Referee Report

Observers, Local Measurements, and Topology

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Summary and recommendation

The manuscript proposes an operational route from measurements made along an observer's worldline to topological information about an observer-accessible part of a fluctuating spacetime. Sections 2 and 3 review relational observer algebras, their proposed type-II completion, and measurements by effects. In Section 4, a detector i is declared active when the probability $p_i(\tau)$ in equation (4.1) exceeds a threshold. Each measurement interval I_α gives a vertex and a set S_α of active detector labels. The central object is the complex in equation (4.6): a finite collection of events is a simplex when all events share at least one active detector and their total proper-time span is at most Λ. The disk and hole examples in Section 4.2 compute the homology of two stipulated activity patterns. Section 5 then varies the probability threshold θ or the temporal scale Λ and interprets persistent classes as candidate spacetime features. Appendix A gives a formal nested-commutator expansion for interacting relational observables.

There is an appealing research program here. It is useful to ask which observer-relative data survive fluctuating geometry, and the combination of gravitational observer algebras with topological data analysis appears to be a novel conceptual juxtaposition. Several mathematical steps are sound. The face-closure argument following equation (4.6) does define an abstract simplicial complex; the boundary matrices and Betti numbers in the disk and hole examples are consistent with the stated activity sets; and the Λ-filtration in equation (5.4) is valid when the event windows are held fixed. The first-order expression in equations (2.10)–(2.19) and its formal Dyson continuation in Appendix A have the expected nested-commutator structure, although their adiabatic limit requires a nontrivial spectral solvability condition that is not currently stated.

The central physical conclusion, however, is not established. Equation (4.6) computes the homology of an event–detector incidence relation. A common detector label is not, without additional hypotheses, equivalent to a nonempty contractible intersection of spacetime regions. The manuscript therefore has not shown that its complex is the nerve of a spacetime cover, or that its homology is invariant under changes of sources, detectors, sampling, opacity, lensing, or measurement dynamics. The examples choose detector patterns that already contain the desired disk or circle homology rather than deriving those patterns from response operators and a state. Persistence of a feature in the two chosen parameter filtrations does not repair this identifiability problem.

Recommendation: major revision. The paper is not suitable for PRD or JHEP in its present form. I would reconsider a substantially revised version that supplies either (i) a conditional reconstruction theorem with a precise target space, a map from measurements to a good cover, and controlled sampling hypotheses, or (ii) a nontrivial field-theoretic detector calculation in a spacetime of known topology, including false-positive and false-negative tests. The measurement protocol, the filtration, and the role of the type-II algebra also require significant clarification. Without one of these new pieces, the work remains an interesting analogy and proposal, rather than a demonstrated result at the standard expected for publication.

Assessment of correctness and logical structure

The manuscript's logical chain has two distinct parts. The first constructs a perfectly legitimate complex from thresholded data. If the event labels form the finite set X and detector labels form Y, define a relation

$$\alpha \, R \, i \quad \Longleftrightarrow \quad i \in S_\alpha.$$

Ignoring the temporal cutoff, equation (4.6) is the relation (or Dowker) complex on X: a set of events is a simplex exactly when it has a common witness in Y. The Λ condition intersects this relation complex with a proper-time diameter complex. This observation explains why the simplicial identities and the toy homology calculations work.

The second part identifies that data complex with the topology of spacetime. No implication establishing this identification is proved. The discussion around equations (4.5)–(4.6) moves from "the same detector is active" to "the associated regions intersect," but neither the regions nor the map between these predicates is defined. The good-cover assumption invoked later in Section 4.1 concerns hypothetical spacetime regions; it does not show that the detector incidence relation is their nerve. The temporal cutoff can remove genuine nerve simplices as well as suppress accidental matches. Consequently, the correct algebraic calculations establish the topology of the recorded incidence data, not the asserted topology of an underlying spacetime.

There are also two internal issues in the operational setup. First, the event intervals are defined by above-threshold activity in equation (4.2), yet Section 5 varies the same threshold while treating the event labels and intervals as fixed. Second, the large-entropy argument for reusing the same state after sequential measurements is not valid in general: a small entropy change does not bound changes in simple local expectation values. These points affect the claimed filtration and the input probabilities, rather than only the exposition.

Novelty and significance

The potentially interesting advance is the attempt to connect relational gravitational observables to an operational topology-recovery protocol. However, each component presently used after equation (4.1) is standard and does not depend visibly on gravity: thresholded one-point probabilities, relation/coactivity complexes, simplicial homology, and one-parameter persistence. Effects and the Born rule are available for general von Neumann algebras, including type III algebras; a trace is needed for the particular density representation in equation (3.1), not for the probability $\omega(\mathcal{E}_i)$ itself. The manuscript should therefore identify a specific feature of the gravitational type-II construction that changes, constrains, or enriches the topology inference.

The closest comparison made in the paper is the Curto–Itskov neural coactivity construction, but it receives only one sentence. Equation (4.6) should also be positioned relative to relation/Dowker complexes, witness and coactivity complexes, and the distinctions among nerve, Čech, and Vietoris–Rips constructions. The persistence material in Section 5 is a standard review; novelty would instead lie in a recovery or stability result tailored to gravitational measurement data. The cited Kajuri–Siddh calculation of Unruh–DeWitt response in toroidal spacetime appears to offer a particularly natural benchmark. A direct calculation of equation (4.1) in such a model would be substantially more informative than the hand-assigned activity patterns in Section 4.2.

Major comments

1. **Establish the missing reconstruction theorem, or explicitly limit the claim to the topology of measurement data.**

   Equation (4.6) does not by itself approximate the nerve of a cover. Indeed it has a simple non-identifiability property. Given *any* finite simplicial complex L on the event vertices, introduce one detector label for every maximal simplex of L and declare that detector active precisely at the vertices of that maximal simplex. For Λ larger than the full time span, equation (4.6) returns exactly L, independently of the spacetime. Conversely, one detector active at every event makes the complex a full simplex and removes all positive-degree homology. Thus unrestricted response operators can manufacture or erase arbitrary topology.

   A publishable reconstruction statement needs precise hypotheses that prevent this. The author should define spacetime subsets U_α associated with events and prove when

   $$\bigcap_{\alpha\in\sigma} S_\alpha \neq \varnothing \quad \Longleftrightarrow \quad \bigcap_{\alpha\in\sigma} U_\alpha \neq \varnothing.$$

   The nonempty intersections must also be contractible if the nerve theorem is to be invoked. An alternative formulation could use detector response regions and the two Dowker complexes of the event–detector relation, but then the effect of the finite-Λ restriction and the required sampling conditions must be proved. The current statement near the end of Section 4.1 that the measurements are "sufficiently fine" simply assumes the result that the paper needs to establish.

2. **Specify the target topology and demonstrate it with a physical response model.**

   The text alternates among the topology of spacetime M, the timelike envelope or causal region accessible to the observer, and a spatial section in the $(2+1)$-dimensional example. These spaces generally have different homology. The target should be fixed, including its boundaries and the observation-time dependence. If the intended object is a spatial slice, the relational rule that associates worldline data to that slice must be given; if it is a causal domain or timelike tube, its topology should be used consistently throughout.

   The activity sets in equations (4.10)–(4.11) and the corresponding hole example are selected so that the desired Betti numbers follow. No effect $\mathcal{E}_i$, state ρ, or probability from equation (4.1) is actually computed. The example therefore checks elementary boundary-matrix algebra, not topology recovery. A revision should calculate detector responses in at least one field theory on two geometries with controlled topology, run the same reconstruction protocol on both, and quantify its dependence on source placement and sampling. The toroidal detector model already cited in the Introduction is an obvious starting point.

   The assertion in Section 4.2 that a black hole is included as a topological hole because radiation cannot escape the horizon should also be removed or made precise. Signal obstruction is a causal property, not by itself a homology class. An opaque body, source anisotropy, lensing, or a horizon can produce similar response data, while the homology depends on the chosen spacetime region and dimension. A topology claim must address these degeneracies rather than identify non-detection directly with a hole.

3. **Replace the state-reuse argument with a controlled measurement protocol.**

   The paragraph following equation (4.1) argues that a measurement changes only a tiny fraction of the entropy of a large state and that simple later effects therefore cannot resolve the change. This implication is false. For example, take a maximally mixed qubit tensored with an arbitrarily large bath. Before a projective measurement, a Pauli observable has expectation zero; conditioned on the "up" outcome, the same simple observable has expectation one. The entropy changes by only $\log 2$, independently of the bath entropy. The same finite-dimensional example can be embedded in a type-II_1 factor.

The author should formulate the measurements by quantum instruments and use the resulting sequential state, or impose and justify a non-demolition, weak-coupling, reset, or independent-ensemble regime. In addition, the paper works with exact theoretical probabilities although the observer records single outcomes. Estimating $p_i(\tau)$ requires repeated preparations, stationarity/ergodicity assumptions, or finite-sample confidence bounds. In a single cosmological history these are substantive assumptions. Without such a protocol, the construction is not yet operational in the sense claimed.

4. **Define event windows independently of the threshold and repair the θ-filtration.**

   Equation (4.2) defines I_α as an interval during which at least one detector is active. The intervals are otherwise arbitrary: splitting one interval into two or merging adjacent intervals changes the vertex set and can change homology. The existential condition "there is a $\tau \in I_\alpha$" can also combine unrelated short activations inside a coarse window. A canonical rule is needed, for example a threshold-independent sampling grid or fixed detector-resolution windows.

   This issue becomes an inconsistency in equations (5.1)–(5.3). As θ changes, above-threshold time components can appear, disappear, split, merge, and move their endpoints. The proof of $\mathsf{K}_{\theta_2,\Lambda} \subseteq \mathsf{K}_{\theta_1,\Lambda}$ instead assumes a common vertex set and fixed windows. With a threshold-independent master set of windows, allowing empty activity sets, the inclusion can be restored. The paper should adopt such a definition before drawing conclusions from θ-persistence.

5. **Distinguish persistence from evidence of physical correctness.**

   Equations (5.3) and (5.4) provide two monotone parameters, while equation (5.5) reviews ordinary one-parameter persistence. If both θ and Λ are varied, the natural object is a bifiltration, with reversed order in θ and ordinary order in Λ; a generic multiparameter persistence module does not have the barcode decomposition displayed in equations (5.6)–(5.7). The author should either restrict the analysis explicitly to one-parameter slices or use appropriate multiparameter invariants.

   More fundamentally, a long bar is only a feature stable under the chosen filtration. The arbitrary-complex construction in comment 1 can generate arbitrarily persistent but completely instrument-induced classes. To support the physical interpretation in Sections 5.2–5.3, the paper needs a noise and sampling model and a stability/consistency statement controlling perturbations of $p_i(\tau)$, sampling times, detector calibration, and event partitioning. The admissible scale $\Lambda_{\max}$ should be derived from a causal or response model rather than selected after inspecting the barcode.

6. **Clarify why a type-II algebra and a trace are needed.**

   Section 3 correctly notes that an effect is an operator $0 \leq \mathcal{E} \leq 1$, but the probability of an effect is defined by any normal state $\omega(\mathcal{E})$. Type-III algebras have abundant effects and projections and support the Born rule; what they lack is a trace density representation of every normal state in the form used in equation (3.1). Consequently the statement in Section 4.1 that the fixed-background case would require a reformulation is misleading, and none of equations (4.1)–(4.6) structurally requires a type-II factor once $p_i = \omega(\mathcal{E}_i)$ is used.

   This point affects both scope and novelty. The author should either formulate the construction for general algebraic states and present gravitational observer algebras as an application, or show a concrete topological datum that depends on the trace, the continuous dimension of projections, or another specifically type-II feature. At present the lengthy gravitational algebra discussion and the incidence construction are only loosely coupled.

7. **Resolve the inconsistency in the interacting-observer model of Section 2.2.**

   The perturbation V in equations (2.4)–(2.5) is treated as a bounded element of the bulk algebra that commutes with the observer energy q. The perturbed vector in equation (2.21) is therefore obtained by applying a bulk operator to a bulk–observer product vector and remains a product. This does not model a detector interaction and cannot support the later statement that the interaction induces observer–bulk correlations. A realistic localized coupling would contain an observer or detector operator, for example a monopole operator multiplied by $\phi(\gamma(\tau))$. Such a coupling need not commute with q or its positive-energy projection, and it would require the fixed-point and modular arguments to be redone.

   Even for a purely bulk bounded perturbation, the hypotheses from Araki perturbation theory should be stated precisely. Being annihilated by the candidate Liouvillean in equations (2.22)–(2.25) is not on its own a proof that the vector is cyclic, separating, KMS, and has the asserted modular operator. The paper also asserts, rather than demonstrates, that the compressed fixed-point algebra in equation (2.27) is type II_1 and that equation (2.28) defines its trace. The relevant cocycle/crossed-product isomorphism and the normalization after compression should be supplied. If this subsection is intended only as review and is not used later, it would be better shortened and its assumptions stated explicitly.

8. **State and verify the solvability conditions for the perturbative constraint.**

   The $i\varepsilon$ prescription in equations (2.11)–(2.13) regulates the inverse of $\mathrm{ad}_{\widehat{H}_0}$ but does not remove its kernel. For any spectral projection P_E of $\widehat{H}_0$ one has

   $$P_E[\widehat{H}_0, X]P_E = 0.$$

   Consequently equation (2.10) can have a solution only if

   $$P_E[V, \Phi_s^{(0)}]P_E = 0$$

   in every energy block (with the analogous statement for continuous spectral subspaces). Although $\Phi_s^{(0)}$ commutes with $\widehat{H}_0$, it need not be scalar inside degenerate sectors, so this condition is not automatic. A nonzero zero-frequency component of the source makes equation (2.11) diverge as $1/\varepsilon$; one cannot then simply "remove ε" after equation (2.18).

   The same obstruction recurs at every order in the recursion of Appendix A. The author should impose and prove the relevant cohomological/spectral conditions order by order, or formulate the result in terms of a Møller operator for a genuinely switched interaction and state hypotheses ensuring that it exists. Until this is done, Appendix A supplies a formal series, not an all-order demonstration that the interacting observer algebra survives.

9. **Reposition the result relative to the relevant topology and measurement literature.**

   The relation to Curto–Itskov should be explained in detail, not merely cited as inspiration. The author should discuss relation/Dowker duality, witness and coactivity complexes, and why equation (4.6), including its time cutoff, is different from or improves upon those constructions. The claims of measurement operationality should be compared directly with the Fewster et al. measurement-scheme paper already cited. The use of persistent homology should be distinguished from Carlsson's general framework and from the author's previous persistence applications. Finally, the paper should explain what genuinely quantum or gravitational information is added beyond a classical sensor-network construction. Calling the shared-label condition a “correlation” is currently misleading: it is coactivation after thresholding one-point probabilities, not an operator or sequential quantum correlation.

10. **Align the abstract and conclusions with what is actually proved.**

    The abstract says that the complex captures topological information about spacetime, and Sections 4 and 6 repeatedly say that the observer can compute the homology of the accessible region. Section 6 simultaneously lists a rigorous proof of precisely this identification as future work. Until a reconstruction theorem or detector calculation is added, the manuscript should state that it defines a topology of measurement-incidence data and conjectures conditions under which this may reflect spacetime topology. The claims about generic closed-universe algebras being type II and admitting maximal-entropy thermal states should likewise be separated clearly from the established de Sitter examples and labeled as assumptions.

Minor comments

1. In Section 3, a general normal state on a semifinite algebra is represented by a positive L^1 density affiliated with the algebra; it need not be a bounded element $\rho \in \mathcal{A}$. Either restrict the class of states or use the standard noncommutative L^1 formulation.

2. Equation (4.1) begins with detector-dependent thresholds θ_i, whereas equation (4.2) and the filtrations use a common θ. State the simplifying specialization and its consequences explicitly.

3. In the disk example the basis contains e_{40}, while the following sentence says that the orientation convention is $a < b$. The displayed boundary of $[v_0 v_3 v_4]$ is consistent with $e_{40} = [v_4 v_0]$, so the stated orientation convention or the basis label should be corrected.

4. The interval decomposition in equations (5.6)–(5.7) should state the finite/tame one-parameter hypotheses under which it is being used. The word “otherwise” in equation (5.7) should be set with `\text{otherwise}`.

5. In equation (A.6), ϕ_s is undefined. It should be replaced by the previously defined $\Phi_s^{(0)} = \phi(p+s)$, or defined explicitly.

6. The suppressed $i\varepsilon$ factors in Appendix A are material to the existence of the integrals and the treatment of the kernel of $\mathrm{ad}_{\widehat{H}_0}$. State whether the all-order series is only formal or give sufficient convergence and adiabatic-limit conditions.

7. Please correct recurrent typographical and grammatical errors, including “two-ouctome,” “the answer can be find,” “bluk modular operator,” “wordline,” “the state describe,” “they develops,” and “an the observer.”

8. The manuscript should distinguish a detector label, a physical source, and a measurement outcome. The worked example assumes a one-to-one source–detector relation, whereas the general definition of S_α does not. This distinction is central to whether a shared label has a spacetime interpretation.

9. The statement that measurement backreaction can change the complex but not its homology is false without qualification: threshold crossings can add or remove simplices and alter homology while spacetime topology stays fixed. Replace the assertion with a quantitative stability condition.