Referee Report

Quantum Gravity Corrections to Giant Graviton Correlators

arXiv:2607.25054

Recommendation: Major revision. The paper contains a novel and potentially publishable one-loop result for maximal-giant correlators, and the unitarity, Mellin-space, and position-space constructions provide substantial mutual support. Before publication, however, the authors must correct an apparent large-index asymptotic error in equation (4.22), propagate the second tree-level ambiguity into the anomalous dimensions in equation (5.16), and give reproducible or analytic support for the central all-order resummations and Mellin formula.

Summary and overall assessment

The manuscript studies the correlator of two maximal giant gravitons and two stress-tensor-multiplet operators in the supergravity limit of $\mathcal{N} = 4$ SYM. Dividing by the heavy two-point function and treating the two determinant operators as a zero-dimensional defect turns the light-light-heavy-heavy four-point function into a defect two-point function. The authors focus on the reduced $k_1 = k_2 = 2$ correlator and construct its leading $1/c$ correction.

The calculation has four principal parts. First, Section 3 applies defect AdS unitarity to the lower-order bulk and defect OPE data. Matrix "unmixing" identities reduce the required averaged one-loop OPE coefficients to generalized-free and tree-level data. The resulting block sums are resummed into the closed defect- and bulk-channel leading logarithms in equations (3.21) and (3.36). Second, Section 4 reconstructs a Mellin amplitude from the simultaneous-pole ansatz

$$\widetilde{\mathcal{M}}^{(2)}_{22}(\delta, \gamma) = \sum_{m,n\geq 0} \frac{c_{mn}}{(\delta + n)(\gamma - 3 - 2m)}$$

and proposes the hypergeometric expression (4.4) for all c_{mn}. Regularizing the lower-log double sum leaves a constant contact ambiguity c_0. Third, Section 5 develops a position-space bootstrap. In addition to the functions familiar from defect-free one-loop correlators, the answer requires the new element $\mathcal{Q}_8$ in equation (5.4), and hence the enlarged alphabet $\{z, \bar{z}, 1 - z, 1 - \bar{z}, V, 1 - V\}$. The final result is compactly written as equation (5.6), with the fourth-order operator (5.7) acting on the explicit pre-correlator (5.8)–(5.9). Finally, the authors compare the large-Mellin limit with a reduced flat-space D3-brane loop integral and extract lowest-twist defect-channel anomalous dimensions in equation (5.16).

There is a substantial result here. The extension of defect unitarity to the zero-dimensional giant-graviton setting is nontrivial, the position-space completion addresses an open problem left by the earlier surface-defect analysis, and the compact answer and its new function-space letters should be useful in further work. The overlap of the bulk and defect leading logarithms, the reproduction of subleading logarithms from Mellin residues, and the highly constrained Mellin/position comparison are meaningful checks.

The present version nevertheless does not yet establish all of its headline claims at the required level. Two concrete internal issues directly affect the final physical conclusions. The arbitrary addition of the tree-level correlator, acknowledged in Sections 5.3 and 6, changes the one-loop anomalous dimensions for every spin but is absent from equation (5.16). Separately, direct evaluation of the exact coefficient formula (4.4) does not agree with the simultaneous large-m, n limit stated in equation (4.22); the discrepancy is a sign and an apparent factor of four. In addition, several central exact formulas are obtained by recognizing finite series patterns and checking them to unspecified order. These problems motivate major rather than minor revision, but they appear remediable and do not presently suggest rejection.

Correctness and logical structure

The logarithmic bookkeeping in Sections 2 and 3 is logically sound. Bulk double-particle anomalous dimensions begin at order $1/c$, whereas the giant-plus-light defect composites receive anomalous dimensions already at order $1/\sqrt{c}$. It follows that the one-loop reduced correlator can contain at most one $\log U$ and two powers of $\log V$, as summarized in Figure 2. The defect-channel identity

$$\widehat{\mathbf{B}}\widehat{\mathbf{\Gamma}}^{2}\widehat{\mathbf{B}}^{T} = \widehat{\mathbf{\Omega}}\widehat{\mathbf{N}}^{-1}\widehat{\mathbf{\Omega}}$$

and its bulk-channel counterpart are the correct way to remove the degeneracy without explicitly diagonalizing every family. As a useful low-twist check, equation (3.19) at $m = 0$ gives

$$\frac{1}{8}\widehat{A}_s^{(0)}\big(\widehat{\gamma}_s^{(1)}\big)^2 = \frac{2}{s+1},$$

in agreement with $\widehat{A}_s^{(0)} = s + 1$ and $\widehat{\gamma}_s^{(1)} = -4/(s + 1)$ in equation (5.11).

The Mellin logic is also plausible but depends on assumptions that should be stated as such. Simultaneous poles reproduce the joint $\log B \log^2 D$ discontinuity, and the subsequent agreement of the full $\log B$ and $\log^2 D$ data is strong evidence against additional single poles at the same thresholds. This does not, by itself, prove the absence of every Mellin term invisible to those discontinuities. The conclusion that only a constant regular ambiguity remains needs explicit locality and large-Mellin growth assumptions. Similarly, the successful eight-function position-space ansatz establishes that the proposed basis is sufficient; a completeness claim requires a sharper characterization of the allowed singularities, branches, and growth.

The position-space OPE extraction is algebraically consistent for the particular representative displayed in equation (5.6). It is not, however, invariant under the second ambiguity that the manuscript itself retains. Consequently, the spin-s binding-energy correction in equation (5.16) is not yet a unique observable. This issue is separate from the contact ambiguity c_0, which affects spin zero only.

Novelty and significance

The appropriate novelty claim is both strong and narrower than some of the current language suggests. The zero-dimensional-defect description and the full family of tree-level maximal-giant correlators are established in the authors' earlier work. The AdS unitarity method for defect two-point functions and the simultaneous-pole Mellin architecture were introduced for a six-dimensional surface defect. The principal advance here is the first strong-coupling supergravity-loop calculation for the maximal-giant $k = 2$ correlator, together with the position-space realization of defect unitarity, the new $\mathcal{Q}_8$ letter structure, and the extraction of one-loop defect data.

These are significant results for the holographic-correlator and giant-graviton communities. They demonstrate that the defect framework retains considerable power beyond tree level even at the endpoint of a zero-dimensional boundary defect. They also supply a concrete target for integrated-correlator and localization data that could fix the remaining contact and protected-sector ambiguities.

The manuscript should distinguish this result more explicitly from quantum giant-graviton correlators at weak 't Hooft coupling, including the cited finite-loop integrand bootstrap, and from the earlier six-dimensional surface-defect calculation. Statements such as "the first quantum correction" should consistently read "the first strong-coupling supergravity-loop correction" for the specified maximal-giant, $k_1 = k_2 = 2$ observable. Likewise, "the full one-loop correlator" should be qualified by the two unresolved tree-like additions until they are fixed.

Major comments

1. **Propagate the tree-level ambiguity into the one-loop CFT data.**

   Equation (5.6) displays a particular representative,

$$\mathcal{H}_{22}^{(2)} = \widehat{\Delta}^{(4)}\mathcal{P}^{(2)} + \frac{1}{2}\mathcal{H}_{22}^{(1)}.$$

   The text immediately below equation (5.9), and again Section 6, correctly states that an arbitrary additional multiple of $\mathcal{H}_{22}^{(1)}$ remains unfixed. Write this freedom as

$$\mathcal{H}_{22}^{(2)} \longrightarrow \mathcal{H}_{22}^{(2)} + a\,\mathcal{H}_{22}^{(1)}.$$

   From the perturbative block expansion (5.10), the tree-level logarithm is $\frac{1}{2}\widehat{A}_s^{(0)}\widehat{\gamma}_s^{(1)}\log V$. Therefore the above addition changes the extracted one-loop anomalous dimension by

$$\widehat{\gamma}_s^{(2)} \longrightarrow \widehat{\gamma}_s^{(2)} + a\,\widehat{\gamma}_s^{(1)} = \widehat{\gamma}_s^{(2)} - \frac{4a}{s+1}.$$

   This affects every spin, not only $s = 0$. Equation (5.16) is thus a scheme- or convention-dependent representative unless a has already been fixed by protected data not displayed in the paper.

   The authors should either determine a from the order-$1/c$ protected or free-theory correlator, or include it explicitly in equations (5.6), (5.15), (5.16), and (6.1). The abstract and conclusion must then state that the binding-energy corrections are obtained only up to this protected-sector ambiguity. It would also be valuable to explain whether the integrated giant-graviton correlators proposed to fix c_0 can simultaneously fix a, or whether a separate OPE normalization is required.

2. **Correct the large-m, n asymptotic formula and revisit the flat-space check.**

   Equation (4.22) appears inconsistent with the paper's exact formula (4.4). For example, setting $m = n = 100$, all three hypergeometric series in (4.4) terminate and can be evaluated without a convergence prescription. The result is

$$c_{100,100} \simeq -0.269928,$$

   whereas equation (4.22), with $x = y = 1$, predicts

$$\frac{12}{5^{3/2}} \simeq +1.073313.$$

At $m = n = 200$, equation (4.4) gives approximately -0.269131, so this is not a low-index effect. More generally, direct evaluations at fixed m/n are consistent with

$$c_{\Lambda x,\Lambda y} \longrightarrow -\frac{3x\sqrt{y}}{(4x+y)^{3/2}},$$

namely $-1/4$ times the expression printed in (4.22).

The authors should verify this discrepancy analytically. If the correction above is confirmed, the normalization and sign of the continuum integral (4.23), the Euler relation (4.24), the functions in (4.25)–(4.28), and the comparison equations (4.34)–(4.35) must be propagated consistently. The nonlocal functional agreement with the flat-space loop may well survive, because the overall Mellin/flat-space normalization was left free, but the displayed equations are currently not mutually consistent.

This correction is also tied to the regulator in equation (4.12). With the negative asymptotic above, the leading contribution of the pole term, summed over a shell $m + n = r$, can cancel the leading shell contribution of d_{mn}. With the positive coefficient printed in (4.22), the signs do not have that property. A derivation of the asymptotic behavior would therefore clarify both the flat-space test and the convergence of the regularized Mellin amplitude.

3. **Provide an analytic derivation or reproducible verification of the central closed formulas.**

   Equations (3.21) and (3.36) are obtained by expanding near an OPE limit, recognizing a bounded-polynomial pattern, and checking the proposed answer "to high order." The general coefficient formula (4.4) is found "through trial and error." These formulas carry most of the new content of the paper, so unspecified finite-order agreement is not enough to support repeated claims of exactness.

   Ideally, the authors should derive recurrences or generating functions that prove the two block resummations and equation (4.4). A sufficient alternative would be to prove that both sides satisfy the same finite-order difference or Casimir equations with enough initial data. At minimum, the paper should state the exact ranges in m, n, twist, spin, and OPE order that were checked; specify whether the comparisons were exact rational, symbolic, or numerical; and provide machine-readable coefficient data and the code used for the resummation, Mellin residues, and position-space matching.

   The equality of the $\log U \log^2 V$ overlap obtained from the two channel resummations is an excellent check and should be quantified. The authors should likewise report how many independent coefficients, beyond the seven unknown parameters, were used in the Mellin/position overmatch in Section 5.2. Listing seven residue pairs for seven remaining parameters does not by itself make the degree of overconstraint transparent, even though each residue contains several independent logarithmic structures.

4. **Define equation (4.4) on the full summation domain and make the regularization precise.**

   The Mellin sum in equation (4.1) begins at $m, n = 0$, but the individual Gamma functions, rational factors, and hypergeometric parameters in equation (4.4) have apparent poles or $0/0$ forms at several boundary points, including $(m, n) = (0, 0)$. The low-lying rows (4.2)–(4.3) are finite, so a limiting prescription evidently exists. The paper should state it explicitly. One clean presentation would define the exceptional rows and columns separately and use (4.4) only on its manifestly regular domain; another would specify the ordered analytic continuation that reproduces (4.2)–(4.3).

For equation (4.11), the order of the double summation must be defined. Please distinguish pointwise large-r behavior from the contribution of an $m + n = r$ shell, which contains $r + 1$ terms. The footnote following equation (4.12) currently says that a summand improving from $1/r$ to $1/r^2$ makes the remaining sum over r convergent; as written, this does not account for the shell multiplicity. The authors should establish absolute convergence, or state the summation prescription and demonstrate that changes of regulator alter the result only by the displayed constant C. This is particularly important because the lower logarithms and the position-space ambiguities are fixed using these regulated residues.

5. **State and justify the completeness assumptions in Mellin and position space.**

   The agreement of all bulk-channel $\log B$ terms and defect-channel $\log^2 D$ terms is strong evidence that no additional single poles at the assumed locations are required. The manuscript should explain why a single-pole term, a pole at another admissible location, or an entire Mellin function cannot evade these tests while changing lower-log or rational terms. Similarly, the statement that the only regular Mellin ambiguity is a constant needs a supergravity power-counting or Regge-growth argument excluding higher polynomials and other contact contributions. The assumptions of locality, two-particle thresholds, and the derivative order of allowed counterterms should be listed together.

   On the position-space side, please justify the degree-14 polynomial numerators, the maximum $(1-V)^{-7}$ denominator, and the claim that the eight functions in equations (5.1) and (5.4) exhaust the relevant function space. The current analysis proves that this ansatz admits a successful solution, but not yet that every allowed one-loop solution belongs to it. Branch conventions for $\mathcal{Q}_8$, especially across $V = 1$, should be given, and its crossing property should be stated in a definite Euclidean domain before analytic continuation.

6. **Calibrate and strengthen the flat-space comparison.**

   After correcting equation (4.22), Section 4.3 should make clear which part of the flat-space statement is actually tested. The comparison presently leaves the overall normalization unfixed and quotients out local polynomial ambiguities by differentiation. It therefore establishes agreement of the non-polynomial kinematic dependence, rather than equality of normalized amplitudes. This distinction should be retained in the abstract and Section 6.

   The relation between the $p = 0$ conformal defect used in the Borel formula (4.15)–(4.16) and the four-dimensional D3-brane worldvolume with $d_\perp = 6$ in equation (4.29) deserves a short derivation. The kinematic region and analytic continuation should also be stated: ρ, u, the logarithms, and the dilogarithm in equations (4.25)–(4.28) are multivalued. Finally, Figure 4 represents one reduced integral after the supersymmetric prefactor has been stripped; the text should avoid implying that it is the complete ten-dimensional one-loop amplitude unless the reduction from the full diagram set is shown.

7. **Sharpen the separation between inherited ingredients and new results.**

   Section 3.2 says only that "some results" were reported previously. Please identify formula by formula which lower-order OPE coefficients, unmixing data, and anomalous dimensions are inherited from the tree-level giant-graviton paper, and which resummations are new here. A compact comparison with the earlier six-dimensional surface-defect calculation would be especially informative: pole locations, hypergeometric building blocks, ambiguity spaces, and the new position-space input can all be contrasted directly.

The weak-coupling giant-correlator literature is currently confined mostly to an introductory footnote. Since recent work determines maximal-giant loop integrands through several gauge-theory loops, the manuscript should state plainly that its novelty is the local strong-coupling supergravity-loop correlator, not the first quantum treatment of the same heavy operators in any regime. With these qualifications, the novelty remains fully adequate for publication.

Minor comments

1. The labelled display $\mathbf{\Omega} = \mathbf{\Lambda}\mathbf{\Gamma}\mathbf{\Lambda}^T$ immediately after equation (3.27) contains a `\nonumber` command but is cited later. It consequently has no stable printed number and produces a duplicate equation anchor. It should be numbered normally or its label and cross-references should be removed.

2. The summation range in equation (3.33), $\sum_{m=-k,\,\ell\,\mathrm{even}}^{\infty}$, is ambiguous. Please give independent ranges for m and ℓ, explain the occurrence of negative m, and state the domain on which the floor/Pochhammer formula (3.34) applies.

3. In equation (5.5), replace $\sum_{n,m}^{14}$ by explicit lower and upper bounds and specify how diagonal terms are counted. This will also make the parameter count in the bootstrap reproducible.

4. The operator ordering in equation (5.7) is visually ambiguous: $\partial_z\partial_{\bar{z}}(z-\bar{z})$ can be read as differentiating the parenthesis rather than as an operator acting on everything to its right. Parentheses or composition symbols should make the intended action unambiguous.

5. State the contour-separation conditions for the Mellin transform (2.22) and for the regulated sum (4.11), particularly when poles collide at the exceptional low-index values.

6. The paper alternates between calling c_0 a counterterm ambiguity, a regulator ambiguity, and part of the CFT data. A sentence relating these descriptions, and specifying which change of scheme leaves separated-point data invariant, would be helpful.

7. Several prose errors should be corrected, including "operators becomes" in Section 2, "give difference slices" and "pattern ... continue" in Section 4.1, "These result" in Section 4.2, and "corelator" in Section 5.3.

8. It would be helpful to include low-lying numerical values of $\widehat{\gamma}_s^{(2)}$, with both ambiguity parameters shown, and to state their large-spin behavior. This would make the physical binding energy result easier to compare with future integrated or semiclassical calculations.

Recommendation

I recommend **major revision**. The paper makes a substantive and original contribution: it extends holographic defect unitarity to a maximal-giant correlator, obtains compact Mellin- and position-space representations, identifies a new position-space alphabet, and extracts genuinely interesting defect CFT data. The core unmixing construction and the multiple channel checks make the result credible.

Publication should follow once the authors correct the large-index asymptotic and all equations that depend on it, expose or fix the tree-level ambiguity in the anomalous dimensions, and put the empirical closed-form formulas on a reproducible or analytic footing. The Mellin regularization, ansatz completeness, flat-space normalization, and relation to prior surface-defect and weak-coupling work should also be clarified. These revisions are significant but have a clear path, and a corrected manuscript should be suitable for a journal such as JHEP or PRD.