

Referee Report

on “Anti-scrambling and euclidean folds from observer correlators in de Sitter space”
arXiv:2607.14215

Summary of the paper

The manuscript studies a sign reversal in the gravitational response of a de Sitter cosmological horizon relative to a black-hole horizon. A positive-energy perturbation that crosses a black-hole horizon produces the familiar time delay and scrambling effect, whereas matter emitted through a cosmological horizon can make a later signal reach an observer earlier. The authors call this effect “anti-scrambling.” Their aim is both to calculate the effect in controlled low-dimensional models and to identify what kind of microscopic quantum description could reproduce the resulting observer correlators.

Sections 2 and 3 develop the gravitational picture. Three-dimensional Einstein gravity with rotationally symmetric dust is reduced to positive-cosmological-constant JT gravity with two timelike zero-dilaton boundaries. The one-shell solution gives the time advance in equation (3.34). The two-shell solution is more dramatic: the invariant controlling the post-collision dilaton changes sign at a separation of order twice the scrambling time, and the corresponding region ends on a spacelike big-crunch surface. Section 3.5 then leaves the rotationally symmetric sector and solves for a localized shock in a conical-defect background. Equation (3.51) neatly displays both the observer-recoil term and the gravitationally induced angularly averaged time advance.

Section 4 translates this physics into observer correlators. For a massless scalar with Dirichlet boundary conditions, the normal derivative on the observer worldline has a thermal two-point function with inverse temperature 2π . The leading late-time four-point function is written as an eikonal scattering integral with a negative phase, equation (4.22). Equations (4.31)–(4.35) are intended to resum the enhanced eikonal contributions and exhibit the sign opposite to the standard black-hole answer. A two-sided continuation, equation (4.36), gives a normalized correlator that initially exceeds one, and equation (4.37) is used to contrast this behavior with a positivity bound in ordinary thermal quantum mechanics.

Section 5 proposes a more systematic observer-worldline theory. The boundary mass is promoted to a dynamical variable and paired with a physical clock. After integrating over the mass fluctuation, the remaining Euclidean path integral is expressed as an integral over closed curves on a sphere, weighted by minus their length. Expanding about the equator gives a set of boundary modes analogous to the Schwarzian mode. The authors compute the leading $1/b_0$ correction to a general primary two-point function in equations (5.30)–(5.31), and reproduce the perturbative anti-scrambling four-point function in Section 5.3.

Finally, Section 6 observes that the normalized de Sitter four-point function is related to the usual scrambling answer by $w \mapsto -w$. The proposed microscopic interpretation uses a Hamiltonian bounded both above and below and folds the operator contour backward in Euclidean time. Equations (6.9) and (6.12) give positive- and negative-temperature versions of this continuation. The authors acknowledge that the integer labeling the fold is not selected, that the rule is ordering-dependent, and that Stokes phenomena may obstruct continuation of the semiclassical answer.

Evaluation

The paper contains an interesting and potentially publishable collection of results. The cleanest contribution is the common physical thread connecting the one-shell time advance, the two-shell crunch, the sign of the eikonal phase, and the initial growth of the observer four-point function. The low-dimensional setup is concrete, and the localized shock profile usefully distinguishes ordinary recoil from the genuinely gravitational contribution. At leading order, the classical scattering calculation and the boundary-mode calculation also point to the same sign. These are substantive results of interest to the de Sitter quantum-gravity community.

The present version is not yet reliable enough for publication, however. Several displayed equations central to the argument are mutually inconsistent. Most importantly, equation (4.32) drops a factor of w^2 , so its closed form does not have the large- w behavior asserted two equations later. The derivation of the eikonal regime also relies on a nonuniform proper-time expansion whose stated all-orders property is false. The worldline path integral is divergent on the real contour, while no steepest-descent contour or nonlinear integration cycle is specified. In addition, the normalization in equation (5.13) contains the operator insertions themselves. These problems look repairable, but they affect the main quantitative claims rather than peripheral presentation.

The Euclidean-fold idea is thought-provoking, but it is currently an observation about a particular four-point analytic continuation, not a microscopic realization of de Sitter anti-scrambling. No Hamiltonian or operator algebra is supplied, no rule is given for general orderings or higher-point functions, and the integer ambiguity is unresolved. A bounded-spectrum thermal system also has important finite-dimensionality and short-distance consequences that are not addressed. The abstract and conclusions should distinguish sharply between the gravitational results, which are concrete, and this speculative interpretation.

The work is sufficiently novel and interesting for a journal such as JHEP or PRD if the central formulas and logical scope are repaired. The revision must also compare the results to very closely related work that appeared essentially simultaneously, especially arXiv:2607.13137 and arXiv:2607.13665. The former already obtains a tree-level rate $2\pi/\beta_{\text{dS}}$ and an initially growing de Sitter OTOC in general dimension, while the latter analyzes anti-scrambling, failure of the KMS-based quantum-mechanical picture, and observer algebras. A careful comparison should make the distinctive contribution of the present manuscript much clearer.

Recommendation

I recommend **major revision**. I would be favorable toward publication after the authors correct the eikonal and JT formulas, supply a well-defined perturbative contour and normalization for the worldline theory, delimit the Euclidean-fold claim, and position the result against the concurrent de Sitter OTOC and anti-scrambling literature. The central gravitational time-advance mechanism appears physically credible, but the current manuscript does not yet establish all of the conclusions drawn from it.

Major comments

1. **The closed form of the central four-point function is algebraically inconsistent.**

Equation (4.26) gives

$$I(w) = \frac{1}{4} [1 - (1 + w)e^w E_1(w)].$$

It follows that the first equality in equation (4.32) implies

$$F(w) = -4w^2 I(w) = w^2 [(1 + w)e^w E_1(w) - 1].$$

The final expression printed in equation (4.32) omits the prefactor w^2 . This is not compatible with equation (4.34): without the prefactor, the function decays as w^{-2} , whereas with it one obtains the advertised expansion $1 - 4/w + \dots$. Equation (B.7) gives the same check, since at $\Delta_V = \Delta_W = 1$ it reduces to $w^2 U(2, 1, w) = w^2 [(1 + w)e^w E_1(w) - 1]$.

The first equality in equation (4.32) suggests that this is a dropped factor rather than a failure of the integral calculation. Nevertheless, F is used in equations (4.31), (6.2), and throughout the fold discussion, so the authors should correct its definition everywhere and recheck every asymptotic limit and normalization that depends on it. It would also help to state the branch of E_1 and the continuation dictated by the Wightman regulators explicitly.

2. The proper-time expansion used to justify the eikonal regime is not uniform.

The footnote following equation (3.34) says that the linear time-advance formula is valid to all orders in $(\omega/b_0)e^{t_1}$. This does not follow from the exact trajectory. Solving equation (3.30), with

$$s = \sqrt{1 + 2\omega/b_0}, \quad q = e^{-t_1},$$

gives

$$e^{t'_1} = \frac{s + 1 + q(s - 1)}{s - 1 + q(s + 1)}.$$

In the double-scaling limit $\omega/b_0 \rightarrow 0$ at fixed $A = (\omega/b_0)e^{t_1}$, this yields

$$t'_1 - t_1 \rightarrow -\log(1 + A/2),$$

not $-A/2$. Thus equation (3.34) is only the first term in the enhanced variable A .

The eikonal phase may still be correct, but it needs a different derivation. In Kruskal coordinates the exact trajectory gives

$$e^{-t'_1} - e^{-t_1} = \frac{(s - 1)(1 - e^{-2t_1})}{s + 1 + e^{-t_1}(s - 1)} = \frac{\omega}{2b_0}(1 - e^{-2t_1}) + O((\omega/b_0)^2),$$

which is uniform at late time. Translating the shell to t_2 can then lead directly to equation (4.22) through the stationary-phase relation, without treating the proper-time shift linearly in A . The authors should present this controlled argument, state which local shell-energy expansion is being made, and tie the upper limit near $2t_{\text{scr}}$ to the change of sign of the post-collision invariant rather than to the incorrect footnote. The statement that the universe is destroyed should also be qualified as a result for the symmetric closed-universe two-shell model and its stated energy regime.

3. The covariant JT equation and several gravitational normalizations must be corrected.

The first line of equation (3.8) has the wrong sign. Equations (3.4) and (3.7), or direct variation of equation (3.10), give

$$-\nabla_\mu \nabla_\nu \Phi + g_{\mu\nu}(\nabla^2 \Phi + \Phi) = T_{\mu\nu},$$

not the equation with $+\nabla_\mu\nabla_\nu\Phi$ printed in the paper. The null constraint in Appendix A and the jump conditions appear to use the corrected sign, but this should be checked systematically.

In Section 3.5 the shock ansatz in equation (3.49) contains an extra factor dx^+dx^- after the conformal prefactor and is not a metric as written. More importantly, the heavy-observer discussion after equation (3.53) defines the horizon entropy as $S = \log(\pi r_0/2G)$ and then sets $t_{\text{scr}} = \log S$. The entropy is instead

$$S_{\text{dS}} = \frac{\pi r_0}{2G} = 2\pi b_0,$$

so the printed text introduces an erroneous double logarithm. These corrections are important for comparing the recoil, gravitational, and scrambling scales.

4. The Euclidean observer-worldline path integral is not defined on the contour that is written.

After equation (5.11), the effective action is minus b_0 times the length of the boundary curve. For a real fluctuation about the equator, equation (5.16) has quadratic eigenvalue proportional to $-(n^2 - 1)b_0$ for every $|n| \geq 2$. Consequently the real- ρ integral is divergent: $e^{-\hat{S}}$ rewards increasingly long, rapidly oscillating curves. The negative propagator in equation (5.17) already presupposes a rotation of these modes to a steepest-descent contour, but no such contour is stated.

The authors should specify the mode-by-mode contour, the treatment of the $n = \pm 1$ isometry zero modes and its Faddeev–Popov factor, and the nonlinear integration cycle whose perturbative expansion is intended. They should also state whether self-intersecting curves are included and how the measure is defined. Without this information the claim that the formalism permits calculations to all orders in G is too strong; at present Section 5 defines, at most, a formal one-loop saddle expansion.

There are two additional normalization problems. Equation (5.13) defines $\hat{Z}$ with the same matter operator insertion as the numerator of equation (5.11), which would normalize the correlator to one. The partition function must be insertion-free. In equation (5.16), the constant term should be 2π , consistently with the equatorial length and the action $-2\pi b_0$, rather than (1). Both errors should be corrected before the perturbation theory is presented as a quantitative derivation.

5. The one-loop correlators require a more complete renormalization and conformal-map argument.

The parameter q introduced before equation (5.27) encodes a choice of composite-operator renormalization. Setting $q = 1$ to preserve a 2π clock period is reasonable as a condition, but the manuscript should identify the local counterterms that implement it and distinguish scheme-independent information from the constant and contact terms in equations (5.30)–(5.31). This is especially important if these formulas are proposed as data for a microscopic dual.

Appendix C determines the boundary conformal map only to first order in ρ , while equation (5.24) is expanded through second order. A second-order map generally contains a contribution generated by evaluating the first-order holomorphic map on the displaced boundary. If its expectation value is pure gauge or vanishes in the chosen scheme, that cancellation should be shown. Otherwise it must be included. The matter Weyl anomaly and any boundary anomaly should likewise be shown to cancel in the normalized correlator or be incorporated into the effective action.

There is also an explicit pairing error in equation (5.33). For the ordering $V(\tau_4)V(\tau_3)W(\tau_2)W(\tau_1)$, free factorization gives $\langle V_4V_3\rangle\langle W_2W_1\rangle$, not $\langle V_4V_2\rangle\langle W_3W_1\rangle$. The latter pairing belongs to the alternating ordering considered next. The authors should recheck the notation and derivation of equations (5.32)–(5.36).

6. The Euclidean-fold construction is not yet a realization in a standard quantum system.

Equations (6.9) and (6.12) show that one can change the sign of the semiclassical variable w by assigning the four insertions to a folded complex-time contour. This is a useful observation. It does not, by itself, define a quantum model. The integer n is arbitrary, the rule depends on the chosen operator ordering, and the two temperature prescriptions are not selected by any dynamical principle. No consistency condition is checked when adjacent operators are fused, the ordering is changed, or a fifth insertion is added. The possible Stokes obstruction noted by the authors is also central rather than incidental, since the continuation crosses between different semiclassical branches.

There is a further structural issue. If H is bounded above and below and $e^{-\beta H}$ has a finite trace, then the Hilbert space must be finite-dimensional: every eigenvalue of $e^{-\beta H}$ is bounded away from zero. Exact thermal correlators of ordinary finite-dimensional operators are entire and finite at coincident Euclidean time. They therefore cannot exactly equal the singular CFT-like two-point function in equation (6.1). A large finite system might reproduce that expression in a semiclassical time window, with the singularity smoothed at the microscopic scale, but this cutoff and order of limits must be part of the proposal.

Unless the authors give an explicit bounded-spectrum model and a contour rule for general correlators, the abstract should say that a folded correlator can *mimic the four-point anti-scrambling structure*, not that anti-scrambling has been realized in a standard quantum system. A conclusion separating established gravitational results from this conjectural microscopic interpretation would substantially improve the paper.

7. The quantum-mechanical bound and the phrase “saturating the chaos bound” need sharper hypotheses.

Equation (4.37) assumes that V is Hermitian and unitary. The scalar primaries used in the gravitational calculations are not automatically bounded or unitary operators, so the manuscript should exhibit an operator choice for which both the gravitational calculation and the norm inequality apply. Expanding the commutator square also yields a bound on the real part of the regulated cross term unless an additional reflection property makes it manifestly real. This should be shown together with the precise $i\epsilon$ prescription.

The normalization also deserves an explicit modular argument. Equation (4.36) divides by a two-sided correlator $\langle V_R V_L \rangle$, whereas the norm expansion in equation (4.37) naturally contains $\langle V^\dagger V \rangle_\beta$. Their identification requires a definite mirror-operator convention and imaginary half-period translation; it should not be left implicit.

The initial rate in the gravitational answer equals $2\pi/\beta_{\text{dS}}$, but the same correlator is then argued not to obey the thermal analyticity and positivity assumptions entering the usual chaos bound. Calling the rate “saturation of the chaos bound” is therefore potentially misleading. It would be more precise to say that the semiclassical exponent has the numerical value $2\pi/\beta_{\text{dS}}$, while the sign and analytic structure place the correlator outside the class to which the standard bound applies.

8. The novelty comparison must include the closest concurrent work.

The introduction says that the earlier observer-correlator papers all find $4\pi/\beta$, whereas the present paper finds $2\pi/\beta$. The revised manuscript must discuss arXiv:2607.13137, “Out-of-time-ordered Correlators in de Sitter Revisited,” which appeared immediately before the present submission and finds the tree-level value $2\pi/\beta_{\text{dS}}$, the same initial growth, and a time-advance interpretation in general dimension. That work also identifies zero-mode, large-diffeomorphism, and loop-level infrared issues that are highly relevant to the claim in Appendix B that the higher-dimensional computation is identical to the JT calculation.

The manuscript should also compare with arXiv:2607.13665, “A de Sitter Anti-Scrambling Algebra.” That paper discusses the same anti-scrambling terminology, the sign of the OTOC, failure of a KMS-based quantum-mechanical interpretation, and observer-algebra consequences. These submissions may be coordinated or genuinely simultaneous; the issue is not priority but the need to state what is distinct. A compact table comparing the spacetime dimension, observer dressing, recoil sector, gravitational approximation, operator ordering, exponent, and proposed microscopic interpretation across these works, Aalsma–Shiu, Kolchmeyer–Liu, and Narovlansky would be valuable. The bibliography already contains Susskind’s arXiv:2205.00315 on scrambling in double-scaled SYK and de Sitter, but it is not cited in the text and should be included in the microscopic discussion.

9. Appendix B is not adequate support for the advertised three-dimensional generalization.

The appendix contains a placeholder figure caption and several inconsistent wavefunctions. In the expression for ψ_3 , the Fourier variable is called v while the integrand uses x^+ , the denominator is missing the closing parenthesis in $\Gamma(\Delta_W)$, and the prefactor contains t_4^* even though the wavefunction is constructed from t_3^* . These are not harmless in a derivation whose time assignments and phases determine the OTOC sign.

More conceptually, the sentence that the three- and two-dimensional theories are equivalent needs qualification. The reduction in Section 3.1 establishes equivalence only in the rotationally symmetric dust sector. Appendix B should explain why a worldline insertion projects onto precisely this sector, how the angular measure and conical-defect normalization enter the horizon wavefunctions, and why no impact-parameter dependence remains. Equation (B.7) should then be derived with all normalization factors, branches, and regulator assignments shown. The appendix is an important part of the paper because it broadens the result beyond the simplest JT model; it should be brought to the same standard as the main calculation.

Minor comments

1. In the first paragraph of Section 3.4, $O_2(t_2)$ is said to emit a shell of energy ω_1 ; this should be ω_2 . The threshold equation shortly below uses b where the surrounding formulas use b_0 .
2. Equation (4.29) writes the time-reversal relation with an undefined G on the right-hand side; it should presumably be G_4 . The analytic continuation and complex conjugation of the regulators should be included.
3. In equation (6.10), the last right-hand side is self-referential: it contains $\tau_{43}^{QM} + \tau_{21}^{QM}$ where the de Sitter differences are intended.
4. The continuity equation in Appendix A immediately after equation (A.2) is missing its equality to zero. The proper-time condition in equation (A.7) should contain $4\dot{X}^+\dot{X}^-(1 - X^+X^-)^2 = 1$, as follows from equation (3.11), rather than the printed factor 2.
5. In Appendix C, the second endpoint factor in the second line of the conformal transformation formula uses $f'_\rho(u_2)$ twice; the second occurrence should be $f'_\rho(u_1)$. The sentence following equation (C.13) should say that $\zeta_n = 0$ for $n < 0$, not that $\xi_n = 0$, as the next equations themselves make clear.
6. Figure 9 has the caption “Caption.” All figures and captions in the appendix should be completed, and the overlong one-sided time assignment near equation (B.8) should be typeset readably.

7. The manuscript would benefit from a short final Discussion or Conclusions section summarizing the domains of validity: low dimension, spherical symmetry except in Section 3.5, small local shell energy, the eikonal window in equations (4.28) and (4.30), and the formal status of the worldline contour.
8. There are numerous correctable language errors, including “their could be,” “flaps it wings,” “quantitatively,” “sigmal,” “prefector,” “could repeated,” and “time-seperation.” The title should capitalize “Euclidean.”