Referee Report

"Gravitational Waves from Multiple Cosmic Superstrings and the Overshoot Problem"

Summary and recommendation

This manuscript studies a post-inflationary cosmology containing a rolling Calabi–Yau volume modulus and three populations of subhorizon string loops: fundamental strings, D3-branes wrapped on two-cycles, and NS5-branes wrapped on four-cycles. Their tensions depend exponentially on the modulus. The authors first generalize an earlier autonomous-system treatment to include all three loop species, and then add gravitational-wave (GW) losses from the loops and the corresponding radiation source. They argue that the heavier D3- and NS5-string loops rapidly convert their energy into a GW bath whose Hubble friction can prevent modulus overshoot, whereas the longer-lived F-string component can return enough energy to the modulus to destabilize the minimum. For a benchmark evolution they also compute a present-day, high-frequency, multi-peaked GW spectrum, first for a monochromatic loop population and then for an imposed log-normal width.

The central idea is interesting. In particular, treating the GW energy released by several time-dependent-tension species as part of the backreacting cosmological system is a useful extension of the one-species and no-emission analyses cited as Refs. [6–8]. The competition between promptly decaying wrapped-brane strings, long-lived F-strings, and the volume modulus could lead to distinctive dynamics. If made quantitatively sound, the work could be relevant to a journal such as JHEP or Physical Review D.

In its present form, however, I do not think the principal numerical results are established. Most importantly, equation (43), which governs the dimensionless emission rate and therefore feeds every GW-backreaction trajectory, does not follow from the preceding definitions. There are additional normalization and typesetting ambiguities in the potential and initial emission rates, the benchmark is not demonstrably inside the adiabatic regime invoked to justify the radiated power, and the spectrum calculation contains an error in the log-normal kernel. The numerical work is also not documented sufficiently to determine which equations were actually integrated or to reproduce the figures. These are correctable issues, but they require a substantive reanalysis rather than editorial changes.

Recommendation: major revision. A revised manuscript should correct and independently validate the dynamical system, rerun all affected scans and spectra, specify the physical and numerical assumptions needed to reproduce the benchmarks, and moderate the phenomenological conclusions where they depend on an illustrative rather than self-consistent loop population.

Synopsis and logical structure

Section II.A assumes isolated, noninteracting loops whose energy is $E_{\rm loop} = \mu\ell$, whose number density redshifts as a^{-3}, and whose length varies as $\ell \propto \mu^{-1/2}$ in the absence of GW emission. Together these assumptions give equation (6). Section II.B introduces the fractional variables X^2, Y^2, Z_j^2, and W^2 in equations (11)–(14), constrained by equation (16), and derives the autonomous equations (19)–(22). The loop–modulus interaction is encoded by the parameters β_j.

Section II.C specializes the scalar to the canonically normalized LVS volume modulus and assigns $\beta_F = 1/2$, $\beta_3 = 1/3$, and $\beta_5 = 1/6$ from the tension scalings in equations (25)–(29). Sections II.D and II.E then use phase-space trajectories and scans to argue that NS5-string loops are most effective at preventing overshoot, that D3- and F-string populations can instead inject kinetic energy into the modulus, and that an externally specified radiation component can either help or hinder stabilization depending on when the system leaves the radiation saddle.

Section III introduces the standard loop power $P_{\rm GW} = \Gamma G\mu^2$ in equation (31), assumes the tension changes adiabatically, and obtains the length and loop-density loss laws in equations (35) and (37). Equations (38), (41), and (43) are intended to describe the loop fractions, radiation fraction, and emission variables $\delta_j = \Gamma G\mu_j/(H\ell_j)$. Section III.B uses numerical benchmarks to conclude that the initial F-string fraction must be suppressed for stabilization and that a GW-dominated epoch is common. Section III.C maps the emitted power to a present-day spectrum, accounts for a subsequent modulus-dominated era, and compares monochromatic and phenomenologically smeared loop lengths. Appendix A argues that an order-one GW energy fraction need not imply an order-one metric perturbation when the waves are subhorizon.

This organization is sensible and the manuscript is generally readable. The difficulty is not the overall logic but the quantitative implementation of its central links.

Major comments

1. **Equation (43) is inconsistent with equations (2), (17), (35), and (39), and this affects the core results.**

   Using the definitions printed in the manuscript, one finds

   $$\frac{\mu_j'}{\mu_j} = -6\beta_j X, \qquad \frac{\ell_j'}{\ell_j} = 3\beta_j X - \delta_j,$$

   where the second relation follows directly from equation (35). Moreover, equation (17), together with the constraint (16), gives

   $$-\frac{H'}{H} = \frac{3}{2}\left(1 + X^2 - Y^2 + \frac{W^2}{3}\right).$$

   Therefore differentiating equation (39) yields

   $$\delta_j' = \delta_j\left[\delta_j - 9\beta_j X + \frac{3}{2}\left(1 + X^2 - Y^2 + \frac{W^2}{3}\right)\right].$$

   In contrast, equation (43) contains $(1 - 9\beta_j X)\delta_j$ inside the square bracket. This multiplies the $-9\beta_j X$ term by an extra factor of δ_j and is not algebraically equivalent to the expression above.

   This is not a peripheral formula. The decay hierarchy, the conversion of loop energy into radiation, the stabilization maps in Figs. 12–14, the benchmark in Fig. 15, and both spectra in Figs. 16 and 17 depend on the evolution of δ_j. The authors should state explicitly which equation was implemented in the numerical calculation, correct the analytic derivation, and rerun every result that used the incorrect version. If the code used the correct equation and only the manuscript contains a typo, that should be demonstrated by providing the relevant evolution equation and reproducible numerical output. As a basic validation, the revision should show the residual of the Friedmann constraint and the cancellation of the loop-loss and radiation-source terms along representative trajectories.

2. **The LVS potential and the initial emission-rate normalizations must be made internally consistent.**

   As typeset, the second term of equation (24) is $\exp[-2\sqrt{(3/2)\Phi}]$, with Φ under the square root. The canonical LVS scaling and the surrounding discussion instead appear to require $\exp[-2\sqrt{3/2}\,\Phi]$. These functions have completely different large-field behavior, and the former is also incompatible with the stated use of a canonically normalized dimensionful field unless additional units are supplied. Since all stabilization scans depend on the potential, the authors must correct equation (24), state the exact function used in the numerics, and regenerate the plots if the implemented function differs.

   There is also an explicit normalization tension between equations (25) and (45). With the manuscript's convention $G = (8\pi M_{\rm p}^2)^{-1}$ and

   $$\mu_F = \frac{\sqrt{g_s}\, M_{\rm p}^2}{4\pi\mathcal{V}},$$

   equation (39) gives

   $$\delta_F^{\rm in} = \frac{\Gamma\sqrt{g_s}}{32\pi^2 H_{\rm in}\ell_F^{\rm in}} e^{-\sqrt{3/2}\,\Phi_{\rm in}/M_{\rm p}},$$

   not the coefficient shown in equation (45). Analogous wrapped-brane normalizations should be derived rather than inferred only from scaling powers. Factors of g_s, 2π, cycle volumes, and $M_{\rm p}$ are especially important here because they determine the ordering and time of the three decays and hence both axes of Fig. 16. The revision should present one consistent convention from the ten-dimensional brane tensions through equations (25)–(29) and (45)–(47), including all factors actually used in the code.

3. **The adiabatic justification for equation (31) is not established for the displayed benchmark, and competing radiation channels need to be addressed.**

   The manuscript obtains the bound $|\dot{\mu}/\mu|T \lesssim 6\beta H\ell$ below equation (32). Yet the main spectrum benchmark in Fig. 15 takes $H_{\rm in}\ell_j^{\rm in} = 0.3$ for all species. During a kinetic regime with X near unity, the actual parameter is $6\beta_j|X|H\ell$; for F-strings this can be approximately 0.9, which is not "much smaller than one." Merely being below the horizon does not guarantee the adiabatic hierarchy required by equation (32).

   For every benchmark used in Figs. 12–17, the authors should plot or quote the maximum of $6\beta_j|X|H\ell_j$ up to evaporation and demonstrate convergence as the initial sizes are reduced. At minimum, the principal figures should be repeated for values securely in the adiabatic regime. If the result changes as $H\ell$ is lowered, that dependence belongs in the main analysis.

   In addition, wrapped D3- and NS5-brane strings and time-dependent-tension F-strings can in principle radiate into moduli, axions, internal excitations, or other light fields. Equation (40) assigns the entire modeled loss to a GW radiation bath and adopts the same constant $\Gamma = 50$ for all species. The authors should justify this branching assumption for the proposed compactification, or introduce species-dependent GW branching fractions and show how stabilization and the observable spectrum change. This is particularly important because the claimed mechanism relies on the identity of the radiation as GWs, not merely on generic relativistic decay products.

4. **The assumed loop fluids and their initial populations require quantitative consistency checks.**

   The analysis postulates three pre-existing, isolated, noninteracting, monochromatic loop fluids with adjustable energy fractions and sizes. The condition $H\ell \ll 1$ only establishes that an object is subhorizon; it does not by itself establish that the population is dilute, disconnected from a long-string network, or noninteracting. Given $\rho_{\rm loop} = \mu\ell n_{\rm loop}$, a useful diagnostic is

   $$n_{\rm loop}\ell^3 = \frac{3\Omega_{\rm loop}M_{\rm p}^2(H\ell)^2}{\mu}.$$

   This quantity, the number per Hubble volume, the mean separation relative to ℓ, and the reconnection or collision rate should be evaluated for each population in the benchmarks. The normalization ambiguity noted above can change these tests appreciably.

   The successful examples also require a suppressed F-string abundance relative to wrapped-brane strings. The paper should explain whether a known post-inflationary production mechanism can plausibly yield $\Omega_3^{\rm in} = \Omega_5^{\rm in} = 2 \times 10^{-2}$ and $\Omega_F^{\rm in} = 10^{-3}$ together with the assumed common size. At present these are useful phenomenological initial conditions, but not yet a demonstrated outcome of the type IIB scenario. The simultaneous existence and stability of the relevant wrapped branes, possible orientifold projections or tadpole constraints, and the origin of their loop-size distributions deserve at least a concrete compactification-level discussion. Claims of generic string-theory signatures should be qualified unless this step is supplied.

5. **The numerical analysis is not reproducible or sufficiently validated.**

   The stabilization claims are almost entirely numerical, but the manuscript does not specify the integration algorithm, tolerances, independent variables evolved, handling of the constraint, event used to declare overshoot or stabilization, treatment when $\ell_j \to 0$ and $\delta_j \to \infty$, scan resolution, or robustness of the colored phase boundaries. Nor is it possible to determine from the paper whether equation (43) or its algebraically correct counterpart was used. Since the system becomes stiff near loop evaporation and near modulus oscillations, these omissions are material.

   The authors should provide code or, at least, complete pseudocode and a machine-readable benchmark table. For representative stabilizing and overshooting solutions, give all dimensionful inputs and initial values, the stopping conditions, the maximum constraint error, and convergence under smaller time steps and tighter tolerances. The phase diagrams should include grid resolution and a convergence check near each nontrivial boundary. Loop evaporation should be implemented with a clear event rule that removes the exhausted population without allowing negative lengths or a singular source. An independent integration in the original variables $(\Phi, \dot{\Phi}, \rho_j, \ell_j, \rho_{\rm rad})$ would be a valuable check on the phase-space system.

6. **The spectrum calculation needs two analytic corrections and a clearer statement of its scope.**

   First, whenever the map

   $$f_{\rm obs}(N) = \frac{a(N)}{a_0}\frac{2k}{\ell(N)}$$

   is non-monotonic, the delta-function integration leading from equation (61) to equation (63) must include a sum over *all* roots N_i satisfying $f_{\rm obs} = f_{\rm obs}(N_i)$. The manuscript itself attributes oscillatory features to the region where $|1 - \ell'/\ell|$ becomes small, precisely where folds and multiple roots can occur. Equation (63) displays only one $N_{\rm em}$. The authors should write the full root sum and explain how it was evaluated numerically; otherwise peak amplitudes can be missed or double counted.

Second, inserting the normalized log-normal distribution in equation (67) into equation (58) preserves the exponent

$$-\frac{[\ln f_{\rm obs} - \ln \bar{f}_k(N)]^2}{2\sigma^2}.$$

Equation (68) instead contains $1/\sigma^2$, while retaining the $1/(\sqrt{2\pi}\sigma)$ prefactor. Thus the kernel in equation (68) is not the normalized transform of equation (67). The formula and Fig. 17 should be corrected, with the convention for σ stated unambiguously.

More broadly, the authors appropriately acknowledge that Fig. 17 imposes an instantaneous width on the single-length background rather than evolving a distribution. This caveat should also constrain the abstract and conclusion: the smeared curve is a sensitivity test, not a self-consistent prediction. The revision should state whether $\sum_k \Gamma^{(k)} = \Gamma$, justify the omission of higher harmonics for the plotted frequency range, and distinguish robust broad peaks from features dependent on the assumed distribution.

7. **The conversion to a present-day observable is under-specified.**

   The frequency and amplitude in Fig. 16 require the absolute scale of the LVS potential, the onset and end of modulus domination, the modulus decay rate, the visible-sector branching fraction, and the entropy history. The caption gives $\Phi_{\rm in}$, fractional densities, $H_{\rm in}\ell$, g_s, and c, but not a complete numerical set including V_0, $H_{\rm in}$, H_Φ, $H_{\rm rh}$, m_Φ, the reheating temperature, or the relativistic degrees of freedom used between emission and today. Without these inputs neither the 10^8–10^{12} Hz placement nor the normalization can be reproduced.

   Please give a benchmark table with all these quantities and show explicitly how equation (66) and the analogous frequency-redshift relation are evaluated, including changes in entropy and energy degrees of freedom. The enhanced width in equation (65) should be tied to a definite validity regime and checked against BBN reheating requirements. The integrated spectrum should be compared with CMB/BBN bounds on extra radiation, and the discussion of detectability should identify realistic high-frequency proposals or be phrased as a target for future concepts rather than an imminent observational signature.

8. **Appendix A supports a plausible scaling argument, but not yet the stronger consistency statement made in the text.**

   The qualitative point that a high-frequency background can carry large energy density while its strain remains small is correct in spirit. However, the appendix should define whether h_{ij} is the dimensional perturbation of g_{ij} or the conventional dimensionless tensor γ_{ij} in $g_{ij} = a^2(\delta_{ij} + \gamma_{ij})$. The factors of a, polarization normalization, and numerical coefficients in equations (A9)–(A12) then need to be tracked consistently. For example, setting $\rho_{\rm GW} = 3M_{\rm p}^2 H^2$ in equation (A11) produces an order-one factor not shown in equation (A12). This does not change the parametric H/ω conclusion, but the equations should not be presented as exact at that level.

   More importantly, small strain is only one requirement. The Isaacson description assumes a short-wavelength, statistically coarse-grained background. The authors should verify $\omega/H \gg 1$ throughout the GW-dominated interval, explain why the number and angular distribution of loops yield an approximately homogeneous and isotropic radiation fluid, and state the averaging scale. The background Friedmann equations may include an order-one Isaacson stress tensor while the metric perturbation remains small, but that separation of scales should be articulated rather than inferred from amplitude alone.

Assessment of novelty, significance, and presentation

Relative to the literature described in the manuscript, the simultaneous treatment of F-, D3-, and NS5-string loop components is a natural extension of Refs. [6,7], while the inclusion of their GW energy loss and a backreacting radiation component extends the high-frequency spectrum analysis associated with Ref. [8]. The potentially novel message is that the wrapped-brane populations may solve the overshoot problem indirectly: they disappear quickly, but their relativistic decay products provide the friction. The proposed ordering of several high-frequency peaks is also interesting as a qualitative discriminator of multiple species.

At present the significance is limited by two layers of uncertainty. The first is mathematical: the evolution equation actually printed is wrong, and it is unclear which version underlies the figures. The second is physical: the initial populations, common loop sizes, purely gravitational branching, and monochromatic evolution are assumed rather than derived. The work need not solve the full cosmic-superstring network problem to be publishable, but it must cleanly separate controlled results from phenomenological illustrations and establish that its conclusions survive reasonable variations of the assumptions.

The prose and section ordering are mostly clear, and the figures convey the intended qualitative regimes. The revision would benefit from reducing repetition in Sections III.B and IV and using that space for derivations, benchmark tables, and numerical validation. With the central corrections and a transparent account of uncertainties, the paper could become a useful contribution. Without them, it is not yet suitable for publication in a high-energy theory journal.

Minor comments

1. In Section II.B the equation of state is written $\rho_{\rm f} = \omega p_{\rm f}$, whereas equations (9), (17), and (22) use the conventional $p_{\rm f} = \omega \rho_{\rm f}$. Please correct the text.

2. “Friedman” should be “Friedmann” throughout.

3. The notation alternates between ϕ and Φ, and exponents sometimes include $M_{\rm p}$ and sometimes do not. State once whether all fields are made dimensionless in Planck units and use that convention consistently, especially in equations (23)–(29) and (44)–(47).

4. Table I gives existence conditions but the discussion repeatedly calls particular points final attractors or saddles. Either provide the relevant eigenvalue/stability conditions for the multi-species system or state clearly that those classifications are numerical and parameter specific. Absolute-value qualifications may also be needed if negative λ or β_j are not excluded explicitly.

5. The manuscript should state how $\Gamma^{(k)}$ is distributed among harmonics and how its normalization relates to the total Γ in equation (31).

6. The text below equations (45)–(47) contains $g_s \sim H_0 \ell_5^{\rm in} \sim 0.1$; this presumably means $H_{\rm in} \ell_5^{\rm in}$.

7. Equation (31) defines a power with a positive sign, while the loop equations use it as a loss. Calling it the emitted positive power and writing $\dot{E}_{\text{loop}}|_{\text{GW}} = -P_{\text{GW}}$ would prevent sign ambiguity.

8. State whether $n_{\text{loop}}(N_{\text{em}})$ in equation (58) means $n_{\text{loop}}(N)$ evaluated at the integration point; the mixed notation is unnecessary.

9. In equation (66), use either $\Omega^{\text{in}}_{\text{loop}}$ or $\Omega^{\text{in}}_{\text{loops}}$ consistently.

10. The relation $f_{\text{em}} = 2k/\ell$ in equation (56) uses cyclic frequency, whereas Appendix A uses an angular frequency ω. Keep the factors of 2π explicit when relating the two.

11. Explain whether the high-frequency spectrum includes only the postulated initial loop population or also any loops produced later. The limits t_{in} and t_f in equation (53) should be specified for each species.

12. The phase diagrams should label the precise numerical threshold used to distinguish stabilization from overshoot and indicate points whose fate has not converged by the final integration time.

13. Several broad statements should be softened: "all possible ingredients" in the Introduction omits interactions, loop production, and non-GW radiation channels, while "general prediction" and "generic features of string theory" overstate what follows from the benchmark model.

14. Correct minor prose issues, including "F-string survive," "arising due the production," "distribution,we," and the inconsistent use of "initial," "in," and "i" subscripts.

15. Please ensure all figure labels, tick marks, and color distinctions remain legible in a two-column print version and are interpretable without relying only on color.

Final recommendation

The manuscript contains a promising mechanism and a potentially useful multi-species extension of recent work, but its primary conclusions depend on an incorrectly stated central evolution equation and on numerical and physical assumptions that are not yet documented well enough to test. I therefore recommend **major revision**. A publishable revision should correct equations (24), (43), (45)–(47), and (68) as appropriate; rerun and validate all dependent figures; establish the adiabatic and population consistency of the benchmarks; and provide a complete, reproducible mapping from the string compactification parameters to the present-day spectrum.