Referee Report

The coupling flow for supergravity

Federico Arrighi, Saurish Khandelwal, and Olaf Lechtenfeld

arXiv:2607.25714

Recommendation: Major revision.

Summary and overall assessment

The manuscript studies the coupling-flow approach to a Nicolai map for four-dimensional $\mathcal{N} = 1$ Poincare supergravity. Its central proposal is to replace the minimal Poincare off-shell formulation used in earlier work by the superconformal construction with a chiral compensator. In this formulation the authors write the superconformal action as a Q-variation, equations (2.5)–(2.6), and use supersymmetry and BRST Ward identities to derive a formal flow in the gravitational coupling κ. After all fields except the vierbein are integrated out, the flow consists of two first-order functional differential operators, R^{inv} and R^{gf}, together with a multiplicative term Z generated by the noncommutativity of supersymmetry and BRST transformations. The authors then argue that such multiplicative insertions may sometimes be converted into derivative terms inside the final free correlator. An axial-gauge super-Yang–Mills example is used to illustrate this mechanism.

The paper next develops the perturbative supergravity construction. It derives free propagators in the superconformal gauge-fixed theory, evaluates the leading gauge-invariant flow operator, and claims that the apparently singular κ^{-1} contribution cancels in Landau gauge. Finally, it compares the structure with a previously constructed leading on-shell Nicolai map and argues that a term missing from a restricted fermionic ansatz may again arise by a Wick contraction of a multiplicative insertion.

The superconformal viewpoint is interesting and potentially useful. In particular, the use of the compensator multiplet to obtain the supervariation in equations (2.5)–(2.6) is a meaningful response to the first obstruction identified in the authors' previous work. The separation of the exact formal coupling response into R^{inv}, R^{gf}, and Z also gives a helpful description of where the remaining difficulty lies. These ideas are relevant to the specialized but active program of extending Nicolai-map methods from globally supersymmetric gauge theory to locally supersymmetric theories.

However, the principal new technical check is not correct as printed. The longitudinal term in the graviton propagator in equation (5.5) has the wrong sign: the displayed propagator does not satisfy the harmonic gauge condition in the claimed Landau limit. This is not an isolated typographical error. The same sign produces the p-dependent coefficient in equation (6.14) that is adjusted to obtain the cancellation in equation (6.15). With the sign required by the gauge condition, the Wick-contracted result is independent of p, and the displayed cancellation does not hold for a general Q-supersymmetry gauge parameter. Moreover, a potentially finite P-gauge contribution has been discarded before taking the Landau limit. Thus Sections 5 and 6 must be rederived before the claimed removal of the third obstruction can be assessed.

There are also important conceptual gaps between the formal Ward-identity flow and the stronger all-order claims. The equality established at $\kappa = 0$ does not by itself justify deleting the entire κ^{-1} operator from the interacting expansion; the Wick conversion is an equality inside suitably defined Gaussian correlators, not an operator identity; the quantum equivalence between the gauge-fixed superconformal path integral and Poincare supergravity is not shown; and the comparison in Section 7 does not actually derive the order-κ flow from the new formalism. These are substantial matters rather than presentational details.

I therefore do not regard the manuscript as suitable for publication in its present form. A corrected and substantially expanded version could become publishable, because the underlying superconformal strategy is novel enough to merit further development. The essential revisions are described below.

Correctness and logical structure

The broad formal logic is clear. Section 1 distinguishes an ordinary Nicolai map, whose infinitesimal action is a derivation, from a more general coupling flow containing a multiplicative term. Section 2 supplies the superconformal variables and gauge choices. Section 3 derives the formal flow and isolates the consistency condition (3.17). Section 4 correctly emphasizes that an equality inside the final path integral can be weaker than an identity in the effective outer-field theory. Sections 5 and 6 then try to verify the consistency condition explicitly, and Section 7 tests whether the known leading map can be represented by the proposed type of flow.

This organization is sensible, but the conclusions currently outrun what the calculations establish. Even apart from the propagator error, the manuscript proves a formal coupling-derivative identity under assumed Ward identities; it does not yet construct a distributive all-order Nicolai map. The authors acknowledge the latter point, but statements about an all-order Poincare-supergravity flow and a regular iterative expansion need to be separated more carefully from the leading checks and from the unproved conversion of Z.

Major comments

1. **The graviton propagator in equation (5.5) fails the stated Landau gauge condition.** Let

$$P_{ab,cd} = \eta_{ac}\eta_{bd} + \eta_{ad}\eta_{bc} - \eta_{ab}\eta_{cd},$$
$$L_{ab,cd} = \eta_{ac}\partial_b\partial_d + \eta_{ad}\partial_b\partial_c + \eta_{bc}\partial_a\partial_d + \eta_{bd}\partial_a\partial_c,$$

so that equation (5.5) reads

$$D_{ab,cd} = \frac{3\mathrm{i}}{2}\left[P_{ab,cd}\Box^{-1} - \left(\frac{1}{\xi_P} - 1\right) L_{ab,cd}\Box^{-2}\right].$$

The linearized gauge functional in equation (5.2) is

$$\mathcal{F}_b(h) = \partial_b h - 2\partial^a h_{ab}.$$

Writing $S_{b,cd} = \eta_{bc}\partial_d + \eta_{bd}\partial_c$, direct contraction gives

$$\mathcal{F}_b(P\Box^{-1}) = -2S_{b,cd}\Box^{-1}, \qquad \mathcal{F}_b(L\Box^{-2}) = -2S_{b,cd}\Box^{-1}.$$

Consequently, the propagator printed in equation (5.5) obeys

$$\mathcal{F}_b(D) = -3\mathrm{i}\left(2 - \frac{1}{\xi_P}\right) S_{b,cd}\Box^{-1}.$$

It is therefore not transverse to $\mathcal{F}_b$ as $\xi_P \to \infty$; indeed, it remains nonzero in precisely the limit that is supposed to impose the harmonic gauge strictly.

With the conventions of the manuscript, the longitudinal term must instead have the opposite sign,

$$D^{\text{corr}}_{ab,cd} = \frac{3\mathrm{i}}{2}\left[P_{ab,cd}\Box^{-1} + \left(\frac{1}{\xi_P} - 1\right)L_{ab,cd}\Box^{-2}\right],$$

for which $\mathcal{F}_b(D^{\text{corr}}) = -3\mathrm{i}\,\xi_P^{-1}S_{b,cd}\Box^{-1}$. This test is independent of a Fourier-transform or metric-signature convention because it checks the same gauge functional and propagator used in the paper.

The authors should re-invert the full bosonic quadratic form in equation (5.3), including the compensator–trace mixing, and verify all inverse relations before repeating Section 6. The corrected propagator should be checked both against the kinetic operator and against every advertised Landau limit.

2. **The error in equation (5.5) invalidates the adjustable Wick cancellation in equations (6.14)–(6.15).**

   The dependence on the integration-by-parts parameter p in equation (6.14) is a direct consequence of the wrong longitudinal sign. Repeating the contraction of equation (6.12) with the sign required by the harmonic gauge gives, using $\omega_{ab} = \partial_a\partial_b/\Box$,

$$Z_0 \;\longrightarrow\; \int \mathrm{d}^4x\, h\left(\frac{1}{4}\eta_{ab} + \frac{1}{2}\omega_{ab}\right)\frac{\delta}{\delta h_{ab}},$$

   within the relevant free correlator. In particular, the result is independent of the split $p + q = 1$. This independence is required on general grounds: p merely distributes terms related by integration by parts, so a correct Gaussian integration-by-parts identity cannot acquire physical tunability from it.

   Combining this corrected expression with equation (6.3), with $R_0^{\text{gf}} = 0$ on the harmonic slice, leaves

$$\left(\frac{1}{2} - \frac{1}{2\xi_Q}\right)\int \mathrm{d}^4x\, h\,\omega_{ab}\frac{\delta}{\delta h_{ab}}$$

   in $R_0^{\text{inv}} + R_0^{\text{gf}} + Z_0 - E$. Thus the terms displayed in the manuscript cancel only for $\xi_Q = 1$, rather than for arbitrary ξ_Q by a choice of p. This is also in tension with the preferred $\xi_Q = \frac{1}{2}$ gravitino gauge selected below equation (5.4) and used in the propagator quoted in equation (7.9).

   Even the possible special-gauge cancellation at $\xi_Q = 1$ is not yet a complete result. The $A = P$ term in equation (6.7) is discarded because $F_P = 0$ on the gauge slice. But the text below equation (2.14) correctly warns that the Landau condition may be imposed only after transformations and free-field contractions. The product $\xi_P\langle F_{P,b}h_{cd}\rangle_0$ has a finite Landau limit, so the P-gauge part of Z_0 may contribute by exactly the mechanism illustrated in Section 4. The authors should retain all $\xi_A F_A$ terms until after the contractions, recompute Z_0, and then test equation (3.17) without any adjustable integration-by-parts artifact.

3. **Equation (3.17) removes a possible pole at one point; it does not justify the all-order operator expansion in equation (4.18).**

   Define

$$A_0 = R_0^{\text{inv}} + R_0^{\text{gf}} - E + Z_0.$$

   Equation (3.17) states $\langle A_0 Y\rangle_{\kappa=0} = 0$ for an allowed functional Y. Even if this identity is established, it does not permit one simply to omit $\kappa^{-1}A_0$ from the finite-κ flow. If $f_Y(\kappa) = \langle A_0 Y\rangle_\kappa$, then

$$f_Y(0) = 0 \qquad \text{but} \qquad \frac{f_Y(\kappa)}{\kappa} = f_Y'(0) + O(\kappa),$$

and the finite term $f'_Y(0)$ is generally nonzero. The interacting measure does not obey the free Gaussian identity used in Section 4 without additional insertions.

The leading condition is sufficient to test the absence of a literal $1/\kappa$ divergence, but it is not sufficient to replace the singular flow by equation (4.18) with coefficients beginning at $n = 1$. The authors need either an all-κ Ward identity that makes A_0 null inside the interacting correlator, or an explicit perturbative reorganization showing how the finite terms generated by $\kappa^{-1} A_0$ are incorporated into R_1, Z_1, and subsequent coefficients. The super-Yang–Mills identity in Section 4, which relies on global supersymmetry in an axial gauge, does not supply this missing supergravity argument.

This distinction should also be reflected in the abstract and conclusion. The exact formal derivative of a path integral with respect to κ, the regularity of its perturbative coefficients, the existence of a first-order derivational flow, and the existence of a distributive Nicolai map are four different statements. At present only the first is established generally.

4. **The status of the multiplicative-to-derivative conversion must be formulated precisely.**

   The operation used in Sections 4 and 6 is Gaussian integration by parts, equivalently a free Schwinger–Dyson identity. It yields an equality inside a specified free expectation value; it is not an identity between Z and a first-order operator in the effective theory. The manuscript should state the result in a form such as

$$\langle Z_0 Y \rangle_{0,c} = \langle R_{Z,0} Y \rangle_{0,c}$$

   for a defined class of functionals Y, with the meaning of the subscript c, the treatment of contact terms, and the regulator made explicit.

   This is especially important because the derivation below equation (6.12) discards $\langle Z_0 \rangle_0 \langle Y \rangle_0$, whereas Section 1 explicitly says that the correlators need not be normalized. The subtraction is automatic for connected or normalized correlators, but not for the unnormalized objects used in the initial definition. If vacuum contractions are set to zero by dimensional regularization or by normal ordering, that prescription must be stated and used consistently, including for $Y = 1$.

   There is also a structural all-order issue. Gaussian integration by parts turns a general polynomial insertion into a functional differential operator whose order reflects the number of uncontracted fields. A Nicolai flow, by contrast, must be a first-order derivation obeying the Leibniz rule. At leading order Z_0 is quadratic, and a selected contraction can happen to produce a first-order operator. This observation alone does not imply that every higher-order Z_n can be converted into a first-order derivation. A useful revision would state and prove a general lemma specifying when such a reduction is possible, or else explicitly limit the claim to the leading examples actually demonstrated.

5. **The supervariation and the quantum superconformal reduction need enough detail to support the central formal claim.**

   Equations (2.5)–(2.6) are the conceptual cornerstone of the paper, yet the manuscript says only that superspace indicates the identity and that the authors have verified it. The composite Lorentz, special-conformal, and S-supersymmetry connections are not given. The authors should add a derivation or a detailed appendix showing $\widetilde{\mathcal{L}}_{\mathrm{SUSY}} = \widetilde{\delta}_\alpha \widetilde{\mathcal{M}}_\alpha$, including conventions for the summed spinor index, integrations by parts, boundary terms, and the cancellation of the auxiliary remainder encountered in the earlier Poincare formulation. Primary references for the relevant superconformal density formula should be cited. It should also be made clear which part is a standard tensor-calculus identity and which part is the new application to the coupling flow.

Similarly, equations (2.11) and (3.2)–(3.4) depend on the soft superconformal BRST algebra, but the needed ghost transformations are not provided. The Ward-identity derivation assumes invariant measures and the absence of supersymmetry, BRST, super-Weyl, and $U(1)_R$ anomalies. For an all-order quantum statement, the regulator and the status of these assumptions must be discussed. The relation between the constant-parameter Q transformation used before conformal gauge fixing and the compensated residual Poincare supersymmetry after imposing $\widetilde{\phi} = 1$, $\widetilde{\rho} = 0$, and $\widetilde{b}_\mu = 0$ should also be explained.

The manuscript itself notes after equation (2.14) that the gauge-fixed superconformal and Poincare theories may differ quantum mechanically for generic gauge parameters. Consequently, integrating out the compensator and conformal ghosts is not by itself a proof of an all-order flow for quantum Poincare supergravity. The authors should show how the strict Landau limits decouple the extra BRST quartets, including measure factors and Jacobians, or describe the result more narrowly as a formal superconformal representation. Since supergravity is treated as an effective field theory, it should also be stated whether the flow concerns a bare formal series or a renormalized action with the required higher-curvature counterterms and additional running couplings.

Finally, the footnote following equation (3.7) acknowledges that eliminating F and A_μ in the gauge-fixed theory generates ghost bilinears and quartic ghost interactions contributing at order $\hbar^2$, which are then ignored. That omission is compatible with a leading classical check, but not with unqualified all-order formulas. The exact construction must retain these terms, or the scope of the claims must be reduced accordingly.

6. **Section 7 does not yet demonstrate agreement between the new flow and the on-shell Nicolai map.**

   Equation (7.2) is taken from earlier work, and equation (7.6) is read off from that known map. The manuscript does not independently compute $R_1^{\text{inv}} + R_1^{\text{gf}} + Z_1$ from equations (3.12)–(3.15). The subsequent failure of the restricted ansatz (7.8) to produce the symmetric term (7.12) is therefore not a comparison of two completed calculations.

   Nor is the ansatz shown to be exhaustive. Expanding the inner-field average at order κ generates not only a term schematically of the form $\langle \mathcal{M}_1 \, \delta h \rangle_0$, but also the order-$\kappa$ correction to the inner propagators, equivalently connected insertions of $\mathcal{M}_0$ with the first interaction in the action. Section 5 itself states that the full propagators in the vierbein background must be expanded. Derivatives acting on vierbeins, compensator terms, composite connections, and more general ghost structures are also absent from equation (7.8). The inability of that local matrix ansatz to generate equation (7.12) is therefore not a no-go theorem for a derivational flow.

   The authors should either calculate the complete leading flow from the superconformal formalism, including the converted Z_1, or weaken the claims of agreement and necessity. A term-by-term comparison with the published order-κ^2 construction cited as Ref. [19] would be valuable. Such a comparison should identify the gauge transformation from the seven-term Feynman-gauge representative to the four-term Landau-gauge representative, separate the longitudinal ambiguity in equation (7.3), and state which constraints follow from the free-action and determinant-matching conditions. Reliance on private communication for the remaining four-parameter ambiguity is not sufficient for a central published claim.

7. **The novelty and demonstrated payoff should be stated more sharply.**

   The genuine advance is presently structural: the standard superconformal compensator formulation supplies a fully supersymmetric variation for the gauge-invariant action and leads to the formal decomposition of the coupling response. The manuscript does not yet produce a new higher-order Nicolai map or a nontrivial supergravity correlator. Ref. [19], as described by the authors, already constructs the map directly through order κ^2 with loop constraints. The introduction should therefore distinguish the new formal organization from an improvement in explicit perturbative order.

A compact comparison of the previous Poincare flow, the direct construction of Ref. [19], and the present superconformal method would clarify the literature position: formulation, gauge, order in κ and $\hbar$, conditions checked, and remaining obstruction. The bibliography should also include primary sources on superconformal tensor calculus and density formulas, BRST or BV quantization of local supersymmetry, compensator gauge fixing, and anomaly or regularization issues. One supergravity textbook is not an adequate reference base for the all-order quantum steps.

To establish practical significance beyond the formal rearrangement, the authors should compute either the full R_1 from the new flow or one nontrivial correlation function and verify it against ordinary supergravity perturbation theory. Without such a test, the conclusion that the flow provides an alternative perturbative computational scheme remains prospective.

Minor and specific comments

1. Equation (3.10) repeatedly contains $Y[\phi_0]$; the intended symbol appears to be $Y[\phi_o]$.

2. The scalar solution in equation (6.1) should be written unambiguously as

$$\phi_1 = -\frac{h}{3\sqrt{2}(1+\xi_D)}.$$

   The printed expression places $\sqrt{2}(1+\xi_D)$ multiplicatively after the slash by 3, which is inconsistent with the equation of motion and the subsequent Landau limit.

3. In the limit attached to equation (6.5), the text says that the M, K, and D gauges are being taken to Landau gauge, whereas the displayed subscript lists M, D, and Y. The indices should be made consistent.

4. In equation (6.10), the final mixed propagator contains $\gamma_b \not{\partial} - 4\partial_a$ although its free vector index is b. The last derivative should apparently be ∂_b, as in equation (5.4).

5. Equation (6.15) omits the arbitrary functional Y. The intended statement should be written as an equality of complete correlators, for example

$$\left\langle (R_0^{\mathrm{inv}} + R_0^{\mathrm{gf}} + Z_0 - E)Y \right\rangle_0 = 0,$$

   with connected or normalized brackets specified.

6. The contraction annotation for the middle spinorial ghost propagator in equation (5.6) is inconsistent with the fields displayed in the contraction. This should be corrected before those propagators are used in higher-order calculations.

7. Equation (3.14) omits the leading Lorentz-ghost term in $sh_{a\mu}$, although the Lorentz ghost appears in the quadratic ghost action in equation (5.3). If the term is removed only after restricting to a symmetric vierbein, that projection and the convention for $\delta/\delta h_{ab}$ should be stated explicitly.

8. The statement following equation (2.12) that every gauge-fixing function except F_P is linear is not literally true before expansion: raised indices and curved gamma matrices introduce vierbein dependence in F_K and F_Q. The authors should specify whether "linear" means only linearized around the chosen background.

9. The manuscript suppresses explicit factors of $\hbar$ in the flow equations after Section 1 and restores loop counting in Section 7. Since the distinction between classical and loop-decorated maps is central, the convention should be stated where the suppression begins.

10. The treatment of auxiliary fields is difficult to follow. The text before equation (3.15) says that F has been eliminated, equation (3.7) uses $F = 0$, but F and A_μ reappear in equations (5.3) and (5.7). The authors should state at each stage whether auxiliaries are retained for off-shell inversion, integrated out classically, or integrated out with their ghost-induced terms.

11. The sentence below equation (1.6) that the r-loop condition begins at order g^r should be reconciled with the statement below equation (7.1) that an r-loop contribution begins at order κ^{2r} after a singular constant at κ^{2r-1}. The coupling counting depends on field normalization and the interaction structure and should not be presented as universal without qualification.

12. The introduction describes the result associated with equation (1.16) both as a one-loop construction and as solving conditions through $\hbar^2$. The relation between the powers displayed in the map and the loop order of the matching conditions should be explained.

13. The DOI in Ref. [18] contains the year 2025, while the journal citation printed immediately beside it is JHEP 02 (2026) 185. The DOI entry should be corrected.

Recommendation

The superconformal organization of the coupling flow is a worthwhile idea, and the manuscript addresses a difficult problem of genuine interest within the Nicolai-map and supergravity communities. Nevertheless, the sign error in equation (5.5) invalidates the central regularity calculation as written, and the omitted Landau-limit contribution prevents an immediate repair by choosing a special value of ξ_Q. The all-order interpretation also requires a more careful treatment of the Ward identities, Gaussian conversion, gauge-fixed quantum equivalence, and auxiliary ghost terms.

I recommend **major revision**. A revised manuscript should, at minimum, rederive the quadratic propagators and all of Section 6, retain every term until after the Landau limit, distinguish pole cancellation from the finite perturbative reorganization, and either compute the complete order-κ flow or substantially moderate the claimed comparison with the direct Nicolai map. If these issues are resolved, the conceptual advance could meet the standard for publication in a high-energy theory journal.