

# Referee Report

“New exact bispectrum shapes in multifield inflation”

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## Summary and overall assessment

The manuscript studies the effective field theory of inflation with a canonically normalized Goldstone fluctuation  $\pi_c$  derivatively mixed with an isocurvature scalar  $\sigma$  through  $\rho\dot{\pi}_c\sigma$ . Rather than treating this quadratic mixing as an interaction, the author includes it in the free Hamiltonian. This produces the dressed mass  $m_{\text{eff}}^2 = m^2 + \rho^2$  and the index  $\nu_{\text{eff}}^2 = 9/4 - m_{\text{eff}}^2/H^2$ . Building on the exact linear solutions of Ref. [1], the manuscript rewrites each mixed curvature mode as an integral over two channels  $a = \pm$ , Eqs. (8)–(10), and recovers the exact power-spectrum enhancement  $R$  in Eq. (11).

The nonlinear application is the contribution of the independent  $\lambda_2\dot{\pi}_c^3$  operator. The proposed channel representation turns its in-in integral into the one-dimensional Schwinger integral (13), whose three factors are the leg kernels (12). The manuscript then studies the shape cosine in Fig. 1, derives the small- $\beta$  kernel tails in Eq. (15), and uses them to obtain the squeezed coefficients (16), the heavy-field clock (17), the weak-mixing expansion (18), and the strong-mixing scaling (19). The main physical claim is an interpolation from the equilateral contact shape at weak mixing to a collider-dominated shape with an exponentially enhanced soft signal at strong mixing.

There is a substantial and potentially important result here. An exact arbitrary-mixing representation for the  $\dot{\pi}_c^3$  contribution, including the clock amplitude and phase rather than only its frequency, would be useful to the primordial-correlator community. Several internal checks also work. The Legendre transform leading to Eqs. (4)–(6) is consistent; the power spectrum (11) follows from Eqs. (10) and (22); and the  $\lambda \rightarrow 0$  limit of Eq. (13) reproduces the standard  $\dot{\pi}^3$  normalization. In the conventions of the paper, the eight channel assignments and  $\int_0^\infty d\xi \xi^2 e^{-\xi} = 2$  give

$$S \longrightarrow 3\lambda_2 f_\pi^2 e_1 e_2 e_3,$$

as expected. The large- $u$  calculation also gives the powers displayed in Eq. (15), once a one-power error in the preceding prose is corrected.

However, the integral representation is not mathematically defined as written, the claimed light-field domain contains an additional endpoint divergence, and the printed weak-mixing coefficient (18) appears to be wrong by a factor of two. The strong-mixing amplitude is also presented without the nonlinear control condition required by its own exponential scaling, and its most dramatic asymptotics follows a deeply bare-tachyonic parameter trajectory. Finally, the mandatory  $\sigma\pi_c^2$  contribution is omitted and the signed cosine in Fig. 1 is interpreted incorrectly as a measure of template independence. These issues prevent publication in the present form, but they do not appear to invalidate the core heavy-field idea if it is reformulated and checked carefully.

## Recommendation

I recommend **major revision**. Before publication, the author should define and validate the analytically continued spectral integrals, correct or restrict the light-field claims, resolve the factor-of-two discrepancy in Eq. (18), establish a controlled strong-mixing parameter region, and recast

the observational conclusions in terms of the complete EFT bispectrum and appropriate template projections. A revised paper that does these things could make a valuable contribution.

## Correctness, novelty, and significance

The genuinely new part is narrower than some statements in the abstract and conclusion but remains significant: it is the exact arbitrary- $\lambda$  channel representation for the strongly mixed  $\dot{\pi}_c^3$  bispectrum and its soft amplitude and phase. The dressed frequency and Eq. (11) are inherited from the exact linear analysis of Ref. [1]. Spectral, Fourier, and Laplace factorizations of cosmological time integrals have also recently been developed in Refs. [23]–[25]. The manuscript should state more precisely what is special about the present strongly mixed kernel, rather than treating the general plane-wave/Schwinger philosophy itself as new. Likewise, earlier work includes analytic information in nonperturbative or low-speed limits as well as numerical strong-mixing results. The defensible priority claim is therefore the first exact arbitrary-mixing representation for this particular two-field bispectrum, not the first analytical access of any kind to  $\lambda \gtrsim 1$ .

The paper is potentially suitable for a journal such as PRD or JHEP because it connects an exact solution of a nontrivial coupled system to a phenomenologically relevant observable. The compact representation could also be valuable for template construction. That significance depends, however, on supplying a well-defined prescription and demonstrating that the advertised shapes occupy a controlled and genuinely independent part of parameter space. Work-in-progress citations [17] and [42] cannot carry an essential proof, a dominance argument, or a priority claim for the present manuscript.

## Major comments

1. **The spectral integrals in Eqs. (8), (9), (12), and (21) are not ordinary convergent integrals.**

At the lower endpoint, the weight in Eq. (9) has the universal behavior

$$\omega_a(u) = \frac{u^{-1+ia\lambda/2}}{\Gamma(ia\lambda/2)} [1 + O(u)].$$

For real nonzero  $\lambda$ , the integral  $\int_0^\epsilon du u^{-1+ia\lambda/2} F(u)$  has no improper limit. Its primitive contains  $\epsilon^{ia\lambda/2}$ , which keeps oscillating as  $\epsilon \rightarrow 0$ . Neither the plane wave in Eq. (8) nor  $e^{-\beta u}$  in Eq. (12) changes this endpoint. The Bunch–Davies  $i\epsilon$  prescription regulates the large- $u$ /early-time end, not  $u = 0$ .

The source of the problem is also visible in Appendix A. The standard integral representation of Tricomi  $U(A, B, z)$  invoked before Eq. (21) initially requires  $\text{Re } A > 0$ , whereas here the relevant parameter is  $A = ia\lambda/2$  and has zero real part. The likely intended object is the analytic family  $u_+^{A-1}/\Gamma(A)$ , continued from  $\text{Re } A > 0$ . For example, for a smooth test function one can define the endpoint by

$$\int_0^1 du u^{A-1} [F(u) - F(0)] + \frac{F(0)}{A},$$

and then analytically continue in  $A$ . Other equivalent contour or finite-part prescriptions may be possible, but one of them must be stated. The change  $u = t/(1 - t)$  mentioned after Eq. (22)

leaves the same  $t^{-1+ia\lambda/2}$  behavior at  $t = 0$  and therefore does not by itself make the integrand “well behaved.”

This is not a cosmetic issue, since Eq. (12) is advertised as an always convergent, readily tabulated kernel and the limit  $\omega_a \rightarrow \delta(u)$  is used in deriving Eq. (18). The revision should define the analytic continuation or contour precisely, establish the allowed interchanges of the  $u$ ,  $\xi$ , and channel sums, demonstrate regulator independence, and explain the numerical implementation used for Figs. 1 and 2.

**2. For real light  $\nu_{\text{eff}}$ , the Schwinger integral has a second endpoint divergence that is not discussed.**

Equation (15) gives, for the more slowly decaying branch,

$$W_2^a(\beta) \sim 4\mathcal{W}_+^a \beta^{-1/2-\nu_{\text{eff}}}.$$

At any finite triangle, all three arguments in Eq. (13) approach zero with  $\xi$ . Its integrand therefore behaves generically as

$$\xi^2 \prod_{j=1}^3 W_2^{-a_j}(2e_j\xi) \sim \xi^{1/2-3\nu_{\text{eff}}} \quad (\xi \rightarrow 0).$$

The integral converges only for  $\nu_{\text{eff}} < 1/2$ , is logarithmically divergent at  $\nu_{\text{eff}} = 1/2$ , and is power divergent above that value, unless a nontrivial channel cancellation or subtraction is supplied. No such cancellation is exhibited, and the leading channel coefficient is not generically zero. The same behavior occurs in the hard integral  $J_+^{b_2b_3}$  in Eq. (16).

This has a physical counterpart. The slowly decaying branch gives schematically  $\dot{\pi}_c \sim a^{-3/2+\nu_{\text{eff}}}$ , so the interaction accumulates as

$$\int dt a^3 \dot{\pi}_c^3 \sim \int dt a^{-3/2+3\nu_{\text{eff}}}.$$

For  $\nu_{\text{eff}} \geq 1/2$ , a finite end time, secular resummation, or boundary subtraction is required and the result can depend on that choice. The heavy-field results with  $\nu_{\text{eff}} = i\mu_{\text{eff}}$  do not suffer this particular divergence, but the assertions of validity for any mass and the light-field statement after Eq. (17) must be revised. The coalescing cases  $\nu_{\text{eff}} = 0$  and, more generally, integer  $2\nu_{\text{eff}}$  also require logarithmic limiting formulas: the separate  $\Gamma(\pm 2\nu_{\text{eff}})$  branches in Eq. (15) are singular there even when their sum has a finite limit.

**3. The weak-mixing coefficient in Eq. (18) appears to be too small by a factor of two.**

This follows directly from the displayed channel normalizations. Define

$$p_a = e^{a\pi\lambda/2} r_a, \quad q = \frac{1}{2} + \nu,$$

and let  $\mathcal{S}(\nu)$  be the digamma combination printed in Eq. (18). Expanding Eqs. (15) and (22) gives

$$\sum_{a=\pm} p_a \mathcal{W}_+^{-a} = \frac{\Gamma(2\nu)}{\Gamma(q)} \mathcal{S}(\nu) \lambda^2 + O(\lambda^4).$$

On the two hard legs, the manuscript states that  $p_a \rightarrow 1$  and  $W_2^a \rightarrow 1$ . Consequently

$$\sum_{a_2, a_3} p_{a_2} p_{a_3} J_+^{-a_2, -a_3} \longrightarrow 4\Gamma\left(\frac{5}{2} - \nu\right).$$

The explicit factor 1/2 in Eq. (16) then yields

$$\mathcal{C}_+ = 2 \frac{\Gamma(2\nu)\Gamma(\frac{5}{2} - \nu)}{\Gamma(\frac{1}{2} + \nu)} \mathcal{S}(\nu)\lambda^2 + O(\lambda^4),$$

whereas Eq. (18) prints the same expression without the leading 2. An additional undisclosed factor 1/2 in a channel sum would conflict with Eqs. (10), (12), (13), and the stated  $W_n^a \rightarrow 1$  limit. The author should resolve this discrepancy, state whether Fig. 2 used the printed or the corrected coefficient, and check the normalization against the direct diagram with two perturbative mixing insertions.

#### 4. The exponentially enhanced result needs a stronger perturbativity and cutoff analysis.

The condition  $H/\Lambda_\star \ll 1$  is necessary but not sufficient once the mode functions are exponentially enhanced. Equation (19) implies that the actual nonlinear expansion parameter contains

$$f_{\text{NL}}^{\text{squeezed}} \Delta_\zeta \sim \left(\frac{H}{\Lambda_\star}\right)^2 \lambda^{3/2} e^{\pi\lambda/2} g(\nu_{\text{eff}}).$$

Perturbative in-in evolution requires this combination to remain well below unity over the configurations used. Loop corrections can inherit its square and further momentum enhancements. The manuscript acknowledges enhanced loops but postpones the question, even though the claimed large amplitude is a central result.

In addition, the relevant excitation scale should lie below every cutoff. For fixed nonnegative bare mass at strong mixing,  $m_{\text{eff}} \simeq \lambda H$ , so one also needs a condition such as  $m_{\text{eff}}/\Lambda_\star \ll 1$ , not only  $H/\Lambda_\star \ll 1$ . The revision should give  $g(\nu_{\text{eff}})$  explicitly or numerically, map the region where the signal is large while  $f_{\text{NL}}\Delta_\zeta$ , the leading loop correction, and all energy-to-cutoff ratios are small, and substantiate the cancellation that changes the nominal  $e^{3\pi\lambda}$  channel coefficient into the  $e^{2\pi\lambda}$  surviving term. Without this analysis, Eq. (19) is a formal linear-mode asymptotic rather than a controlled prediction.

#### 5. The exponential fixed- $\nu_{\text{eff}}$ trajectory is deeply bare-tachyonic, and the stated stability interpretation is not established.

For the heavy case,

$$\frac{m^2}{H^2} = \frac{9}{4} + \mu_{\text{eff}}^2 - \lambda^2.$$

Thus taking  $\lambda \rightarrow \infty$  at fixed  $\mu_{\text{eff}}$ , as in the derivation of Eq. (19), necessarily sends the bare mass squared to a large negative value. For example, the  $(\mu_{\text{eff}}, \lambda) = (2, 4)$  curve in Fig. 2 has  $m^2/H^2 = -9.75$ . This is qualitatively different from increasing  $\lambda$  at fixed nonnegative  $m^2$ , where  $\mu_{\text{eff}}$  grows and Eq. (11) has only the stated polynomial enhancement.

The conclusion says that the bare instability acts only on sub-Hubble scales. That statement is not generally correct. In a locally static approximation the two coupled frequencies obey

$$\omega_\pm^2 = p^2 + \frac{m_{\text{eff}}^2}{2} \pm \frac{1}{2} \sqrt{m_{\text{eff}}^4 + 4p^2 p^2}, \quad \omega_+^2 \omega_-^2 = p^2(p^2 + m^2).$$

For  $m^2 < 0$ , the lower branch is unstable for  $p^2 < |m^2|$ , and at small momentum

$$\omega_-^2 \simeq \frac{m^2}{m_{\text{eff}}^2} p^2.$$

When  $|m| \gg m_{\text{eff}}$ , Hubble damping competes with this growth only around  $p \sim H m_{\text{eff}}/|m| \ll H$ . The fluctuations may ultimately freeze, but the exponential enhancement is itself evidence that the intervening instability is dynamically important.

The author should separate fixed-bare-mass and fixed-effective-mass trajectories, show amplitude and shape results along  $m^2 \geq 0$  curves, and analyze the duration of the tachyonic phase, backreaction, nonlinear stabilization, and compatibility with the EFT cutoff. The non-tachyonic part of Fig. 1 suggests that interesting shape deformation may survive this restriction; demonstrating that would make the physical claim stronger.

**6. The plotted  $\dot{\pi}_c^3$  diagram is not the complete bispectrum of the EFT in Eqs. (1)–(3).**

The interaction

$$\sigma \left[ \dot{\pi}_c^2 - \frac{(\partial_i \pi_c)^2}{a^2} \right]$$

is fixed by the same mixing and cannot be removed from the model while keeping  $\rho \dot{\pi}_c \sigma$ . After resumming the quadratic mixing, this vertex gives another one-vertex contribution to the curvature bispectrum. Its coefficient grows with  $\lambda$  and it may dominate or interfere strongly with the component studied here.

It is reasonable for a Letter to focus on one independent Wilson coefficient, but then every figure and phenomenological statement must be described as the  $\lambda_2 \dot{\pi}_c^3$  component, not as the predicted bispectrum of the minimal EFT. Alternatively, the author should include the mandatory component. At a minimum, a controlled hierarchy should be exhibited in which the present diagram dominates while the cutoff and nonlinear conditions of comment 4 hold. Referring this question to the unpublished companion [17] does not establish the physical relevance of the present templates.

**7. The signed cosine in Fig. 1 is misinterpreted, and the data-analysis claim needs a more appropriate projection.**

The coefficient  $\lambda_2$  is free in sign, and an equilateral estimator fits a signed amplitude. Therefore  $\cos(S, S_{\text{eq}}) = -0.9$  means 90 percent anti-correlation, not strong decorrelation: an optimally signed equilateral template captures  $\cos^2 = 81$  percent of the shape norm. Only the near-zero strip in Fig. 1 is genuinely orthogonal to the equilateral direction. Statements that the shape “decorrelates” when the cosine becomes strongly negative, and that the whole blue band motivates a new estimator, overstate what the displayed diagnostic shows.

The paper should plot  $|\cos|$ , the residual fraction  $1 - \cos^2$ , or an explicit best-fit residual. It should also project onto at least the joint equilateral–orthogonal subspace and calculate the asserted lack of overlap with the orthogonal template. The uniform primordial weight and the arbitrary  $x_1 \geq 10^{-3}$  cutoff used after Eq. (14) should be varied, especially because the novelty is concentrated in squeezed oscillations. A CMB- or LSS-relevant inner product would be preferable for a detectability claim. Finally, the left panel normalizes to the equilateral value precisely where that value can be suppressed or change sign; absolute norms, equilateral amplitudes, and residual norms over the  $(\lambda, \mu_{\text{eff}})$  plane are needed to distinguish a large physical signal from a small normalization denominator.

**8. The general factorization claim and the numerical results need a self-contained validation.**

Appendix B proves the elementary identity (23) for a time integrand proportional to  $(-\tau)^N$  only when  $N$  is a nonnegative integer as written; its natural gamma-function extension requires  $\text{Re } N > -1$ , as well as explicit early- and late-time prescriptions. Scale invariance by itself does not ensure these conditions. No  $\sigma$  channel mode or  $\sigma$  leg kernel is displayed, so the assertion that all interactions containing  $\sigma$  follow immediately is not demonstrated. The relevant fact is a nonzero unequal-time mixed propagator, not a nonzero equal-time canonical commutator. The broad statement that “every tree-level bispectrum” reduces to the same construction should be restricted to the leading connected three-point function with all quadratic terms in the free Hamiltonian, local vertices of the stated scale-invariant EFT, and time powers for which the regulated integrals exist, unless a fuller proof is supplied.

The numerical evidence is also entirely internal to the new representation. Figure 2 compares Eq. (13) with a squeezed expansion derived from Eq. (13), so it does not test the spectral prescription, absolute normalization, or channel cancellations. Representative points should be checked against direct numerical solutions of the original coupled mode equations followed by the original in-in time integral. The benchmark set should include weak and strong mixing, non-tachyonic and tachyonic bare masses, and at least one convergent real- $\nu$  case. Quadrature regulators, interpolation errors, precision, momentum grids, and timing hardware should be reported, and releasing the kernel tables or code would make the “fraction of a second” statement reproducible.

## Minor comments

1. Immediately before Eq. (15), the large- $u$  behavior of the kernel integrand should be  $u^{n-5/2\pm\nu_{\text{eff}}}$ , not  $u^{n-3/2\pm\nu_{\text{eff}}}$ . Indeed,  $\omega_a \sim u^{-3/2\pm\nu_{\text{eff}}}$  and  $(1+2u)^n/(1+u) \sim 2^n u^{n-1}$ . Equation (15) itself uses the corrected power for  $n=2$ .
2. In the Regime of Validity paragraph, the dimension-five operator combines  $\dot{\pi}_c^2$  with  $(\partial_i \pi)^2$ . The latter must also be written in terms of  $\pi_c$ ; otherwise the two terms have inconsistent normalization and dimensions.
3. Since  $M_3^4$  and  $\lambda_2$  can have either sign, define  $\Lambda_\star = |\lambda_2|^{-1/2}$  and retain  $\text{sgn}(\lambda_2)$  explicitly in the bispectrum. This also makes the discussion of signed template correlations unambiguous.
4. The claim that collider oscillations are unavoidable for  $\lambda \geq 3/2$  should use a strict inequality for nonnegative bare mass. At  $m=0$  and  $\lambda=3/2$ , one has  $\mu_{\text{eff}}=0$ , not an oscillatory clock.
5. In the three limits following Eq. (11), distinguish the bare-mass index  $\nu = \sqrt{9/4 - m^2/H^2}$  from  $\nu_{\text{eff}}$  consistently. The fixed-bare-mass and fixed-effective-mass limits should be labeled explicitly in every formula.
6. The phrase “closed form” for Eqs. (16)–(17) is too strong while  $J_\pm^{b_2 b_3}$  remains an unevaluated integral over kernels that are themselves integral-defined. “Exact integral representation” or “one-dimensional quadrature” would be accurate.
7. Replace “bearing cancellations” below Eq. (11) by “barring cancellations.” Quantify qualitative phrases such as “mildly squeezed,” “large,” and “fraction of a second.”

8. State that  $\lambda$  may be taken nonnegative without loss of generality by an appropriate field redefinition, or display explicitly how the channel formulas transform under  $\lambda \rightarrow -\lambda$ .
9. When calling the clock a purely multifield soft feature despite the  $\pi_c$ -only cubic vertex, cite and discuss the single-clock consistency conditions represented by the already listed Maldacena and Creminelli–Zaldarriaga references. The quadratic mixing is what invalidates the single-clock premise.
10. The Introduction should distinguish exact arbitrary-mixing control from the analytic limiting descriptions and nonperturbative numerical approaches in Refs. [13]–[16]. The discussion around Refs. [23]–[25] should compare the precise kernels, domains, and factorization claims rather than only saying that the representations are “related but different.”