles are not excluded. The later heavy-light analysis around equations (2.82)–(2.83) usefully shows how one saddle enters one lateral contour in a representative regime, but it does not establish equation (2.55) throughout the stated mass domain.

The admissible domain should also be stated. The dual Euclidean tetrahedron requires the four face triangle inequalities and an appropriate sign of the Cayley–Menger determinant. Outside that domain, equations (B.6)–(B.8) define complex continuations rather than a real spherical saddle. For the symmetric masses, the manuscript identifies several branch points, but it should say which real interval supports each Euclidean configuration and how the relevant thimbles continue across the branch loci. In the main text, equation (2.53) defines d_{ij} as the shortest endpoint distance, whereas the same-side discussion speaks as though d_{13} and d_{24} themselves exceed π. Appendix B has the correct distinction between the endpoint distance $b \leq \pi$ and the trajectory length $2\pi - b$; the main text should adopt it.

The matching in equations (2.68)–(2.69) remains valuable at the level of the leading semiclassical exponent. If this is the intended claim, equation (2.55) and the surrounding prose should be qualified accordingly. If a quantitative amplitude is intended, the thimble weights and one-loop factors are needed.

7. **The wrong-sign de Sitter JT model does not yet define the nonperturbative contour used in Section 2.3.**

   Appendix A explicitly notes that the immersion integral (A.7) is nonconvergent and treats fluctuations around the hemisphere by Wick rotation. This is useful perturbative information, but the choice of rotations for the infinitely many unstable modes is not a nonlinear integration cycle. Matching the leading four-point correction (A.24)–(A.25) fixes the negative coefficient $C = -S_m/(2\pi)$ in equation (A.26), but by itself does not prove that the complete eikonal measure and contour are identical to ordinary JT with $C \to -C$. Because the two lateral answers are the central phenomenon, the authors should either derive this reduction for the boosted modes or present the model explicitly as a perturbative toy whose nonperturbative definition remains open.

   The real two-dimensional integral displayed in equation (A.14) is itself divergent: $\exp(-\delta\beta\,\delta E)$ grows in two quadrants. The value $-\mathrm{i}$ is therefore a contour assignment, not the value of the written real integral. The rotation directions and regulator should be given before it is compared with the inverse-Laplace contour in equation (A.15), especially because this phase is being used to discuss positivity.

   The gauge quotient in equations (A.4)–(A.7) should include the constraints on the Möbius parameters, and the measure terms listed in the footnote after (A.7) should be discussed to the extent needed for the one-loop phase. These details need not produce a full nonperturbative theory, but the claims based on the model should match what has actually been defined.

8. **The higher-dimensional recoil derivation contains a direct inconsistency.**

   Equation (E.6) states

   $$S_{\text{worldline}} \simeq 2m\,\vec{y}_+ \cdot \vec{y}_+,$$

   which contains no $\vec{y}_-$. It therefore cannot make both $\vec{y}_+$ and $\vec{y}_-$ integrals Gaussian as claimed in the next sentence, nor can it generate the mixed phase $p_+p_-\cos\phi/(2m)$ in equations (E.9)–(E.10). The intended quadratic action appears to be proportional to $2m\,\vec{y}_+ \cdot \vec{y}_-$, with a sign fixed by the Fourier conventions. This equation must be corrected, and the short Gaussian integration should be shown. The powers of $-p_\pm$, the ordering signs s, s', and the overall phase hidden in C_0 should then be checked so that the reduction to the $d = 3$ formulas is unambiguous.

9. **The relation to simultaneous work must be made substantive.**

   The three entries cited as "To appear" are no longer adequate for assessing novelty. In particular, the simultaneous work of Milekhin, Narovlansky, and Xu (arXiv:2607.13137) also computes de Sitter OTOCs, finds initial growth and a chaos-bound conflict, and emphasizes an $\ell = 1$/vector-sector infrared problem. The present observer defect lifts the corresponding zero modes and produces the linear Ge^t term in equation (2.42). The authors should explain whether this is the physical regulator of the large-diffeomorphism issue in that work and why the two treatments can differ in their odd-power structure.

   Likewise, Cui and Kolchmeyer (arXiv:2607.13665) reach a related conclusion about the Hartle–Hawking state and a trace using algebraic/KMS reasoning and de Sitter JT shocks. The present paper's separate contribution appears to be the explicit recoil/backreaction profile, the two-resummation and tetrahedron analysis, and the Cauchy–Schwarz positivity formulation. This division should be explained in the Introduction and Discussion, with complete references. The manuscript should also distinguish the previously known initial-growth observation from these genuinely new results.

Minor comments

1. State the physical range of the conical-defect parameter when equation (2.8) is introduced. In the regime used later one needs $0 < \alpha < 2\pi$, equivalently $0 < 4Gm < 1$. The zeros and sign changes of $h(\phi)$ should be noted because they control the lateral prescriptions.

2. Define the V-field mass parameter ν' alongside ν at the start of the four-point discussion. In the JT subsection, say explicitly when the two external dimensions are set equal in equations (2.14)–(2.20).

3. The analytic continuation from the orbifold values $2\pi/\alpha \in \mathbb{Z}$ to a general conical angle, used immediately before equation (2.32), should specify the scalar boundary condition at the defect. The continuation is plausible but is part of the definition of the matter theory coupled to the observer.

4. The assertion following equation (2.42) that every perturbative coefficient is positive deserves the one-line sign argument: at antipodal separation $\Upsilon^n_\nu(\pi) < 0$, while $(-1)^n \int h^n > 0$ for $0 < \alpha < 2\pi$.

5. The table in equation (B.25) should state its row-wise domains. For real $\omega > 0$, equation (B.23) has the sign pattern associated with the $\mathcal{F}_{14}$ rows, whereas the $\mathcal{F}_{12}$ rows require $\omega < 0$. The definition $v = \log(4|\omega|/(\nu^2 - \omega^2))$ is useful on the real axis but should be replaced by branch-sensitive analytic functions when discussing continuation. A Gram-signature or embedding check would also clarify in what sense each configuration is Lorentzian rather than genuinely complex.

6. Equation (A.18) uses $\theta_1(u_1)$ and $\theta_2(u_2)$, although only a single function $\theta(u)$ has been defined. These should presumably be $\theta(u_1)$ and $\theta(u_2)$.

7. In Appendix G, the equation-style reference command used after the word "figure" produces a parenthesized figure number. Use an ordinary figure reference. The placement of the conjecture label before equation (1.1) similarly causes some references to display "conjecture (1)" rather than equation (1.1).

8. Equation (G.2) loses the inherited i0 or lateral contour prescription after the p_+ integration. Its denominator can vanish in the signaling regime illustrated in Figure 12. The pole prescription and

square-root branch must be stated, and the numerical curves in Figures 8, 11, and 12 should be accompanied by enough contour, cutoff, and convergence information to make them reproducible.

9. Remove the duplicate bibliography keys `Gibbons:1977mu`, `Gu:2021xaj`, and `Gao:2000ga`. The discussion associated with the Gao–Wald-type time-delay figure should also cite the directly relevant source consistently.

10. A short table distinguishing the finite-G dS_3 regime, pure recoil, heavy-light limit, wrong-sign JT model, and higher-dimensional extension would improve readability. Section 2 presently moves among several different hierarchies of m, ν, ν', G.

11. Please correct typographical errors including "gravitional," "momenum," "diffemorphism," "an unique," "triangles in below," and "By Taylor expand." There are also subject–verb errors in the paragraphs around equations (2.55), (2.68), and (F.1).

12. Equations near (2.33) mix the manuscript's roman i convention with a literal italic i. The phase convention should be uniform, especially in formulas whose complex conjugation properties are important.