

Referee Report

Cosmological Collider Signals at Strong Mixing

arXiv:2607.14891

Summary and recommendation

The manuscript studies a constant-turn two-field inflationary system in which the quadratic derivative mixing between a canonically normalized curvature mode q and an isocurvature mode σ is treated nonperturbatively. Starting from the exact linear solutions obtained previously by Huenupi, Muñoz, Palma, and Sypsas, the authors construct the mixed late-time external leg $K_{\sigma q}$ in equations (3.5)–(3.7). For a cubic σ^3 interaction, this reduces the tree-level in-in bispectrum to the single integral (4.3). In the squeezed configuration the late-time nonanalytic branches of the soft leg give

$$B_{q,\text{nonan}}(k, k, ck) \propto c^{-3/2} |C_{\lambda,\rho} J_{\lambda,\rho}| \sin(\rho \ln c + \theta_{\lambda,\rho}),$$

where the soft coefficient $C_{\lambda,\rho}$ is a finite gamma-function sum and the hard coefficient $J_{\lambda,\rho}$ is expressed in equation (4.17) as an absolutely convergent one-fold series. The manuscript also derives a fixed-mixing, large- ρ endpoint/JWKB expansion for the amplitude and phase.

This is a potentially useful analytic advance. The logarithmic oscillation and its $c^{-3/2}$ envelope are standard quasi-single-field results, and the exact linear basis and power spectrum are imported from earlier work. The distinctive content here is therefore the nonperturbative mixing dependence of the σ^3 amplitude and phase, the reduction of the hard integral to a convergent series, and the proposed large-mass expansion. The compactification and factorization leading from equation (D.5) to (4.17) appear internally consistent at fixed $\lambda > 0$ and $\rho > 0$, and the algebraic combination of the displayed large- ρ formulas is also consistent within a genuinely fixed- λ limit.

The present version nevertheless falls short of the standard required for publication in JHEP or PRD. A significant part of the parameter grid is not physically characterized; the squeezed expansion is used nonuniformly under an integral without a regions argument; the crucial weak-mixing check is absent; and the plotted raw coefficient is not normalized by the strongly mixing-dependent power spectrum. The stated domain of the JWKB result is incorrect, and its global endpoint dominance is not yet demonstrated. Appendix C contains explicit inconsistencies in both momentum scaling and the Wightman commutator. Finally, the branch prescriptions and numerical checks are not detailed enough to validate formulas that involve delicate cancellations and analytic continuation.

Recommendation: major revision. I would be willing to reconsider a substantially revised manuscript. The exact mixed-leg construction and the hard-integral series could form the basis of a publishable result, but the authors must first establish their physical domain, supply decisive analytic and numerical cross-checks, correct Appendix C, and present the signal in an observable normalization.

Assessment of the derivation

Several parts of the calculation are persuasive. Equations (2.3)–(2.10) correctly follow from writing the time-derivative sector as $\frac{1}{2}(\dot{q} - \lambda H \sigma)^2 + \frac{1}{2}\dot{\sigma}^2$. After the decaying integration constant in equation (2.11) is discarded, the apparent $-\lambda^2 H^2$ term in the σ equation cancels, giving the late-time indices in

equation (2.12). The matrix Wronskian (2.15), the Wightman-function dictionary (2.20)–(2.23), and the combinatorial factor and sign in the tree-level in-in expression (4.3) are mutually consistent. The dimensional rescaling from equations (4.3) to (4.6) is also correct.

Conditional on inserting the soft expansion under the integral, the subtraction in equation (4.11) is the right one:

$$\text{Im}\left[Ce^{i\rho}J^+ + C^*e^{-i\rho}J^-\right] = \text{Im}\left[Ce^{i\rho}\{J^+ - (J^-)^*\}\right].$$

This gives the sine and phase in equation (4.13), including its overall normalization. In Appendix D, Pfaff transformation, the cancellation $p_\eta - 2 + \alpha_\eta = 1$, and the factorization of the double integral are coherent. The endpoint estimate $I_{s,n}^{(\eta)} = O(n^{-2})$, together with $(p_\eta)_n/n! = O(n^{3/2})$ in magnitude, indeed gives an $O(n^{-5/2})$ summand and absolute convergence at fixed nonzero parameters. The three powers in the Hurwitz-zeta tail are consistent with this endpoint expansion.

These positive checks do not settle the central questions below. In particular, absolute convergence of the final series does not by itself justify the analytic continuation and contour interchanges used to reach it, nor does agreement with a second evaluation of the same integral representation test the imported mode normalization.

Novelty and significance

The paper should state more sharply what is new. The universal squeezed scaling, the pair of principal-series branches, and the interpretation of ρ as the logarithmic clock frequency are established results. The exact linear modes and equation (2.27) are likewise results of the cited Huenupi et al. paper. The new claim is the exact-in- λ σ^3 coefficient, together with a useful analytic representation and its fixed-mixing asymptotics. That is a worthwhile but narrower advance than the current broad presentation suggests.

The assessment of novelty is also incomplete because the Note added cites two coordinated works merely as *work in preparation*. At least one is now identifiable as Lucas Pinol, *New exact bispectrum shapes in multifield inflation*, arXiv:2607.15251. That work also uses the exact strong-mixing linear basis and derives a one-dimensional representation and an exact squeezed bispectrum, although for a different cubic interaction. The revised manuscript should compare the two constructions, conventions, domains, and squeezed coefficients directly, and should identify precisely what the present σ^3 result adds. The placeholder attribution *Lucas et al.* is also inaccurate for a single-author paper. A similar update is needed for the other coordinated work once it is public. More broadly, the discussion should connect the analytic formulas to the existing numerical strong-mixing literature and to recent analyses of the limitations of cosmological-collider observables at large mixing.

Major comments

1. **Clarify the mass parameters, stability domain, and physical meaning of the plotted strong-mixing region.**

Equation (2.3) contains the net quadratic term

$$-\frac{1}{2}(\mu^2 - \lambda^2 H^2)\sigma^2.$$

Thus, if m^2 denotes the mass parameter before completing the kinetic square, then $m^2 = \mu^2 - \lambda^2 H^2$, while μ is the effective mass that controls the late-time index and $\rho^2 = \mu^2/H^2 - 9/4$. This distinction is central when λ is varied at fixed ρ , but it is nowhere stated.

A local constant-coefficient analysis makes the issue concrete. Ignoring Hubble friction over a short interval and writing $p = k/a$, the two frequency branches following from (2.3) obey

$$\omega_{\pm}^2 = p^2 + \frac{\mu^2}{2} \pm \frac{1}{2}\sqrt{\mu^4 + 4\lambda^2 H^2 p^2}, \quad \omega_+^2 \omega_-^2 = p^2(p^2 + \mu^2 - \lambda^2 H^2).$$

When $\mu^2 < \lambda^2 H^2$, the lower branch has a tachyonic momentum interval. For $\rho = 1$, for example, $\mu/H = \sqrt{13/4} \simeq 1.80$, while Figures 1–4 extend to $\lambda = 3$. A substantial part of those curves therefore describes a transiently unstable rather than an ordinary stable heavy-field regime.

The authors should define bare, entropic, and effective masses unambiguously; state the stability and background-realizability conditions; and mark or remove parameter points outside the intended regime. If transient tachyonic growth is intentional, it should be analyzed as such, including its duration, its effect on the choice of state, and the resulting backreaction and perturbativity bounds. The strong-mixing claims cannot be assessed while stable and unstable regimes are plotted together without comment.

2. Justify the squeezed-limit expansion by a regulated method-of-regions analysis.

The step from equation (4.7) to equations (4.8)–(4.13) replaces $T_{\sigma q}(cx)$ by its small- cx expansion inside an integral over $0 < x < \infty$. This expansion is not uniform: the region $x = O(c^{-1})$ has $cx = O(1)$. The standard physical expectation is that the $x = O(1)$ region produces the nonlocal clock signal, whereas the large- x region contributes analytic/contact terms after the Bunch–Davies prescription is imposed. That expectation must be shown here because the hard integrals are only oscillatory at infinity and because the mixing has been resummed.

The revision should introduce the $i\epsilon$ prescription already in Section 4, split the integration regions, and demonstrate that the $x = O(c^{-1})$ region cannot alter the coefficient of $c^{3/2 \pm i\rho}$. It should also bound the remainder after subtracting the two nonanalytic soft branches. Without this argument, equation (4.13) does not yet follow from the exact bispectrum (4.7), even if the formal coefficient $J_{\lambda,\rho}$ is evaluated correctly.

3. Supply the weak-mixing benchmark, including normalization and phase.

The formulas contain delicate apparent singularities: equation (3.6) has an explicit $1/\lambda$, while equation (4.17) has $1/\lambda^2$, and the finite answer emerges only after the $s = \pm 1$ branches and then $J^+ - (J^-)^*$ are combined. The decisive check is the known perturbative quasi-single-field limit. For the σ^3 diagram one expects

$$T_{\sigma q} = O(\lambda), \quad C_{\lambda,\rho} = O(\lambda), \quad J_{\lambda,\rho} = O(\lambda^2), \quad B_{\text{nonan}} = O(\lambda^3).$$

The manuscript merely warns that the limit must be taken after combining branches.

The authors should derive the leading small- λ coefficient and phase explicitly and compare them, with the same mode and in-in conventions, to the standard three-mixing-insertion calculation. This comparison should test the full numerical prefactor and sign, not only the λ^3 scaling. Direct numerical integration of the original coupled mode equations at small λ would provide an independent check.

The exact-leg derivation should also be made self-contained. Appendix A states the branch expansion for ζ_b^+ and the sums over $A_s^{(b)}$, but reaching equation (A.6) additionally requires the

boundary identity that relates $\zeta_b^+(0) - \zeta_b^-(0)$ to $\sum_s A_s^{(b)} \mathcal{U}_s$. This identity and the origin of $\mathcal{U}_s$ in equation (3.7) are not supplied. Adding them would make the normalization transparent and would provide a parallel derivation of the power spectrum (2.27). The phrase *corrected identities* preceding equations (3.1)–(3.2) should say what was corrected and relative to which published formulas.

4. Present an observable, power-normalized signal and its domain of perturbative control.

Figures 1–4 plot $|J|$ and $|CJ|$, but $|CJ|$ is not by itself an observable clock amplitude. Equation (2.27) shows that the curvature power spectrum can depend very strongly on the same mixing parameters. Once the observed scalar amplitude is held fixed, the relevant reduced bispectrum contains the corresponding powers of the exact power-spectrum ratio. A raw growth of $|CJ|$ can therefore substantially overstate the growth of the observable non-Gaussianity.

The revised paper should give a dimensionless reduced bispectrum or f_{NL} -type clock coefficient, normalized by the appropriate $P_\zeta(k_L)P_\zeta(k_S)$, and replot the parameter dependence after using equation (2.27). The normalization convention for Δ_ζ should be made explicit, since equation (2.25) denotes by Δ_ζ what is often written as $\mathcal{P}_\zeta$ or Δ_ζ^2 .

The same discussion must state the bound on g_{σ^3}/H required for the first-order cubic calculation and for loop corrections to remain small, especially in the amplified region. In a complete constant-turn model, other mixing-related cubic vertices are generally present; the authors should explain when the isolated σ^3 diagram is meaningful and when it dominates. Claims concerning signal enhancement or removal of Boltzmann suppression should be made only for this normalized, perturbatively controlled quantity.

5. Correct the validity condition and complete the justification of the large- ρ expansion.

The sentence following equation (5.4) states that a practical condition is $\lambda/\rho \ll 1$. This is not sufficient for the displayed expansion. The first relative corrections in equations (5.1), (5.2), and (5.3) scale as

$$\frac{6\lambda^2 + 1}{12\rho}, \quad \frac{12\lambda^2 + 1}{24\rho}, \quad \frac{\lambda^2 + 1/8}{\rho},$$

respectively. If λ is allowed to grow with ρ , the displayed ordering requires at least $\lambda^2/\rho \ll 1$, together with control of the higher coefficients. The condition $\lambda/\rho \ll 1$ can hold while the nominal NLO term is order unity.

It follows that the suggestion in Section 5 and the Discussion that the fixed-mixing expansion indicates lifting of Boltzmann suppression for $\lambda/\rho = O(1)$ is unsupported: this regime lies parametrically outside the derivation. That inference should be removed, or replaced by a uniform double-scaling analysis and exact numerics for the power-normalized observable.

There is a second control issue in Appendix E. The local conical/Bessel matching identifies a plausible endpoint contribution, but the manuscript does not prove that it is the complete algebraic asymptotic series. It notes that a global steepest-descent deformation would require cut-lip contributions, yet it does not bound those contributions, the $t_j \rightarrow \infty$ faces and mixed corners, or the error made by extending the endpoint Bessel profile to $u = \infty$ through Abel continuation. In particular, some large- t phase sectors lose their leading oscillation and need an explicit estimate. The authors should supply a matched-regions or global contour argument establishing equation (5.2) and its remainder. Numerical tests at $\rho = 10, 20, \dots$, with $\lambda^2/\rho \ll 1$, should then verify the predicted $1/\rho$ error scaling.

6. Re-derive or remove Appendix C.

Appendix C supports an abstract-level claim about isocurvature Wightman functions, but it is not internally consistent. At fixed conformal times, a normalized massive mode behaves as

$$\sigma_k(\tau) \sim k^{-3/2}(-k\tau)^{3/2\pm\nu},$$

so the three late-time momentum structures in $G_{\sigma\sigma}(k; \tau_1, \tau_2)$ scale as $k^{2\nu}$, $k^{-2\nu}$, and k^0 . Equation (C.6) instead gives $k^{3+2\nu}$, $k^{3-2\nu}$, and k^3 . The latter would correspond to the dimensionless quantity $k^3 G$, not to the Green function defined in (C.1), and it conflicts with the explicit $e^{\pm 2i\rho \ln k}$ dependence in (C.7)–(C.11).

The final commutator formulas are also contradictory. The equation immediately after (C.11) implies $G^> - G^< = 2i \operatorname{Im} P$, whereas the following position-space equation replaces $\operatorname{Im} P$ by P . At equal times P is generally real and nonzero, so the latter formula would violate the equal-time field commutator. Moreover, a leading small- k analytic term cannot be promoted without qualification to an exact delta-local relation over the full momentum range. The text saying that $G^>$ and $G^<$ *coincide* is inconsistent with the difference subsequently displayed, and the z - and τ -arguments are mixed.

The authors should redo the normalization and Fourier transform from (C.1), distinguish the nonanalytic part from local analytic terms, and check the equal-time canonical relations. They should also avoid calling $G^>$ and $G^<$ the full Schwinger–Keldysh bulk-to-bulk kernel unless the time-ordered $++$, $--$, $+-$, and $-+$ components are actually constructed.

7. Make the Mellin–Barnes continuation and the status of the residue expansion precise.

The binomial Barnes representation (B.5) ordinarily requires a strip separating the poles of $\Gamma(a_s + u)$ and $\Gamma(-u)$. With $a_s = i s \lambda / 2$, that real strip collapses. Referring generically to *analytic continuation selected by the Bunch–Davies prescription* does not specify the lateral contour, the regulator, or which poles are crossed when the two-dimensional contour is closed. These details matter for the phase of $C_{\lambda, \rho}$.

There is also an explicit warning sign in the analytic residue families written between equations (B.13) and (B.16): $\mathcal{S}_{m,n}^{(2)}$ contains $\Gamma(-m + n + 1)$, which has poles for infinitely many nonnegative m, n as the formula stands. Intersecting pole divisors require combined or higher-order residues and can generate logarithms; the displayed expression is not an ordinary finite coefficient. Since the paper uses only the two nonanalytic families, the safest revision may be to delete the purported full analytic expansion. Otherwise, the authors must provide the allowed residue cones and a collision prescription, and demonstrate the cancellations explicitly.

Appendix D should likewise state the parameter domain in which each interchange is justified before continuation, distinguish absolute convergence of the final n -series from conditional convergence of the original oscillatory x -integral, and explain the nonuniform $\rho \rightarrow 0$ and $\lambda \rightarrow 0$ limits. These additions are necessary for equation (4.17) to be regarded as an exact representation rather than a formal one.

8. Strengthen the numerical validation and make it reproducible.

The numerical points in Figures 1–4 are described only as an *independent finite-cutoff numerical integration*. No cutoff, $i\epsilon$ regulator, working precision, quadrature transformation, tolerance, or uncertainty is reported. The leading endpoint-tail correction is explicitly not a rigorous error bound, yet no N -dependence or residual is shown. This is insufficient for quantities formed from exponentially ill-conditioned branch cancellations and for phases that jump at zeros.

At minimum, the authors should provide convergence tables in N , cutoff, regulator, and precision; quote relative errors for magnitudes and complex errors before taking an argument; and release the

code or numerical data behind the four plots. The present double-integral check is derived from the same special-function representation as the series. A genuinely independent validation should integrate the original coupled mode equations with canonical Bunch–Davies initial data, compare $K_{\sigma q}$ directly with equation (3.6), and insert the numerical modes into (4.3). A finite- c test of (4.7) against (4.13) would also check the squeezed extraction. Tests should include the weak-mixing regime, points near phase zeros, the boundary between stable and transiently unstable evolution, and a large- ρ sequence that lies inside the corrected JWKB domain.

9. Update and sharpen the comparison with prior and concurrent work.

The revision should separate three layers: the exact linear solution and power spectrum inherited from Huenupi et al.; the standard quasi-single-field nonanalytic structure; and the genuinely new exact-in-mixing σ^3 coefficient, series, and asymptotics. A formula-by-formula comparison with the coordinated Pinol paper is essential, including the relation between its leg-kernel representation and equations (3.6), (D.4), and (4.17), and the different physics of its cubic vertex. Relevant numerical strong-mixing and phase-focused literature should also be discussed. Until this comparison is supplied, the novelty and phenomenological significance of the main result cannot be evaluated reliably.

Minor comments

1. Define $\nu^2 = 9/4 - \mu^2/H^2$ explicitly next to equation (2.12), rather than relying on the heavy-field definition in the Introduction.
2. State whether $\lambda > 0$ is assumed without loss of generality and explain how $\sigma \rightarrow -\sigma$ changes both λ and g_{σ^3} , including the resulting phase convention.
3. Equation (4.1) is a conformal-time interaction density containing a^4 , whereas Section 2 uses cosmic time and a^3 . Label the measure and Hamiltonian convention explicitly.
4. The phase $\arg(CJ)$ is meaningful only away from zeros of CJ . The discussion of Figures 2 and 4 should distinguish principal-branch jumps from a physical change of phase and should quote magnitudes near any apparent jump.
5. Use f'_+ and f'_- , rather than f'_1 and f'_2 , in Appendix C to match the definition in Section 3. Specify the differentiation variable in each two-time expression.
6. Equation (2.15) is the field–momentum Wronskian, but the full two-field canonical algebra also requires the field–field and momentum–momentum isotropy conditions. State that the Bunch–Davies basis enforces and preserves these conditions, or display them explicitly.
7. Correct the typographical error *yiel* before equation (B.16), use a single typographic convention for the imaginary unit, and number the final equations in Appendix C if they are retained.
8. The figure captions should state the actual λ grid, numerical precision, and whether the solid curves interpolate discrete samples or are evaluated continuously.
9. If the bulk-to-bulk material is retained in the abstract, Appendix C must contain a complete, consistent result. Otherwise the abstract claim should be narrowed to the mixed external leg used in the bispectrum.