Referee Report

on “Linking defects via AdS/CFT holography”
arXiv:2607.21994

Summary

The manuscript studies probe branes in the supersymmetric one-parameter deformation of global $AdS_5 \times S^5$ introduced by Gauntlett, Gutowski, and Suryanarayana. The authors first review the type IIB background, including the metric fibration, the five-form flux, and the two preserved supersymmetries. They then claim a general class of κ-symmetric D5-brane embeddings specified by two holomorphic constraints, equations (2.24) and (2.25). A particular member is presented as a connected U-shaped brane that approaches the conformal boundary as an embedding parameter c is made large. The DBI contribution is then discarded in that limit, leaving a proposed topological Wess-Zumino coupling and the finite expression (2.38).

The second part introduces a purported anti-D5 counterpart and a class of D3-brane embeddings. A special D3 configuration is interpreted as a giant graviton dual to a determinant operator. Moving this D3 through a coincident D5–anti-D5 configuration is said to create a fundamental string through a Hanany-Witten transition. The phase in equation (3.10) is proposed to measure the determinant operator's charge under a boundary $U(1)$ symmetry.

The subject is timely. A correct construction of supersymmetric D5 probes in this deformed background, together with a deformation-dependent realization of a symmetry operator, could be useful. Unfortunately, the central chain of the present paper is not established. The concrete D5 and D3 examples are algebraically inconsistent with the complex embedding equations from which they are claimed to follow. The worldvolume action and the topological limit are not derived, the brane–anti-brane physics is omitted, and the proposed charge measurement is not matched quantitatively to a specified boundary charge. Moreover, the symmetry operator associated with the relevant S^5 isometry is not constructed in full.

Recommendation

I recommend rejection in the present form. The problems below concern the defining embeddings and the main physical claim, rather than matters that can be repaired by clarification or a limited revision. A substantially rebuilt paper could merit consideration as a new submission if it provides a consistent D5 solution, the complete symmetry-operator construction, and a quantitative charge test.

Major comments

1. **The concrete D5 embedding does not follow from the stated holomorphic conditions.**

   This is the most immediate technical obstruction. Equation (2.21) defines

$$\Phi_0 = l \cosh\rho\, e^{i\phi_0}, \qquad \Phi_1 = l \sinh\rho \cos\theta\, e^{i\phi_1},$$
$$Z_2 = l \cos\alpha \sin\beta\, e^{-i\xi_2}, \qquad Z_3 = l \cos\alpha \cos\beta\, e^{-i\xi_3}.$$

The example announced in Section 2.2 is $\Phi_0\Phi_1 Z_3^2 = c_1$, together with $Z_2 = 0$. Away from a degenerate coordinate locus, these equations imply

$$\frac{l^4}{2}\sinh(2\rho)\cos\theta\cos^2\alpha\cos^2\beta = |c_1|, \qquad \phi_0 + \phi_1 - 2\xi_3 = \arg c_1,$$

and $Z_2 = 0$ selects $\beta = 0$, not $\beta = \pi/2$.

Equation (2.26), by contrast, contains $\sinh(2\rho)\sin\theta\sin^2\alpha = c$, $\xi_3 = -(\phi_0+\phi_1)/2$, and $\beta = \pi/2$. At $\beta = \pi/2$, one has $Z_3 = 0$, so $\Phi_0\Phi_1 Z_3^2$ cannot equal a nonzero c_1. The radial dependence, the trigonometric factors, the phase relation, and the selected coordinate locus all disagree. This is not a harmless normalization issue. Figure 1, the induced metrics (2.28)–(2.29), and the action calculation (2.30)–(2.38) all depend on the real embedding (2.26), so the paper has not shown that the configuration used for its main argument belongs to the claimed BPS family.

Even if (2.26) is treated as an independent ansatz, the displayed pullback requires correction. At $f = 0$, the coefficient of $d\phi_2^2$ inherited from global AdS_5 is proportional to $\sinh^2\rho\,\sin^2\theta$. The corresponding term in (2.28), after using (2.26), is instead proportional to $\sinh^2\rho\,\cosh^2\rho\,\sin^2\theta$, with an extra factor of $\cosh^2\rho$. In addition, the asymptotic metric (2.29) changes the sign of the time component and drops the ϕ_2 direction without explanation. A direct line-by-line pullback of the ten-dimensional metric is needed.

The authors must state the intended complex equations, derive their real and imaginary parts explicitly, specify all branches and coordinate ranges, and then recompute the induced metric and action. The anti-D5 example compounds the problem: equation (2.45) contains $(\overline{Z}_3)^{-2}$, whereas the proposed D5 condition contains Z_3^2. Those two loci are not coincident copies with opposite orientation. The claimed overlap must be demonstrated from corrected equations, not asserted.

2. **The κ-symmetry analysis is not complete, and it is inconsistent with the later worldvolume gauge field.**

   Equations (2.12)–(2.18) use an unnormalized six-gamma density as Γ_κ, while the standard type IIB D5 projector includes the determinant normalization, orientation, the doublet/Pauli-matrix action, and, when present, the full gauge-invariant worldvolume two-form $\mathcal{F}$. Conventions can of course be chosen so that the determinant is moved to the right-hand side, but the paper must give the complete formula and show that its square and orientation have the required properties.

   More importantly, the passage from the 22 six-form conditions (2.15) to the arbitrary holomorphic family (2.24)–(2.25) is too abbreviated to verify. Setting the coefficients of the anti-holomorphic one-forms and of $\mathfrak{e}^0 - \mathfrak{e}^9$ to zero supplies a possible sufficient ansatz; it does not by itself prove that all terms in (2.15), the volume-form condition (2.18), and the probe equations of motion are satisfied. The missing algebra should be supplied, and the result should be checked directly on the concrete example. The undeformed $f = 0$ limit should also be compared with known D5 classifications and standard $AdS_5 \times S^5$ defect embeddings.

   There is a further incompatibility in equations (2.32)–(2.35). The connection is written schematically as

   $$a \sim \alpha(d\phi + d\psi).$$

   If α is the spacetime coordinate used throughout Section 2, then $da = d\alpha \wedge (d\phi + d\psi)$ is nonzero. It must therefore enter both the DBI determinant and the κ-symmetry projector, invalidating an embedding derived with zero worldvolume flux. If instead α denotes a constant holonomy parameter, then $da = 0$ locally and the coupling $da \wedge P[C^{(4)}]$ in (2.32) vanishes. The notation and physics must be disentangled, with a globally defined connection, flux quantization, boundary conditions, and a new supersymmetry analysis when $\mathcal{F} \neq 0$.

3. **The D5–anti-D5 system and its stability are not analyzed.**

   Reversing the sign in the κ-symmetry equation is not, on its own, a derivation of a coincident anti-brane preserving the needed structure. The coordinate operation in (2.42), which includes $r \mapsto -r$, is especially problematic because r is a nonnegative radial coordinate. The authors should formulate the orientation reversal intrinsically on the worldvolume and determine which background Killing spinors, if any, are preserved by each component.

   A coincident D5–anti-D5 pair has an open-string tachyon and is not the direct sum of two independent BPS probes. Tachyon condensation, partial annihilation, and brane recombination are central to the brane–anti-brane regularization of continuous symmetry defects cited by the manuscript. None of these ingredients appears here. Consequently, the paper does not establish the existence or stability of the proposed pair, its equivalence to a non-BPS D4 object, or its fusion law.

   There is also a conceptual ambiguity. Equation (2.26) and Figure 1 already describe a single connected U-shaped D5 with two asymptotic branches. The later construction places an additional anti-D5 "on top" of this connected brane. The manuscript must explain whether the two branches are themselves the resolved brane/anti-brane pair, whether an independent anti-D5 is intended, and how the resulting Chan-Paton and tachyon data avoid double counting. This issue must be resolved before the Hanany-Witten process in Section 3.1 can be defined.

4. **The large-c limit does not establish a topological defect, and the worldvolume action is not under control.**

   The key step from (2.30) to (2.31) is justified by saying that the near-boundary D5 is heavy, so its DBI term does not contribute. This reasoning is not valid as stated. The D5 tension multiplies both the DBI and Wess-Zumino sectors, and a large induced volume near the conformal boundary normally makes the classical DBI action larger and more divergent. A topological symmetry defect requires a demonstrated insensitivity to deformations of its support, or equivalently control over the displacement operator and metric-dependent fluctuations, after a specified renormalization. Proximity to the boundary alone does not supply that result.

   The complete D5 action should be written with its relative signs, factors of $2\pi\alpha'$, the dilaton and B-field where applicable, and all relevant RR couplings, including $C^{(6)}$ and $C^{(4)} \wedge \mathcal{F}$. The integration by parts from (2.32) to (2.34) also requires careful treatment of the conformal boundary, large gauge transformations of $C^{(4)}$, and the fact that the five-form flux is not represented globally by a single four-form potential. These are precisely the issues that determine whether the proposed phase is gauge invariant.

   Equations (2.36)–(2.38) then assume a factorization into $\gamma \times \Sigma_3 \times \Sigma_2$, although the displayed embedding mixes the radial, angular, and internal coordinates. The cycles, their ranges, orientations, and pullbacks are not given. The claimed quadratic divergence, the counterterm that removes it, and the finite coefficient $96\sqrt{3}\,\pi^5 f l^6$ are stated without an intermediate calculation. The authors should provide a reproducible regulated integral, a gauge-invariant counterterm prescription, and the flux-quantization relation to N, g_s, and α'. They should also explain why the proposed topological operator has an action proportional to the background deformation f and what remains in the undeformed $f \to 0$ limit.

5. **The full boundary symmetry and its bulk operator have not been identified.**

   The gauge field A in equations (2.1) and (2.6) originates from an S^5 isometry and therefore corresponds to a particular R-symmetry current, not to an unspecified generic $U(1)$. The original supergravity solution is dual to $\mathcal{N} = 4$ SYM on a non-conformally-flat background with nonzero R-current data. The manuscript should identify the exact generator and global form of the $U(1)$, state the boundary source and current normalization, specify the physically allowed range of f, and explain how this curved-background state modifies the usual operator dictionary.

This omission is also important on the bulk side. The recent hanging-brane construction cited in the manuscript explains that an isometry symmetry operator in type IIB generally contains both a D5 contribution associated with the five-form sector and a KK-monopole contribution associated with the Einstein-Hilbert sector. In the present background the same gauge field A appears explicitly in both the ten-dimensional metric (2.1) and the five-form flux (2.6). Equations (2.30)–(2.38) retain only the D5/five-form piece. The paper therefore has not constructed the full Gauss-law operator for the purported R-symmetry. The missing KK sector must be included, or a background-specific argument must prove that it is absent or cancels.

Finally, the charged boundary object is not specified precisely. The manuscript should say which of the three complex adjoint scalars is denoted by Z, give its charge under the chosen generator, and state the resulting charge of $\det Z$. Without this dictionary, the proposed measurement has no definite quantity to reproduce.

6. **The special D3 embedding is algebraically inconsistent with its defining holomorphic equations.**

For the ratios in equation (3.3), the definitions (2.21) give

$$|c_3| = \tanh\rho\,\cos\theta, \qquad |c_4| = \tanh\rho\,\sin\theta.$$

Hence, writing $q^2 = |c_3|^2 + |c_4|^2$, one obtains

$$q < 1, \qquad \tan\theta = \frac{|c_4|}{|c_3|}, \qquad r = l\sinh\rho = \frac{lq}{\sqrt{1-q^2}}.$$

Equation (3.5) instead gives a radius proportional to $|c_3|/\sqrt{|c_3|^2-1}$, omits the dependence on c_4, and is not real in the range required by $\Phi_1/\Phi_0 = c_3$. It is also dimensionally inconsistent as printed because r has dimensions of length while the right-hand side is dimensionless.

Moreover, the first equation in (3.3) becomes $Z_1\Phi_0 = 0$ when (3.5) sets $\alpha = 0$, so it requires $c_2 = 0$. At that locus the phase ξ_1 parametrizes a collapsed circle, making the printed phase condition questionable. These issues must be corrected before the induced metric (3.6), the interpretation as a giant graviton, or its placement relative to the D5 profile in Figure 2 can be accepted. A direct substitution into the D3 κ-symmetry projector and equations of motion is needed, particularly because equations (3.1)–(3.2) are asserted in the deformed background without a derivation. The proposed D3 connection in (3.8) is also locally exact, while ξ_1 is degenerate at $\alpha = 0$ and is not one of the displayed D3 worldvolume coordinates. Its claimed global holonomy therefore requires a separate patchwise construction; it is not defined by the expression currently given.

7. **The charge measurement remains a schematic proposal rather than a calculation.**

The internal support Σ_2 of the D5 and the S^3 wrapped by the D3 are never defined as oriented submanifolds with explicit embeddings. Their alleged intersection number in (3.10) is therefore not computable from the manuscript. This is not merely notation: $H_2(S^5)$ and $H_3(S^5)$ vanish, so a naive intersection of closed ordinary cycles cannot provide the required nonzero topological invariant. The recent literature treats isometry charges using appropriate equivariant or relative data. The authors must specify the corresponding construction here and evaluate it for their configuration.

The Hanany-Witten process also needs a charge-conservation analysis. One must track the orientation of the string created when the D5 crosses the D3 and the string created or destroyed when the anti-D5 crosses it. It is not enough to infer the final string solely from Figure 2. The endpoints, the two Chan-Paton gauge fields, the tachyon, and all signs should be included. Table 1 is itself inconsistent with the stated static gauge: the D5 row does not mark the θ direction as a worldvolume direction, even though (2.27) uses θ as a worldvolume coordinate and the created F1 is said to extend along θ.

Most importantly, equation (3.10) should be compared with the known transformation of the determinant operator. The paper does not compute $\Sigma_2 \cdot S^3$, define the periodicity and normalization of $\alpha_\gamma = \int_\gamma a$, or recover the expected representation weight and N-dependence of $\det Z$. Gauge invariance under large Chan-Paton gauge transformations is not discussed. Until these quantities are fixed and the two phases are shown to agree, the central claim of charge measurement is untested.

8. **The novelty relative to the cited recent literature is insufficiently separated from previously established results.**

   The D3–D5 crossing, F1 creation, Chan-Paton holonomy, and intersection phase in Section 3.1 closely follow the construction in Bah, Bonetti, Chitoto, and Leung, "Continuous symmetries and charge measurement of boundary operators in holography" (arXiv:2602.22377), especially their type IIB $AdS_5 \times S^5$ analysis. The manuscript acknowledges that reference but nevertheless presents equation (3.10) as its proposal without identifying a new deformation-dependent prediction. The profile (2.26), the induced metric, Figure 1, and the phase (3.10) display no dependence on f; only the unsubstantiated finite action (2.38) does.

   The authors should isolate what is new beyond the general hanging-brane and charge-measurement mechanism. A potentially interesting contribution would be a verified family of D5 probes specific to the two-supercharge deformed background, together with the complete D5–KK operator and an observable whose f-dependence is calculated. This requires a systematic comparison with the cited hanging-brane, brane–anti-brane, and $U(1)$ R-symmetry constructions, as well as with classic D3/D5 defect holography and known $f = 0$ probe embeddings. In the present version, the technically established content does not yet support the claimed advance.

Minor comments

1. The preamble title refers to a "κ-symmetric M5 brane web" in AdS_7/CFT_6, whereas the title page and manuscript concern D5 branes in AdS_5/CFT_4. The metadata and date should be corrected.

2. The standard terminology is "type IIB", not "type 2b". The notation "anti-$\overline{\text{D5}}$" is redundant; either "anti-D5" or $\overline{\text{D5}}$ should be used consistently.

3. The coordinate ranges after the redefinition $\theta \mapsto 2\theta$ in (2.20) should be stated. Figure 1 uses $0 < \theta < \pi$, while the complex-coordinate parametrization and the original S^3 coordinates suggest a different range. The axis should also retain the factor l in $r = l \sinh \rho$, and the fixed value of α and the dimensions of c should be given.

4. The topology $\mathbb{R} \times \widetilde{S}^2$ is not established by the asymptotic line element (2.29). The coordinate identifications, endpoints, degenerating circles, and conformal representative of the defect metric should be displayed.

5. Equation (3.4) is not a correct indexed expression for a determinant. It should be replaced by the standard product of N matrix elements contracted with two Levi-Civita tensors, with unambiguous indices and normalization.

6. The DBI actions in (2.30) and (3.7) need consistent Lorentzian signs and D-brane tensions. The same notation should be used for F, $\mathcal{F}$, and the Chan-Paton connection throughout.

7. The presentation contains numerous typographical and grammatical errors, including "gravi-tion", "Hannay-Witten", and "transtion". Several bibliography entries have incorrect arXiv numbers or metadata. A careful technical and language edit is needed after the physics has been revised.

Conclusion

The manuscript addresses an interesting problem, and a correct deformation-specific D5 construction could become worthwhile. At present, however, the explicit D5 and D3 embeddings do not follow from their defining equations, the worldvolume and brane–anti-brane dynamics do not justify a topological limit, the full isometry-symmetry operator is missing, and no normalized determinant charge is reproduced. These defects invalidate the principal conclusions. For that reason I do not recommend publication in the present form.