Referee Report

Mirror symmetry in 3d in 3d mirror symmetry

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Summary and overall assessment

The manuscript proposes a broad connection among Calabi–Yau threefold mirror symmetry, Rozansky–Witten boundary conditions, a proposed three-dimensional SYZ construction, and Donaldson–Thomas theory. For a compact Calabi–Yau threefold Y, it considers the scaled complex-structure and stringy Kähler moduli spaces $\mathcal{M}_{cpx}(Y)$ and $\mathcal{M}_{sympl}(Y)$, together with their cotangent bundles and integral-lattice quotients. Section 2 identifies the latter with B- and A-side universal intermediate-Jacobian fibrations and attributes hyperkähler structures to them. Section 3 then proposes Definition 3: a physical 3d A- or B-brane is a complex Lagrangian support carrying a flat or holomorphic family of triangulated dg categories and, with the two conditions interchanged, a holomorphic or flat family of Π-stability conditions.

The main proposal is developed in Section 4. After dropping the integral lattices, the authors set

$$X = T^*\mathcal{M}_{sympl}(Y), \qquad X^! = T^*\mathcal{M}_{cpx}(Y).$$

For fixed Ω, the family of categories $D^b(Y, \Omega)$, with stability condition varying with ϖ, is placed over the zero section of X. If all deformations of this brane are assumed to arise by varying Ω, its moduli space is declared to be $\mathcal{M}_{cpx}(Y)$, and $X^!$ is consequently interpreted as the cotangent bundle of that brane moduli space. Cotangent-fiber and zero-section branes are then paired so that the proposed A- and B-model Hom categories both reproduce $D^b(Y, \Omega)$. Parallel constructions are stated using $Fuk(Y, \varpi)$.

Section 5 specializes to a quintic. The Greene–Plesser/Dwork family is used to propose an inclusion of the mirror complex moduli into the original complex moduli, and the conormal to its graph is used as a Lagrangian correspondence producing further 3d branes. The section closes with a claimed extension to chains of finite Abelian quotient groups. Section 6 proposes DT-branes. On the B-side, a universal holomorphic Chern–Simons functional is intended to generate a Lagrangian in $X^!$ whose intersection with a cotangent fiber recovers DT moduli, invariants, and categories. An A-side functional is similarly proposed for special Lagrangians. The final discussion restores the integral lattice and invokes Bousseau's holomorphic Floer proposal to relate section–fiber morphisms to special-Lagrangian DT data.

The paper contains an ambitious and potentially fruitful organizing idea. It is natural to ask whether category-valued families over complex Lagrangians can connect CY3 mirror symmetry to 3d boundary conditions, and whether DT theories can appear as morphisms between such objects. The attempt to put complex-structure variation and stability-condition variation on opposite A/B footings is conceptually suggestive. A precise version of the DT-brane proposal, or even one nontrivial example in which a Hom category can be calculated independently, could be significant.

In its present form, however, the manuscript does not establish its main claims. The central 3d SYZ statement is conditional on an uncomputed assertion that all deformations of a category-valued brane are parametrized by Ω. The Hom theory needed to test the proposal is not defined, and the displayed Hom equivalence follows by assigning the same category to both sides. The main construction is performed on cotangent covers after discarding the lattices, although the abstract and Section 2 concern universal intermediate Jacobians and Section 6 itself says that restoring the lattice changes the morphism category. There are also concrete errors: a general CY3 intermediate Jacobian is not automatically a principally polarized Abelian variety; the final A-side torus is assigned the B-side $H^3(Y, \mathbb{Z})$ charge lattice despite a rank mismatch; a rational or signed DT invariant cannot in general equal an ordinary vector-space dimension; and the proposed special-Lagrangian quotient and calibration conditions are incomplete. The quintic and finite-group constructions require maps of moduli spaces that have not been defined.

Recommendation: reject in the present form. This recommendation does not reflect a lack of interest in the program. It reflects that the missing steps are the proposed results themselves, rather than technical details that can be repaired locally. A substantially reconstructed paper could merit reconsideration if it formulates a precise brane moduli problem, computes its deformation theory in a controlled setting, treats the lattice quotient and charge systems consistently, and supplies at least one genuine calculation supporting the proposed Hom or DT identifications.

Correctness and logical structure

The manuscript is clearest when reviewing standard structures: the scaled period domain, local special Kähler geometry, the SYZ and HMS viewpoints, and the role of cotangent zero sections and fibers as complex Lagrangians. Those ingredients do suggest a useful research program. The logical status changes, however, when the paper passes from this background to its new claims. Definitions, expectations, assumptions, and asserted consequences appear in a single continuous narrative. No theorem or proposition is stated, and the hypotheses needed for the global moduli spaces, categorical families, stability conditions, and Lagrangian compositions are not isolated.

The key implication in Section 4 is circular at the current level. If the moduli of the brane ${}_A\mathcal{C}_\Omega$ is assumed to be exactly $\mathcal{M}_{cpx}(Y)$, then its cotangent is of course $T^*\mathcal{M}_{cpx}(Y)$. What needs proof is the premise: the tangent and obstruction theory of the brane, the exclusion of support, categorical, schober, autoequivalence, and stability deformations, and the treatment of automorphisms. Similarly, the Hom identity is not a check of mirror symmetry unless the two Hom categories are constructed independently. At present they are both stipulated to be $D^b(Y, \Omega)$ by the same transverse-intersection heuristic.

The manuscript acknowledges that much of the discussion is conjectural. That is appropriate, but it does not by itself make the conjectures well-posed or remove incorrect intermediate statements. A conjectural paper can be publishable when it states mathematically definite predictions and supplies substantial evidence. Here the underlying 2-category, its objects and morphisms, the relevant moduli stack, and the DT categories are not yet specified sufficiently to decide what several central assertions mean.

Novelty and significance

The apparent novelty is a synthesis of several active themes rather than a demonstrated new equivalence. The RW boundary-condition perspective is tied to the cited work of Kapustin–Rozansky–Saulina and later higher-Fukaya proposals. The cotangent-of-brane-moduli version of

3d SYZ is attributed to the authors' two recent papers and, at a decisive point, to an unpublished work described as in preparation. The special Kähler/cotangent geometry, Greene–Plesser family, holomorphic Chern–Simons description of sheaf moduli, and Bousseau's proposed link between holomorphic Floer theory and DT invariants are established or previously proposed ingredients.

What may be new here is Definition 3 with its additional stability data, its application to the two CY3 moduli spaces, the quintic conormal construction, and the packaging of DT categories as 3d brane morphisms. The manuscript should say this explicitly and compare each point with the cited 2024 Chan–Leung papers, the RW boundary 2-category literature, and modern cohomological and categorical DT constructions. At present the paper neither proves these claims nor provides a nontrivial example that distinguishes them from the definitions used to formulate them. Consequently the significance cannot yet be assessed at the standard of a research journal: the proposed bridge is interesting, but it has not produced a new invariant, duality check, category equivalence, or physical prediction.

Major comments

1. **The central 3d SYZ construction is assumed rather than derived, and it does not fit the rank-one setup stated in Section 3.**

   The decisive sentence in Section 4 begins by supposing that all deformations of ${}_{A}\mathcal{C}_{\Omega}$ are of the same form and depend only on Ω. This is exactly the claim needed to identify the brane moduli with $\mathcal{M}_{cpx}(Y)$. No deformation complex is given. In particular, the paper does not analyze deformations of the zero-section support, the local system or schober of categories, the dg category itself, its monodromy representation, its Π-stability data, or its autoequivalence twists. Nor does it discuss obstruction groups, automorphisms, derived structure, or why the result should be an ordinary manifold rather than a stack.

   There are explicit extra or redundant directions even before one considers sophisticated categorical deformation theory. A zero section in a cotangent bundle has local complex-Lagrangian deformations given by graphs of closed holomorphic one-forms, and the manuscript gives no mechanism by which the category or stability data obstructs them. Conversely, $\mathcal{M}_{cpx}(Y)$ includes the $\mathbb{C}^*$-scaling of the holomorphic volume form, while the stated datum $D^b(Y, \Omega)$ depends on the induced complex structure and is unchanged by $\Omega \mapsto \lambda\Omega$. Definition 3 does not include a Calabi–Yau trace or volume-form structure that would distinguish this scaling. Thus the proposed map from the scaled $\mathcal{M}_{cpx}(Y)$ is not even shown to be injective on the brane data as defined.

   There is also an internal mismatch. The 3d SYZ recipe reviewed near the end of Section 3 takes a moduli space of *rank-one* perverse-schober branes whose generic fiber is $Vect_{\mathbb{C}}$. The brane used in Section 4 has generic fiber $D^b(Y, \Omega)$, which is not rank one. The authors need either to extend the earlier proposal to this non-rank-one situation and explain why its mirror is still an ordinary cotangent bundle, or to construct a different rank-one object to which the stated proposal applies.

   A publishable treatment should formulate the brane moduli functor, identify its tangent and obstruction spaces, and prove at least locally that the natural map from complex-structure deformations is an equivalence onto a specified component. It should also decide whether a CY structure is part of the brane datum or whether the complex moduli should be projectivized.

Without this work, the headline SYZ construction is the consequence of a premise chosen to imply it.

2. **Definition 3 and the Hom theory are not sufficiently defined to support the main equivalence.**

   Definition 3 introduces a flat or holomorphic family of triangulated dg categories and a holomorphic or flat family of Bridgeland-type stability conditions over a complex Lagrangian. Each term carries unresolved content. A flat categorical family is not canonically trivial even locally; a holomorphic family requires an enhancement and descent data; singular fibers generally require precisely the perverse-schober conditions that the manuscript says it will neglect. The existence of the required Bridgeland stability conditions is not known for a general compact CY3. Local constancy of a central charge also does not by itself give a global flat family of stability conditions: Bridgeland's central-charge map is a local homeomorphism on a chosen component, with possible changes of slicing, monodromy, and wall crossing.

   The proposed examples inherit these issues. It is not clear that $D^b(Y, \Omega)$ forms the needed global holomorphic family over all of $\mathcal{M}_{cpx}(Y)$, or that one fixed ϖ defines a stability condition throughout that family. Universal families can be stacky or twisted, monodromy acts by autoequivalences, and the geometric stability condition itself is conjectural in the stated generality. Corresponding claims for Fukaya categories require choices of coefficients, gradings, bounding data, and continuation functors. The cited GIT schober results concern restricted settings and do not establish these assertions for an arbitrary compact CY3.

   Most importantly, the manuscript does not define the intersection theory denoted by ${}_{A/B}Hom_X$. The product formulas in Section 3 and the two Section 4 identities are asserted without constructing a bimodule, functor category, derived intersection, or composition law. Thus the equality

   $$ {}_{A}Hom_X({}_{A}\mathcal{C}_{\Omega}, {}_{A}\mathcal{D}_{\varpi}) \simeq {}_{B}Hom_{X^!}({}_{B}\mathcal{C}^!_{\Omega}, {}_{B}\mathcal{D}^!_{\varpi}) $$

   is not an independently verified mirror statement: both sides are assigned $D^b(Y, \Omega)$ by the proposed rule. The B-side formulas in the manuscript also use the ambient subscript X, even though both branes live in $X^!$. The authors should first define a restricted 2-category in which these objects and Hom categories exist, and then calculate one nontrivial Hom from that definition.

3. **Discarding the integral lattice changes the space and the morphisms, so Section 4 does not establish the claims about universal intermediate Jacobians.**

   The abstract, introduction, and Section 2 define

   $$ \mathcal{J}(Y) = T^*_{\mathbb{Z}}\mathcal{M}(Y) \backslash T^*\mathcal{M}(Y), $$

   a torus fibration. Section 4 explicitly ignores $T^*_{\mathbb{Z}}\mathcal{M}$ and works with the cotangent cover. This replacement changes the topology, compactness, allowed local systems, winding sectors, and intersection theory. It also changes the meaning of a cotangent fiber: on the quotient it is a torus, not a vector space.

   The final part of Section 6 makes the problem explicit. There the authors return to $\mathcal{J}_{sympl}(Y)$ and argue that the Hom between its section and a torus fiber is not simply $Vect_{\mathbb{C}}$, because the path-space cover has nontrivial charge sectors. That is precisely the section–fiber pair treated as trivial on the cotangent cover in Section 4. Even if the Section 6 assertion is only conjectural, it demonstrates that the lattice is not a harmless notational simplification within the manuscript's own framework.

The paper must distinguish two claims: a proposal on the cotangent covers $T^*\mathcal{M}$, and one on the universal intermediate-Jacobian quotients. To make the latter, the authors must define the A-side integral structure, show that the category-valued branes descend through the lattice, incorporate monodromy and winding sectors, and recompute the Hom categories. On the A-side the lattice should be related to numerical K-theory/even cohomology, its Euler or Mukai pairing, and the appropriate integral structure; it is not determined merely by copying the B-side H^3 lattice. Until this is done, the abstract should restrict its claims to cotangent covers.

4. **The final section assigns the wrong charge lattice to the A-side torus fibration.**

   For
$$\mathcal{J}_{sympl}(Y) = T^*_{\mathbb{Z}}\mathcal{M}_{sympl}(Y)\backslash T^*\mathcal{M}_{sympl}(Y),$$
   the fiber has complex dimension $1 + h^{1,1}(Y)$. Its first integral homology therefore has rank $2 + 2h^{1,1}(Y)$. The last part of Section 6 instead asserts
$$H_1(F_\varpi, \mathbb{Z}) \simeq H^3(Y, \mathbb{Z}),$$
   whose rank is $2 + 2h^{2,1}(Y)$. These groups cannot be identified in general. For the quintic used in Section 5, their ranks are 4 and 204, respectively.

   This is a conceptual interchange of the A- and B-side charge systems. Coherent-sheaf DT charges lie in numerical K-theory, rationally modeled by $H^{even}(Y)$, which has the rank appropriate to the A-side/stringy-Kähler integrable system. Special-Lagrangian charges lie in $H_3(Y)$, dual to the $H^3(Y)$ lattice of the B-side complex-structure intermediate Jacobian. Consequently the subsequent claims that the critical points form an $H^3(Y, \mathbb{Z})$-torsor, that their differences are classes $\gamma \in H_3(Y, \mathbb{Z})$, and that Bousseau's conjecture supplies all special-Lagrangian DT sectors on $\mathcal{J}_{sympl}(Y)$ do not follow.

   The authors need to reconstruct this argument on the correct integrable system and charge lattice, perhaps using $\mathcal{J}_{cpx}(Y)$, the mirror threefold, or the particular Seiberg–Witten integrable system to which the cited conjecture applies. This cannot be repaired by changing a symbol because it alters the fiber dimension and which BPS objects its charges describe.

5. **The intermediate-Jacobian and special-Kähler geometry in Section 2 needs correction and precise global hypotheses.**

   Section 2 states that the intermediate Jacobian of a projective threefold is a principally polarized Abelian variety. This is false in the stated generality. The Griffiths intermediate Jacobian is a complex torus; it is an Abelian variety only when the weight-three Hodge structure satisfies the additional Riemann positivity needed for algebraicity. For a generic CY3 with $H^{3,0} \neq 0$, one cannot simply assert that condition. The later description of $\mathcal{J}_{sympl}(Y)$ as an Abelian-variety fibration repeats the same problem.

   There is a useful local statement behind the discussion. On an appropriately marked smooth locus, the scaled period image is a Lagrangian cone in $H^3(Y, \mathbb{C})$, and affine special Kähler geometry gives a semi-flat metric on the cotangent bundle. The manuscript must specify the marked cover or stack, smooth locus, lattice embedding, monodromy, and whether the metric claimed is positive, pseudo-hyperkähler, complete, or only semi-flat. In fact, on the conical moduli retaining the Ω-scaling, the natural special-Kähler Hermitian form has indefinite

Hodge–Riemann signature in the $H^{3,0}$ and $H^{2,1}$ directions, so the rigid c-map generally gives a pseudo-hyperkähler metric rather than the positive target metric of a unitary sigma model. Projectivizing changes both the geometry and the dimensions used later.

The A-side object requires even more care. The quantum-corrected stringy Kähler moduli space and its integral cotangent lattice are asserted globally but not defined for a general compact CY3. The GLSM discussion in Section 1 suppresses phase, secondary-fan, quotient-stack, and discriminant issues. The authors should restrict to a local semi-flat/marked setting where the constructions are defined or supply the missing global definitions. The Hodge decomposition in Section 2 also contains $H^{0,3,0}$, which should be $H^{0,3}$, and the identification with a cotangent space is not canonical without specifying the symplectic pairing and complement.

6. **The quintic conormal construction depends on a moduli-space inclusion that is not canonically defined as written.**

   Section 5 begins with an arbitrary smooth quintic but then uses the special Dwork pencil and its Greene–Plesser quotient. This gives a distinguished one-parameter locus only after choices of marking and finite quotient; it does not automatically provide a global literal inclusion of the coarse moduli space of the mirror threefold into the full complex moduli space of the original threefold. Orbifold stabilizers, monodromy, resolution choices, scaling of the holomorphic volume form, and the range of the mirror map all matter.

   The Givental and Lian–Liu–Yau mirror theorems cited in this context identify genus-zero generating functions and period solutions near the large-radius/large-complex-structure boundary; they do not prove the blanket global isomorphism of full moduli spaces and Frobenius structures stated here. The precise coefficients, completion, and split-closure in the cited quintic HMS theorem should likewise be given. At most, the available statements motivate a local, marked mirror map with analytic-continuation and monodromy choices.

   Since the map
   $$\iota : \mathcal{M}_{sympl}(Y) \longrightarrow \mathcal{M}_{cpx}(Y)$$
   is not defined, neither is the global conormal brane built from it. Even after choosing a local or stacky version of ι, the conormal to its graph supplies only a Lagrangian relation between supports. A correspondence from X to $X^!$ should be placed in $X^- \times X^!$ with a clear sign convention, and a categorical kernel is needed to transport the dg-category and stability decorations. The manuscript simply assigns the original fiber after the correspondence; it does not derive that fiber or prove clean composition. This section would be much stronger if it constructed the map and conormal on a marked neighborhood and computed one resulting Hom category.

7. **The finite-Abelian generalization in Section 5 is unsupported and is not true for an arbitrary symmetry chain.**

   The manuscript asserts that a finite Abelian symmetry Γ admits a decreasing sequence $\Gamma_r \supset \cdots \supset \Gamma_0$ such that resolutions of Y/Γ_j and Y/Γ_{r-j} are mirror. Greene–Plesser or Berglund–Hübsch-type duality uses specially related dual or annihilator subgroups, an invariant polynomial, determinant conditions, and appropriate quotient/resolution hypotheses. An arbitrary nested chain does not acquire the displayed $j \leftrightarrow r-j$ mirror pairing merely from being finite Abelian.

   The ensuing moduli inclusions and transports of categories are therefore not established. In the first summary table of the generalization, the A-brane support is denoted by an undefined

$\mathcal{C}$; other entries change among Y, Y_j, and Y_{r-j} without specifying maps that transport Kähler parameters, complex structures, Fukaya categories, or stability conditions. Those maps are precisely the data needed to put the proposed families over a common support.

The authors should remove this generalization or replace it with a precise theorem or conjecture for a recognized dual-group construction, listing all hypotheses and defining the quotient stacks and crepant resolutions. One explicit intermediate-subgroup example would be valuable. The novelty claim should also be separated from standard Greene–Plesser duality and from the authors' earlier 3d mirror proposals.

8. **The DT categorification formulas and holomorphic Chern–Simons construction in Section 6 are not correct or complete as stated.**

The section begins with

$$DT_{(Y,\Omega),\varpi}(\gamma) = \dim H^{DT}_{(Y,\Omega),\varpi}(\gamma), \qquad H^{DT} = HH^*(\mathcal{C}^{DT}).$$

A generalized DT invariant can be rational and can carry signs; it cannot in general be the ordinary dimension of a vector space. Cohomological DT theory refines numerical DT data through vanishing-cycle cohomology, gradings, monodromy, orientation data, and a graded Euler characteristic, weight polynomial, or superdimension. A DT category whose Hochschild cohomology is that vector space is an additional conjectural categorification, not a standard identity. The manuscript must choose a precise DT theory and stability condition, state the moduli hypotheses, and compare its proposal with modern cohomological and categorical DT constructions.

The Chern–Simons sketch does not yet produce the claimed B-brane. For a nontrivial bundle E, writing $d + A$ requires a trivialization or reference connection, and the Chern–Simons functional is generally relative and multivalued under large gauge transformations. Its derivative in the Ω direction correspondingly changes by periods, so the natural target is a lattice quotient rather than the unqualified bare cotangent bundle used in this section. The space of Hermitian connections is a real infinite-dimensional affine space, whereas the graph is claimed to be a complex Lagrangian; the holomorphic construction normally uses $(0,1)$-connections or a derived moduli stack.

One must take a derived critical locus and gauge quotient, account for stabilizers and singularities, impose ϖ-stability, choose orientation data, and compactify. Fixing one smooth vector bundle also misses torsion sheaves, complexes, and non-locally-free semistable objects counted by general DT theory. Even for bundles, $F^{0,2} = 0$ modulo complex gauge gives all holomorphic structures, not the desired stable compactified moduli. Projecting a critical locus does not by itself show that intersection multiplicities equal DT weights, much less that a 3d Hom category equals a DT category. The defining Hom display also uses ${}_B Hom_X$, although both branes are on $X^!$.

9. **The A-side functional does not define a special-Lagrangian DT invariant in its present form.**

The manuscript integrates $\mathbf{u}^*\varpi \wedge e^{F_{\mathbb{A}}}$ over $M \times [0,1]$ and states the boundary Euler–Lagrange equations

$$u^*\omega = 0, \qquad iu^*B + F_A = 0.$$

The degree-four component, normalization, dependence on the background map and homotopy, gauge invariance, and period ambiguity should all be derived. The configuration space is too large: a general map $u : M \to Y$ need not be an immersion or embedding, so the displayed equations include constant and rank-deficient maps that are not Lagrangian submanifolds. The symmetry statement is also incorrect: for a Hermitian line bundle, $C^\infty(M, S^1)$ is already its unitary gauge group; it is not separately the group of Hamiltonian deformations. Reparametrizations and Hamiltonian isotopies require different groups and quotients.

A special Lagrangian requires a phase and calibration condition. After $u^*\omega = 0$, one normally imposes

$$u^* \operatorname{Im}(e^{-i\theta}\Omega) = 0$$

and a positivity/orientation condition on $u^* \operatorname{Re}(e^{-i\theta}\Omega)$. The display $u^* \operatorname{Re}\Omega = 0$ could correspond to a particular phase convention, but the complementary positivity condition and phase are absent, and imposing the special-Lagrangian equation is not an alternative to quotienting by a symmetry group.

Most importantly, general special-Lagrangian moduli are not zero-dimensional. By McLean deformation theory, a smooth compact special Lagrangian has local deformation space of dimension $b_1(M)$, and adding flat line bundles supplies further torus directions. No deformation-invariant rational count follows from setting aside analytic issues. Compactness, singularities, multiple covers, orientations, bounding chains, wall crossing, and stability are central to whether an invariant exists. The proposed number, Floer categorification, and further 3d categorification should be presented as separate conjectures with exact inputs and expected properties, and related accurately to the Joyce, Thomas–Yau, and Bousseau proposals.

10. **The physics interpretation and relation to prior work need a controlled statement and a genuine test.**

Section 2 says that the moduli spaces and intermediate-Jacobian fibrations are Coulomb branches obtained by compactifying a ten-dimensional supersymmetric theory to four and three dimensions. The claim should specify the type-II theory, decoupling or rigid limit, vector- versus hypermultiplet sector, circle dualization, and which perturbative or instanton corrections are retained. The resulting metric is generally not captured globally by a bare semi-flat cotangent quotient. Likewise, BFN construct Coulomb branches of 3d $N = 4$ gauge theories; they do not by themselves furnish a universal mirror for every stack written as $T^*[V/G_{\mathbb{C}}]$. The manuscript also conflates that UV stack with Higgs and low-energy targets.

The authors expressly state that X and $X^!$ are not claimed to be a full 3d mirror pair. They should then formulate exactly which observable consequence qualifies as 3d mirror behavior. An equality obtained by assigning the same category on both sides is insufficient. Possible tests include a comparison of deformation complexes, a calculable hypertoric or torus example, matching numerical invariants of Hom categories, compatibility with composition, or a protected partition function. At least one such test is needed to show that the framework makes a prediction beyond repackaging CY3 HMS data.

Finally, the manuscript should identify what is new relative to the cited Chan–Leung papers and the work in preparation on which the rank-one 3d SYZ construction depends. A journal article must be self-contained enough for its principal new assertion to be evaluated without an unavailable reference.

Minor comments

1. Section 1's HMS display has $D^b(Y^\vee, \Omega)$ on the right-hand side; the complex structure should be denoted by $\Omega^\vee$. The strict-CY condition should explicitly read $H^{k,0} = 0$ for $0 < k < n$.

2. The global GLSM formula $\mathcal{M}_{sympl} \simeq (\mathbb{C}^*)^k \setminus \Delta$, and the claim that the same quotient presentation works for any moment-map parameter, require phase, stack, and secondary-fan qualifications. The geometric and Landau–Ginzburg phases of the quintic already illustrate the issue.

3. In the Landau–Ginzburg discussion, it is the derived critical locus of W that carries the relevant (-1)-shifted symplectic structure; when $W = 0$, this is $T^*[-1]Y^\vee$. The LG pair itself should not be identified with that shifted cotangent space.

4. In Section 3 the gauge-theory sentence introduces a pair (G, Y) but then calls V a representation of G. The intended notation and distinction among the UV cotangent stack, Higgs branch, and BFN Coulomb branch should be corrected.

5. The statement that a flat family of categories is locally and *canonically* quasi-equivalent to one fixed category is too strong. A local trivialization is a choice and can carry monodromy and higher descent data.

6. The transverse-fiber formula $Y_1 \times_X Y_2 = Y_{1,p} \times Y_{2,p}$ requires actual maps $Y_i \to X$, not only fibrations $Y_i \to C_i \subset X$. Its hypotheses and derived nature should be stated.

7. In Section 4 both displays involving the B-model Hom of ${}_B\mathcal{C}^!_\Omega$ and ${}_B\mathcal{D}^!_\varpi$ should carry the ambient subscript $X^!$, not X. The same correction is needed in the Section 6 DT-brane Hom display.

8. In Section 5, quasi-equivariant appears to mean quasi-equivalent. The notation $\Omega_{\varpi'}$ requires a specified local mirror map and marking rather than only the asserted inclusion of coarse moduli.

9. The general finite-group table uses an undefined support $\mathcal{C}$. Its base points, fibers, and stability conditions should be indexed consistently by the same Y_j.

10. The holomorphic Chern–Simons formula should use a reference connection and make clear whether A denotes a full connection or its $(0,1)$-part. The fixed Chern character γ should be connected explicitly to the topology of E and to the additional sheaf types included in the moduli problem.

11. The symbol t is used both in the scaled polyform $\varpi = t \exp(\omega + iB)$ and as the coordinate on $[0, 1]$ in Section 6. One should be renamed.

12. The paper should number its central conjectural displays. At present all mathematical displays are unnumbered, so even the main Hom identity and DT-brane proposal cannot be referenced precisely.

13. The bibliography and prose need a technical copyedit. Examples include Vyes for Vyas, Pasceleff for Pascaleff, and Beasseau for Bousseau, as well as frequent subject–verb and article errors. A conclusion should separate established facts, precise conjectures, evidence, and open construction problems.

Recommendation

The manuscript is not suitable for publication in its present form. I recommend rejection, with the possibility of reconsidering a fundamentally revised and self-contained paper. Such a paper should, at minimum: (i) correct the intermediate-Jacobian, charge-lattice, and DT statements; (ii) define a restricted 3d brane 2-category and its stability data; (iii) compute the deformation theory underlying the Section 4 moduli claim; (iv) distinguish the cotangent cover from the lattice quotient and redo the morphism analysis accordingly; (v) formulate the quintic map locally or stack-theoretically; and (vi) provide one nontrivial Hom, deformation, or DT calculation that tests the proposed mirror behavior. These changes would turn an interesting research agenda into a refereeable mathematical-physics claim.