Referee Report

Macroscopic statistical field from the spectral function of vacuum fluctuations and its gravitational implications

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Recommendation: Reject in the present form.

Summary and overall assessment

The manuscript proposes that gravity emerges as a collective phenomenon of the flat-space Standard Model energy–momentum tensor. Its logical chain is the following. First, the commutator two-point function of $T_{\mu\nu}$ is decomposed into spin-2 and spin-0 sectors. An exponential window is then applied to the spectral density, and the resulting two-point function is used to define a macroscopic symmetric tensor field. Next, the window length σ is promoted to a stochastic order parameter, assumed to evolve between a Planck-scale ultraviolet fixed point and a Hubble-scale infrared fixed point. A Model-A Langevin equation, an effective temperature $T_{\text{eff}} \propto 1/\sigma$, and a fluctuation–dissipation relation are introduced to close this dynamics. Section 3 argues that critical slowing down excludes a smooth stationary spin-2 spectrum and selects an atom $Z\delta(\mu^2)$ at zero invariant mass. Finally, Section 4 interprets that atom as a massless graviton, invokes Weinberg's soft-graviton argument to obtain Einstein gravity, and estimates G, the Hawking temperature, and Λ.

The broad question is important, and the proposed diagnostic—an isolated zero-mass contribution in a stress-tensor spectral measure—could be useful if it were embedded in a controlled nonperturbative calculation. The manuscript also correctly states the elementary conditional fact that an atom $Z\delta(\mu^2)$ in an appropriate positive spectral measure contributes a Z/p^2 term to a scalar spectral integral. These observations do not, however, establish the claimed theory of emergent gravity.

The principal new claim is the formation of the pole. That claim fails when tested against the equations in the manuscript: multiplication by the smooth window in equation (2.9) cannot create new singular support; the injection term in equation (3.2) is not derived; and a proposed $\delta(\mu^2)$ contribution drops out of the dilution term and therefore does not cancel the injection that produced the alleged contradiction. Moreover, the sum rule in equation (3.5) is incompatible with positivity and with the continuum retained in equation (3.6). Independent problems occur in the identification of the tensor pole with a two-helicity graviton and in the normalization of its coupling. The estimates of G and Λ contain dimension, algebra, and sign inconsistencies.

These are foundational issues rather than matters of exposition. Resolving them would require a different, explicitly derived microscopic flow and a new analysis of the massless tensor state and its interactions. I therefore do not think that an ordinary major revision could make the present manuscript suitable for a high-energy theory journal.

Major comments

1. **The spectral object and its analytic continuation must be defined consistently.**

Section 2 starts with the Standard Model on Euclidean $\mathbb{R}^4$, but equations (2.2)–(2.3) use an operator commutator, causal spectral reasoning, and Lorentzian momentum projectors. A commutator spectral distribution is not itself a nonnegative Euclidean covariance. Positivity applies to the positive-energy Wightman spectral measure; the commutator also contains the energy-sign and support structure. Conversely, a Euclidean Schwinger correlator requires reflection positivity and a controlled analytic continuation before statements about particle poles and unitarity can be made. The manuscript needs to choose one formulation, specify all contact terms and subtractions for the composite operator $T_{\mu\nu}$, and derive the other formulation from it.

There are related inconsistencies in the reconstruction. Equation (2.6) introduces two independent measures, $\rho^{(2)}$ and $\rho^{(0)}$, whereas equation (2.10) uses one common density multiplying $P^{\mathrm{TT}} + P^{\mathrm{S}}$. A two-point function can be used to define a generalized free field after positivity has been established, but it does not invoke the full Wightman reconstruction theorem to produce a unique interacting field with the higher vertices later assumed in Section 2.3. Those vertices require higher stress-tensor correlators or an independently defined effective action. The author should give a dimensionally consistent Lorentzian Källén–Lehmann representation for both tensor structures, including the operator-renormalization and contact-term ambiguities.

2. **Equations (2.8)–(2.9) are not the Wilsonian coarse-graining described in the text and cannot generate a pole.**

   Equation (2.9) is simply

$$\rho_\sigma(\mu^2) = e^{-\mu^2\sigma^2/2}\rho(\mu^2).$$

   Multiplication of a measure by a smooth positive function preserves its singular support. If the original measure has no atom at $\mu^2 = 0$, no finite value of σ creates one. Its derivative, equation (3.1), suppresses weight at every $\mu^2 > 0$; it contains no spectral-space current that transports conserved weight toward zero. The statement in Section 3.1 that fluctuations of the window "push spectral weight from high to low momenta" is therefore not a property of the displayed operation.

   There is also a mismatch between invariant mass and Wilsonian momentum. Filtering by $\mu^2 = p^2$ suppresses large invariant masses, not modes with large spatial momentum. An arbitrarily energetic on-shell massless mode still has $p^2 = 0$ and is unaffected by this window. Once σ becomes position dependent in equation (2.13), translation invariance is lost and the coarse-graining cannot be represented by the same diagonal momentum-space multiplier; gradient corrections and a bilocal kernel are unavoidable. A genuine blocking prescription should be defined at the path-integral level and shown to induce a flow for the renormalized stress-tensor correlator.

3. **The stochastic dynamics of σ is postulated and is internally inconsistent.**

   The Planck and present Hubble lengths are asserted to be zeros of a beta function because they are proposed limits of description. This does not derive renormalization-group fixed points. Using ℓ_P is circular when G is later claimed to emerge, while inserting c/H_0 imports a measured, time-dependent cosmological scale into an otherwise flat Poincaré-invariant vacuum. The Hubble radius is not in general the largest separation of points with a common causal past. Equation (2.15) is consequently a phenomenological ansatz, not a result of Standard Model coarse-graining.

   The ansatz also conflicts with the claimed critical mechanism. Sections 2.4 and 3 require a finite ultraviolet point σ_* at which $\gamma(\sigma_*) = \gamma'(\sigma_*) = 0$. Immediately after equation (2.25),

however, the manuscript fixes
$$\gamma(\sigma) = \frac{c}{2\pi\sigma},$$
which is strictly positive at every finite σ and has no such second-order zero. An interior zero of γ would in any case make equation (2.15) vanish there and introduce an additional fixed point, contrary to the repeatedly stated two-fixed-point picture. Equation (2.15) is positive throughout $\sigma_0 < \sigma < \sigma_c$, contrary to the prose saying that β is negative near σ_c. The entire pole argument depends on these incompatible assertions.

Finally, a real-time fixed point of the stochastic variable does not imply $\partial\rho/\partial\sigma = 0$. Critical slowing can make $d\rho/dt = (\partial\rho/\partial\sigma)(d\sigma/dt)$ vanish because $d\sigma/dt$ vanishes while $\partial\rho/\partial\sigma$ remains nonzero. The stationarity condition imposed in Section 3.2 therefore does not follow from the Langevin critical point.

4. **The Model-A, fluctuation–dissipation, and temperature closure do not follow from coarse-graining.**

   Renormalization-group loss of resolution is not automatically physical real-time dissipation. Tracing over vacuum modes generally produces an entangled reduced state and a nonlocal, colored influence kernel; it does not by itself produce a Markovian thermal bath with white noise and detailed balance. If Model A is to be more than an analogy, the author must derive the mobility, noise kernel, equilibrium measure, and stochastic convention from a microscopic influence functional.

   Even at the phenomenological level, equations (2.14), (2.18), and (2.20) do not use a consistent mobility. The same γ is treated as a relaxation rate, an inverse dissipation coefficient, and the coefficient multiplying the noise correlator, while equation (2.14) contains a separate factor c/σ_c. Their dimensions do not agree. The manuscript also uses a pre-existing time variable, a spatial Laplacian, and $\delta(t-t')$ while later claiming that the arrow of time is produced rather than assumed.

   The numerical temperature in equations (2.24)–(2.25) follows from an explicit scaling error. With fixed V, equation (2.23) gives $S_{\text{cg}} \propto V\sigma^{-3}$ and $T_{\text{eff}} \propto \sigma^{-1}$, so
$$T_{\text{eff}}\frac{dS_{\text{cg}}}{d\sigma} \propto V\sigma^{-5}, \qquad \frac{dE}{d\sigma} \propto \sigma^{-2},$$
   not the same power as asserted. If V is instead the coarse-graining cell volume $V \propto \sigma^3$, the displayed entropy is constant and its derivative vanishes. In addition, equation (2.23) is a thermal-radiation entropy formula, not the generic von Neumann entropy produced by a Gaussian momentum filter. The uncertainty estimate in equation (2.21) does not imply a Gibbs state. Thus neither $\alpha = 1/(2\pi)$ nor $\gamma = c/(2\pi\sigma)$ has been derived.

5. **The injection flow and the "smooth contradiction" are not established.**

   Equation (3.1) follows exactly from the definition in equation (2.9). Equation (3.2) then adds an unspecified injection Δ_σ, changes the flow, and calls the result Wilsonian without deriving it from an exact renormalization-group identity. A valid calculation would need a regulator, running vertices, composite-operator mixing, Ward identities, and the appropriate contact terms. The displayed heat-kernel series in equation (3.4) supplies none of this. Seeley–DeWitt coefficients describe local terms in a Euclidean effective action; local contact polynomials do not automatically equal positive spectral injection.

   There are direct consistency checks that the proposed expansion fails. With the manuscript's convention $\rho \sim \mu^4$, the two terms in equation (3.2) do not have the same mass dimension if $\Delta_\sigma \propto \sigma^{-4}$ as in equation (3.4). Furthermore, if A_0, A_1, and A_2 in equation (3.4) are all nonzero as claimed, stationarity of a dilution term beginning at order μ^6 requires all three lower coefficients to vanish, not only A_2. The quoted value $A_0 = 12.33$, the assertion that every A_n is nonzero, and the crucial claim $A_2 \neq 0$ are neither calculated nor supported by a cited result. Nonzero matrix elements of $T_{\mu\nu}$ do not establish those coefficients. Also, the full Standard Model is not asymptotically free because of its Abelian sector, so the claimed ultraviolet free-field control requires a much more careful argument.

6. **The proposed delta function does not solve equation (3.2).**

   Substitution of equation (3.6) into the proposed fixed-point equation gives

$$0 = -\sigma_* \mu^2 \big[Z\delta(\mu^2) + \rho_{\text{cont}}(\mu^2) \big] \Delta_{\sigma_*}(\mu^2).$$

   Since $\mu^2\delta(\mu^2) = 0$, the residue Z drops out. The delta contribution is an arbitrary homogeneous zero mode of the displayed dilution operator; it neither cancels the nonzero analytic injection invoked in equations (3.3)–(3.4) nor provides an evolution equation for Z. For example, an injection proportional to μ^4 is balanced, if at all, by an ordinary continuum term proportional to μ^2, not by the delta atom. Accordingly, the proposed pole is not shown to exist, to be selected, or to be stable.

   The preceding critical-slowing argument does not repair this problem. A $1/\omega^2$ singularity of a stochastic relaxation mode is a zero-frequency response singularity in a system with a preferred time and a dynamic critical exponent. It is not automatically a Lorentz-invariant mass-shell atom $\delta(p^2)$ in the spin-2 stress-tensor measure. The manuscript must derive the relevant real-time spectral function and demonstrate this change of analytic structure. Failure of one smooth ansatz would not, by itself, prove existence of a stationary solution or select a delta function.

7. **The superconvergence argument is incompatible with positivity and with the manuscript's own spectrum.**

   Equation (3.5) states

$$\int_0^\infty d\mu^2\, \mu^2 \rho^{(2)}(\mu^2) = 0.$$

   Together with $\rho^{(2)} \geq 0$, this forces the positive measure to have support only at $\mu^2 = 0$. It therefore forbids the positive continuum retained in equation (3.6), as well as the perturbative multiparticle continuum discussed in Section 2.1. The moment is also ultraviolet divergent for the stated behavior $\rho^{(2)}(\mu^2) \sim (\mu^2)^2$, so it is not an ordinary unsubtracted sum rule. Stress-tensor conservation gives transversality and Ward identities with contact terms; it does not by itself give the displayed vanishing positive moment. The alleged absence of Schwinger terms is asserted rather than established.

   If the author intends a subtracted or sign-indefinite superconvergence relation, its precise dispersion relation and assumptions must be derived; the positivity argument then cannot be used in its present form. The subsequent exclusion of branch points is also incorrect. Massless multiparticle continua, threshold powers, and logarithms are common in perfectly local quantum field theories. Nonanalytic terms in a quantum effective action do not imply nonlocality of the microscopic theory or failure of effective-field- theory renormalizability. Hence a delta atom is not the unique infrared singularity allowed by unitarity and locality.

8. **A scalar $1/p^2$ factor has not been shown to be a physical massless graviton.**

   The projector in equation (2.5), with trace coefficient $-1/3$, is the four-dimensional massive-spin-2 projector and Section 2.3 explicitly assigns it five polarizations. It contains $p_\mu p_\nu/p^2$

and is singular on the very support $p^2 = 0$ proposed in Section 3. A massless graviton instead has two physical helicities and a gauge redundancy. Between conserved sources, massless Einstein exchange has the trace coefficient $-1/2$, modulo gauge terms, rather than the massive $-1/3$ structure. The trace decomposition in equation (2.12) is also not the conserved scalar decomposition associated with equation (2.5), which uses $\theta_{\mu\nu}$, not $\eta_{\mu\nu}/4$.

Appendix A performs the elementary scalar spectral integral but does not resolve the singular projector, derive a gauge Ward identity, reduce five polarizations to two, establish positivity of the physical Hilbert space, or show that the spin-0 sector decouples. Therefore equations (4.1) and (A.10) do not establish equivalence to the Pauli–Fierz action. These properties must be derived before a soft-graviton theorem can be applied.

The manuscript must also confront the Weinberg–Witten obstruction. It begins with a local Lorentz-invariant flat-space theory possessing a Lorentz-covariant conserved stress tensor and proposes a composite massless spin-2 particle that carries energy and couples to that same tensor. These assumptions lie directly in the domain of the no-go theorem. A viable model must identify a concrete evasion—for example nonlocality, failure of exact Lorentz invariance, absence of a covariant local stress tensor, or genuinely emergent spacetime—and show that the evasion remains compatible with the Lorentz invariance, locality, and unitarity later used in Section 4.

9. **Universal coupling and field normalization are not derived.**

   Section 2.5 states that a symmetric rank-2 tensor transforms in the $(\frac{1}{2}, \frac{1}{2})$ Lorentz representation; that is the vector representation. More importantly, Lorentz covariance permits an index contraction but does not fix its coefficient or exclude trace, derivative, improvement, or species-dependent operators. Equation (2.16) lacks the dimensionful coefficient required when $\Phi_{\mu\nu}$ is reconstructed from the stress-tensor spectrum. In a Gaussian theory, a linear source shifts the one-point function but leaves the connected covariance unchanged, so the amplitude rescaling in equation (2.17) does not follow. Equation (2.26) is then another assumption rather than a closure result.

   There is a direct algebraic conflict in Section 4. Equation (2.16) gives $S_{\text{int}} = \int T^{\mu\nu}\Phi_{\mu\nu}$. With the definition $h_{\mu\nu} = \kappa\Phi_{\mu\nu}$, it becomes $S_{\text{int}} = \kappa^{-1}\int T^{\mu\nu}h_{\mu\nu}$, whereas Section 4.2 asserts $\kappa\int h^{\mu\nu}T_{\mu\nu}$. Likewise, $\langle\Phi\Phi\rangle = Z/p^2$ together with $\kappa^2 = 1/Z$ makes $h = \kappa\Phi$ canonically normalized, for which the quadratic action is proportional to $\frac{1}{2}hEh$, not $\frac{1}{2\kappa^2}hEh$ as in equation (4.2). The pole residue of a composite-operator two-point function cannot fix a physical Newton coupling until the source normalization and relevant three-point functions are consistently derived.

10. **The use of Weinberg's theorem is conditional and overstated.**

    Weinberg's soft-graviton argument constrains couplings once a unitary Lorentz-invariant S-matrix containing a genuine massless helicity-2 state has already been established. It does not create that state from a transverse projector, prove that a composite residue has the required normalization, or by itself supply the missing gauge symmetry. Consistent self-coupling arguments yield general relativity as the leading two-derivative interaction, while a cosmological term, higher-curvature operators, nonminimal operators, and additional gapless sectors remain allowed in the effective theory. The dissipative white-noise dynamics used to produce the putative state must also be reconciled with the Lorentz invariance and unitarity assumed in the soft theorem.

    The cited nonperturbative graviton spectral-function studies do not support the specific premise of this paper: those calculations already contain a dynamical gravitational sector, whereas the present claim concerns a graviton atom generated in the matter-only Standard

Model stress-tensor correlator. The manuscript should distinguish these problems and engage directly with composite-gravity no-go and evasion literature. The Amati–Veneziano and recent composite-tensor references present in the bibliography are not substantively compared with the proposal.

11. **The normalization and estimate of Newton's constant are dimensionally and algebraically invalid.**

    With the definitions in Section 2, $T_{\mu\nu}$ has mass dimension four and its momentum-space two-point coefficient has mass dimension four. Therefore the coefficient Z in $Z\delta(\mu^2)$ has mass dimension six. The identification $G = 1/(8\pi Z)$ in equation (4.3) then has mass dimension -6, not the required -2. A different normalization of the emergent field could be introduced, but it must be stated and carried consistently through equations (2.10), (2.16), and (4.1)–(4.3).

    There is also a plain inversion error. The text posits

    $$Z \sim \frac{N_{\text{eff}}}{\sigma_c^2 \hbar c}.$$

    Combined with equation (4.3), this would give $G \sim \sigma_c^2 \hbar c/(8\pi N_{\text{eff}})$, not equation (4.6), $G \sim \hbar c/(\sigma_c^2 N_{\text{eff}})$. In natural units the right-hand side of equation (4.6) has dimension L^{-2}, while G has dimension L^2. It therefore cannot agree with the measured Newton constant.

    The degree-of-freedom count is also inconsistent with its own infrared criterion. At $\sigma_c = c/H_0$, $k_B T_{\text{eff}}$ is of order $\hbar H_0$, approximately 10^{-33} eV. Neutrinos and the Higgs are far too massive, and gluons are not massless infrared asymptotic states because of confinement. The listed contributions neither justify $N_{\text{eff}} = O(100)$ nor determine a spectral residue. A controlled computation of a correctly normalized residue is essential; dimensional analysis with an observed Hubble scale cannot replace it.

12. **The cosmological-constant and black-hole checks are circular or contradict the assumed dynamics.**

    Removing an identity shift from a commutator is standard, but it does not solve the cosmological constant problem. Vacuum energy is a one-point/effective- action problem. Once the manuscript postulates an effective metric coupled to matter, the local operator $\int d^4x\sqrt{-g}$ is allowed and radiatively sensitive. A mechanism that removes a constant from a connected spectral function does not explain why this term is absent or small.

    Equation (4.9) does not provide such a mechanism. Since $\beta = -V'_{\text{eff}}$ and $\beta(\sigma_c) = 0$, $\beta'(\sigma_c)$ is related to the curvature of the potential, not to its vacuum value. Sections 2.4 and 3.1 require $\beta'(\sigma_c) < 0$ for an infrared attractor, while the paragraph following equation (4.9) assumes a positive value to obtain positive Λ. Substituting the observed $\sigma_c = c/H_0$ to obtain $\Lambda \sim H_0^2/c^2$ is a dimensional restatement of the input, not a prediction.

    The Hawking-temperature result is similarly calibrated. Equation (4.8) follows only after adopting $\sigma_c = 2r_S$, including the coefficient needed to reproduce the known answer. Elsewhere σ_c is a universal Hubble-scale fixed point, so it cannot simultaneously equal the object-dependent radius of every black hole. Moreover, equation (4.8) is the temperature measured at infinity; a local static observer near the horizon measures the Tolman-redshifted temperature. Schwarzschild geometry and the Hawking result are consequences of the Einstein theory assumed in Section 4, not independent tests of the proposed microscopic mechanism.

Novelty and significance

If a matter-only, local quantum field theory were shown to develop an isolated positive massless spin-2 atom with the correct two-helicity tensor residue and universal coupling, the result would be highly significant. The manuscript does not supply such a result. Its only established pole statement is the standard Källén–Lehmann implication "delta atom implies scalar $1/p^2$ term." The claimed novelty lies instead in the stochastic cutoff dynamics and the assertion that it produces the atom, but the cutoff field, its two endpoints, the critical zero, the injection kernel, the pole residue, and the source coupling are all assumed or inferred qualitatively.

A potentially publishable project could recast the paper around a precise nonperturbative observable. It should define a positive Lorentzian spectral measure or a reflection-positive Euclidean correlator, derive an exact or controlled approximate flow, and identify a finite-volume scaling test that distinguishes an atom at $\mu^2 = 0$ from a massless continuum. A toy model in which the complete calculation can be performed would be much more persuasive than order-of-magnitude matching to scales already inserted as inputs.

Minor comments

1. Equation (2.1) uses $\sqrt{-g}$ while the surrounding text declares Euclidean signature. The Hilbert stress tensor is not generally identical to the displayed canonical tensor without Belinfante improvement, possible scalar improvement terms, and composite-operator renormalization.

2. The statement that $\rho^{(2)} \sim p^4$ holds to all perturbative orders needs qualification. Massless thresholds, anomalous dimensions, running couplings, and logarithms affect the infrared and ultraviolet forms even when there is no one-particle atom.

3. Section 2.3 says that the Gaussian approximation is automatically valid near the infrared endpoint and also invokes the central limit theorem at the ultraviolet endpoint where no averaging has occurred. Neither assertion is derived from the correlators.

4. The text alternates between "low temperature suppresses quantum fluctuations and forces a new ground state" and the claim that the transition requires high-temperature fluctuations near the ultraviolet repeller. Thermal suppression does not remove zero-temperature quantum spectral weight, and these two proposed mechanisms are mutually unclear.

5. The assertion below equation (3.5) that $\rho \sim \mu^{-\alpha}$ makes the infrared moment divergent for every $\alpha > 0$ is false. With $x = \mu^2$, the integral $\int_0^\epsilon dx\, x^{1-\alpha}$ diverges only for $\alpha \geq 2$.

6. A gapless continuum can generate long-range power-law correlations even without a one-particle pole. The paper should distinguish "not a Newtonian $1/r$ force" from "not long range."

7. Expanding the Einstein–Hilbert action about flat space gives a boundary term at first order and the kinetic action at quadratic order. The statement following equation (4.5) that its first-order expansion yields the linearized Einstein equation should be corrected.

8. The manuscript should state a single convention for c, $\hbar$, field dimensions, Fourier transforms,

and metric signature, and then verify the dimensions of every term in equations (2.14), (2.20), (3.2), and (4.1)–(4.9).

Conclusion

The manuscript addresses an ambitious and worthwhile question, but its main result—dynamical formation of a massless spin-2 pole—is not demonstrated and is inconsistent with several of its own equations. The subsequent identification with a graviton does not establish the required helicity and gauge structure, does not address the relevant no-go theorem, and uses incompatible field and coupling normalizations. The proposed estimates of G, Λ, and the Hawking temperature do not constitute valid predictions. I therefore recommend rejection. A future submission would need a newly derived spectral flow, an explicit positive fixed-point solution, a concrete evasion of the massless composite-spin-2 obstruction, and a dimensionally consistent calculation of the residue and source coupling.