Referee Report

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"Carrollian bosonic supergravity at order α' and the universal cancellation of higher-curvature divergences"

Summary of the paper

The manuscript studies a double-scaled Carrollian limit of the 26-dimensional bosonic-string effective action. Section III starts from the Metsaev–Tseytlin action in equations (1)–(4), decomposes the metric, two-form, and dilaton according to equations (5)–(11), and takes $w = 1/c$ to infinity while keeping $\alpha'_C = w^2\alpha'$ fixed. The leading measure is of order w^{-2}, so a four-derivative Lagrangian must grow no faster than w^4 to leave a finite contribution.

Section IV expands the relativistic Riemann tensor as in equation (12) and the three-form in equations (13)–(15). After introducing the connection in equation (20) and the metricity relations (22)–(25), the author identifies a possible w^6 divergence in the mixed H^2R term, equation (26). The principal first-order claim is that inserting the coefficient $R^{(2)}$ displayed in equation (27) cancels this divergence without making a particular choice of the allowed non-metricities. The R^2 sector had already been treated in Ref. [22].

Section V then imposes the stronger conditions $\nabla_\mu h_{\nu\rho} = 0$ and $\nabla_\mu \tau^\nu = 0$, equation (28), and presents a compact two-derivative action together with the nominally finite R^2, H^2R, H^4, and $(H^2)^2$ contributions in equations (31)–(36). Appendix A spells out the terms containing the three-form $f_{\mu\nu\rho}$.

Finally, Section VI proposes a power-counting criterion for derivative-free invariants containing N Riemann tensors. Once equation (28) has removed the $w^2R^{(2)}$ coefficient, the argument in equations (37)–(40) attributes all remaining positive powers of w to inverse metrics. The manuscript applies this reasoning to selected R^3 and R^4 representatives of the bosonic-string action in equations (41)–(44). It claims that the displayed rational R^4 terms survive while the terms proportional to $\zeta(3)$ vanish.

Assessment of correctness and clarity

The extension of the first-order calculation to the full Metsaev–Tseytlin H-dependent sector is a worthwhile problem. In particular, a transparent proof that the w^6 coefficient in equation (26) vanishes for arbitrary non-metricities consistent with equations (22)–(25) would be useful. The attempt to formulate higher-order power counting in a way that can be applied without expanding every contraction is also potentially useful.

I do not, however, find the main higher-curvature conclusions correct as written. Equation (28), together with the manuscript's own definition of the curvature in equation (21), immediately implies

$$0 = [\nabla_\rho, \nabla_\sigma]\tau^\mu = R^\mu{}_{\nu\rho\sigma}\tau^\nu.$$

This identity makes the R^2 and H^2R expressions displayed in equations (32) and (33) vanish. It also makes both purportedly surviving R^4 contractions in equation (44) vanish. Independently, both R^3

terms in equation (42) contain a Riemann tensor contracted with $\tau^\rho\tau^\sigma$ on its antisymmetric last pair and hence vanish. Thus the manuscript distinguishes nonzero rational contributions from a vanishing $\zeta(3)$ sector even though its displayed rational contributions are themselves zero.

This is not a cosmetic simplification. It changes the claimed explicit actions, the higher-order applications, and the interpretation of the "universal" mechanism. In addition, the essential cancellation in Section IV is asserted after a long component formula rather than demonstrated, and several tensor definitions needed to reproduce it are missing or malformed. The paper therefore requires a new tensor-algebra analysis before its central correctness claims can be assessed positively.

Novelty and significance

The clearest potentially new contribution is narrower than the title and conclusion suggest: it is the completion of the order-α' analysis by the H^2R, H^4, and $(H^2)^2$ terms, especially the claimed non-metricity-independent cancellation in equations (26)–(27). Section III explicitly states that Ref. [22] already established finiteness of the leading action and of the R^2 sector. The priority claim in Section VII that the present work gives the first proof for R^2 is therefore inconsistent with the manuscript's own account and should be corrected.

The higher-order observation would be interesting if formulated as a precise theorem, but equations (37)–(40) presently give a conditional power-counting bound in a strongly restricted geometric sector. They do not demonstrate a universal cancellation among general higher-curvature invariants, and the explicit examples do not survive the elementary curvature identities above. The dependence on the simultaneous $c \to 0$, $\alpha' \to 0$ scaling and on the chosen field-redefinition scheme also needs to be made central. With these issues corrected, the full four-derivative cancellation could still constitute a focused result of interest to the Carrollian and string effective-action communities.

Recommendation

Major revision. The manuscript is not suitable for publication in JHEP or PRD in its present form because the equations asserted to be the surviving pure-curvature corrections vanish under the assumptions used to derive them. I would be willing to reconsider a substantially reworked version that resolves this contradiction, supplies a verifiable derivation of the unrestricted four-derivative cancellation, states the double-scaling and field-basis dependence precisely, and narrows the novelty and T-duality claims accordingly.

Major comments

1. **The compatibility condition annihilates the displayed curvature corrections.**

   The condition $\nabla_\mu\tau^\nu = 0$ in equation (28) has an immediate integrability consequence. With the curvature convention in equation (21),

   $$R^\mu{}_{\nu\rho\sigma}\tau^\nu = 0.$$

   No metric compatibility or additional Riemann symmetry is needed for this conclusion. It follows simply by commuting two covariant derivatives on τ^μ.

Several headline formulas conflict with this identity. Equation (32) contains $R^{\mu}{}_{\nu\rho\sigma}\tau^{\nu}$ in its first factor and the analogous contraction in its second factor. Equation (33), as well as the curvature-dependent expression at the beginning of Appendix A, contains the same zero factor. In the first term of equation (44), for example, $R^{\alpha}{}_{\beta\pi\chi}\tau^{\beta}=0$; all four Riemann tensors are contracted in this way. The second term has the same property. Both terms in equation (42) vanish for an additional, even more elementary reason: they contain factors such as $R^{\mu}{}_{\nu\rho\sigma}\tau^{\rho}\tau^{\sigma}$, which are zero by antisymmetry in ρ,σ.

The author should first simplify every expression using the identities implied by equation (28). If a nonzero result was intended, the connection, curvature convention, or compatibility assumption must be changed and the replacement stated explicitly. Otherwise the correct conclusion appears to be that the displayed pure R^2, R^3, and R^4 finite coefficients vanish in this restricted sector. In that case the claim that the rational R^4 terms survive while only the $\zeta(3)$ terms disappear must be withdrawn. The Ricci term in equation (31) should be checked for the same reason under the standard Ricci contraction.

2. **The cancellation in Section IV needs an actual proof and a complete power inventory.**

   The most promising new result is compressed into the sentence following equation (27): after substituting an eighteen-term expression into equation (26), the divergence is said to vanish. A reader cannot tell which of equations (22)–(25), which antisymmetries, and which differential identities are required. Nor is it clear whether the cancellation is termwise, follows after relabeling dummy indices, or uses integrations by parts. Since this calculation is the basis for the claim that no particular non-metricity is needed, it should be displayed in a factorized form or supplied as a short sequence of tensor identities.

   The section should also tabulate the maximal w-power of every term in equation (3). The previous treatment of R^2 may be cited, but the paper should explicitly explain why antisymmetry bounds the H^4 and $(H^2)^2$ terms by w^4, and why equation (26) exhausts the possible w^6 coefficients. The phrase "without any specific choice of the non-metricities" should be made precise: equations (22)–(25) are already relations inherited from relativistic metricity, so the result concerns arbitrary data within that constrained class. Given the size of equation (27) and Appendix A, an ancillary symbolic-algebra file or an independent adapted-coordinate check would be appropriate.

3. **Equation (28) defines a restrictive subsector, not merely a convenient simplification.**

   The connection has already been specified in equation (20). Imposing $\nabla_{\mu}h_{\nu\rho}=0$ and $\nabla_{\mu}\tau^{\nu}=0$ therefore constrains the Carrollian fields; it is not a freely selectable convention. For a torsionless connection these conditions imply

   $$\mathcal{L}_{\tau}h_{\mu\nu}=\tau^{\rho}\nabla_{\rho}h_{\mu\nu}+h_{\rho\nu}\nabla_{\mu}\tau^{\rho}+h_{\mu\rho}\nabla_{\nu}\tau^{\rho}=0.$$

   Thus the spatial metric has vanishing evolution along the Carrollian time vector. This excludes generic Carrollian geometries and is directly responsible for strong curvature identities, including the one in the first major comment.

   The paper should characterize the existence and integrability conditions for this compatible torsionless connection, give a nontrivial class of backgrounds satisfying them, and state clearly that equations (31)–(44) apply only to this sector. It should also distinguish the general finiteness claim of Section IV from the much more restricted explicit action and higher-order discussion. Ideally, the finite order-α' action should be given before equation (28), or the author should explain why the restricted sector is physically sufficient.

4. **The "universal criterion" must be reformulated as a precise scaling statement.**

   Equation (38) is not a well-defined scalar invariant: its indices are schematic and no complete contraction is supplied. The argument should define the class under consideration, including whether covariant derivatives, Ricci tensors, epsilon tensors, or dimension-dependent identities are allowed. For derivative-free scalars constructed from mixed-index Riemann tensors, one can formulate a clean upper bound by counting inverse metrics. That upper bound establishes finiteness after the chosen double scaling, but it does not establish that the w^{2N} coefficient is nonzero.

   This distinction is exactly what is missed in equations (42) and (44). Riemann antisymmetry and the integrability identity from equation (28) remove the nominal leading power. The paper must calculate the first nonvanishing coefficient, not infer a surviving action from the maximum number of inverse metrics. The use of $\mathcal{O}(w^{2N})$ is also ambiguous here: it is an upper bound and includes terms that grow more slowly or vanish. If a nonzero finite limit is intended, the relevant leading coefficient must be exhibited.

   There is additionally a missing factor in the displayed logic. Equations (39) imply that the action contains $w^{-2N}\mathcal{I}_N$, whereas the left-hand side of equation (40) prints no w^{-2N}. The formula and the meaning of $\mathcal{I}_N$ before and after taking the limit should be corrected. Until a theorem for a clearly defined class is proved, "sufficient power-counting criterion" is more accurate than "universal cancellation."

5. **The limit is a double scaling, and higher-order conclusions are field-basis dependent.**

   The finite terms are retained by taking $\alpha' = \alpha'_C/w^2$. Thus the calculation does not take the Carrollian limit at fixed string length; it sends α' to zero simultaneously with c, keeping a particular combination fixed. This essential qualifier should appear in the abstract, in the list of main results, and in the conclusion. The author should motivate why this is the physically relevant scaling, whether other scalings define distinct Carrollian theories, and which quantities are held fixed dimensionally.

   Beyond first order, local field redefinitions change the representative of the effective action. Equations (41)–(44) use a particular pure-Riemann basis from Ref. [25], while the discussion itself notes that field redefinitions affect the comparison with the non-relativistic limit. The claims about "all purely gravitational contributions" and about the disappearance of the $\zeta(3)$ sector must therefore be qualified by the field basis. The author should explain whether a regular relativistic field redefinition remains regular under the double scaling and whether finiteness or vanishing is invariant under the allowed redefinitions. The pure-curvature truncation should also be separated from the full bosonic-string action containing H, the dilaton, and derivative couplings.

6. **The full order-α' action is not presently reproducible or demonstrably invariant.**

   The shorthand H^4 in equation (3) is never defined as a tensor contraction; equation (4) defines $H^2_{\mu\nu}$ and H^2, but not H^4. This omission prevents a direct check of equations (34) and Appendix A. The connection in equation (20) contains an undefined $\tau^{\rho\sigma}$, and equation (13) has mismatched free indices. Equations (22), (30), and (43) contain additional malformed symmetrizations or tensor notation. These are not harmless presentation issues when the claimed result depends on a long component expansion.

The author should give unambiguous definitions of every contraction, correct the index structure, and show the transformation rules induced by the decomposition $B = b + \tau \wedge A$. In particular, the final action should be checked under two-form gauge transformations, diffeomorphisms, and the relevant local Carrollian boost redundancy. Calling equations (31)–(36) "manifestly covariant" is not enough without specifying these symmetries. A compact notation, together with one direct reconstruction of the relativistic H-sector, would make the appendix far more useful.

7. **The novelty, priority, and T-duality statements should be narrowed and supported.**

   Section III credits Ref. [22] with the leading action and R^2 finiteness, while Section VII calls the present work the first proof for R^2. The author should state exactly which coefficients and assumptions are new here. The abstract discusses R^3 and R^4 applications at orders α'^2 and α'^3, whereas the conclusion refers to R^2 and R^3 and omits the displayed R^4 analysis. A consistent result inventory is needed.

   The sentence after equation (44) also suggests evading the order- α'^3 T-duality no-go of Ref. [26], but speaks of a formulation only "up to order α'." No duality-covariant action or transformation law is constructed. Even a correct disappearance of a subset of R^4 terms would not by itself establish that the obstruction has been avoided. This claim should either be removed or replaced by a concrete comparison with the tensor structures responsible for the no-go. Similarly, the Carrollian/non-relativistic contrast should be presented as a comparison in the stated field variables, not as a scheme-independent obstruction, especially while Ref. [33] is unpublished.

Minor comments

1. Equation (13) should read $\hat{H}_{\mu\nu\rho} = h_{\mu\nu\rho} + f_{\mu\nu\rho}$; as printed, μ is absent and σ is an unmatched free index on the right-hand side.

2. Define $\tau^{\rho\sigma}$ in equation (20), or replace it by $\tau^\rho\tau^\sigma$ if that is intended. The derivation of the connection from equations (5)–(6) should be shown.

3. Equation (27) begins with $\hat{2}R^{(2)}$, which is not meaningful notation. Equations (22) and (30) also need careful re-typesetting of their symmetrized and antisymmetrized indices.

4. Equation (43) contains a malformed α' superscript, $\hat{(}R$, and a mixture of hatted and unhatted metrics. It must be corrected and checked term by term against Ref. [25] before any conclusion is drawn from equation (44).

5. The source uses the same equation label for the Metsaev–Tseytlin action in equations (3) and (36). All labels should be unique, even if the current prose does not expose the duplicate reference.

6. The notation $h_{\mu\nu\rho}$ for the three-form field strength is easy to confuse with the spatial metric $h_{\mu\nu}$. A distinct symbol would substantially improve the long formulas in Section V and Appendix A.

7. The statement that $\nabla_{[\mu}\tau_{\nu]}$ is "arbitrary" after equations (29)–(30) should be reconciled with the fact that the connection has already been fixed and with the integrability conditions following from equation (28).

8. The manuscript should include at least one non-flat compatible background on which equations (31)–(36) are evaluated. Such an example would expose which nominally finite terms are actually nonzero and would provide a useful check of all numerical coefficients.

9. The prose needs a thorough edit. Examples include "ona," "we we," "simplifies considerable," "these contribution," "no divergence arise," and "stablish." Claims of proof should be reserved for statements whose intermediate identities are supplied.