Referee Report

"Toward an Observable Algebra for JT-de Sitter Space:
Gap Protection and Modular Dressings"

arXiv:2607.26194

Recommendation: Major revision. The central geometric and operator-algebraic arguments appear sound under their stated abstract assumptions. Publication should nevertheless await a revision that makes the chiral/full-CFT identification precise, formulates the compressed algebras and their commutants consistently, and restricts the JT-de Sitter and "discontinuity" claims to what has actually been proved.

Summary and overall assessment

The manuscript studies whether the fundamental complement of a gravitating region can be reflected in an operator-algebraic assignment in global de Sitter space. It contains three logically distinct strands.

First, Section 2 gives a geometric analysis on the time-symmetric circle of global dS_2. For an admissible finite union of arcs, Theorem 2.5 and equation (14) identify the fundamental complement with the domain of dependence of the unique complementary gap of angular size at least π, if such a gap exists. Lemma 2.3 correctly includes the threshold case: complete timelike curves may approach the two conformal-boundary points of a half-circle diamond while accumulating infinite proper time. Proposition 2.8 then identifies the higher-dimensional existence threshold as containment of an open hemisphere, without overclaiming a closed formula for the causal completion in higher dimensions.

Second, Sections 3 and 4 place a chiral conformal net in a common regular crossed-product representation with an auxiliary $L^2(\mathbb{R})$ clock. The regional continuous cores are compressed by the common projection $P = \mathbf{1}_{[0,\infty)}(h)$. Theorem 4.2 gives an exact criterion for literal inclusion of two represented cores, or of their common corners: the larger region's modular flow must restrict to the smaller region's modular flow, equivalently there must be a vacuum-preserving conditional expectation. Proposition 4.5 uses vacuum correlations and Reeh–Schlieder to show that this condition fails when the larger region contains an additional open arc spacelike to the smaller one. Proposition 4.9 restores inclusion on one fixed finite atomic family by replacing the vacuum restriction with a split product state.

Third, Section 5 proposes the representation-dependent assignment

$$\mathcal{A}_{\text{model}}(a) = \mathcal{M}(\widetilde{A})' \cap \mathcal{M}_{\text{ref}}$$

in equation (56). The noncommutation argument of Proposition 5.3, the high-frequency compression lemma (Lemma 5.4), and Corollary 5.5 establish that this is a proper subalgebra of the reference factor when the protected gap overlaps the reference arc. Combining that result with the convention for an empty fundamental complement yields Theorem 5.10: adding an antipodal arc of arbitrarily small angular measure changes the output from a proper subalgebra to the full reference factor.

The paper is technically careful, and I did not find a decisive error in the main proofs. In particular, the equality case in the gap theorem, the corner-to-core implication in Theorem 4.2, the conditional-expectation contradiction in Proposition 4.5, and the Hardy-space compression estimate in Lemma

5.4 all withstand close scrutiny. Appendix C also has internally consistent signs and powers of L in the ADM constraints and York transformation.

The principal problem is instead the status and significance of the construction. The operator algebra is built from an abstract chiral net and an auxiliary clock, whereas the JT discussion uses a distinct non-chiral matter theory and only a classical constraint relation. Isotony of equation (56) remains open in the nontrivial case in which both fundamental complements are nonempty; factoriality, nontriviality beyond scalars, commutant recovery, entropy, and a gravitational-clock realization are also open. Moreover, Theorem 5.10 proves a sharp proper-versus-full change, but no topology is specified in which this is a discontinuity. These issues are substantial for a paper whose title and main physical narrative concern an observable algebra for JT-de Sitter space.

Correctness and logical structure

The geometric argument is convincing once the causal-complement identity in equation (8) is supplied with a proof. Equation (10) gives the correct arc domain. Lemma 2.3 constructs complete timelike curves for $\beta \geq \pi$, including $\beta = \pi$, and Proposition 2.4 then performs the required causal completion rather than merely testing for existence of a single curve. The length sum ensures that no two gaps can reach the threshold, so equations (14) and (15) follow. Proposition 2.8 likewise uses the closed-ball characterization in equation (17) and compactness of the round sphere correctly.

The crossed-product conventions in equations (37)–(45) are mutually consistent. With the stated dual action, the spectral density of the canonical trace is proportional to e^{-x}, and the chosen Haar/Fourier normalization gives $\widehat{\mathrm{Tr}}(P) = 1$. In Theorem 4.2, the common dual fixed-point algebra proves the core-inclusion criterion, Takesaki's theorem gives the equivalence with a state-preserving expectation, and the translated projections $P_r \to \mathbf{1}$ strongly recover the uncompressed inclusion from the corner inclusion. These steps are logically sound, subject to the support-P convention discussed below.

Proposition 4.5 is also persuasive. A vacuum-preserving conditional expectation would map the algebra of the added commuting arc into the center of the smaller factor and hence would force product correlations. A centered nonzero local operator would then annihilate the vacuum by cyclicity, in conflict with separatingness. Lemma 5.4 is a useful nontrivial result: modulation by e^{iNq} moves a band-limited witness into the positive spectral subspace while the negative-frequency leakage of multiplication operators vanishes. Thus the pointwise noncommutation found at $q = 0$, which persists on a positive-measure set by continuity, cannot disappear after compression.

These checks support the abstract mathematical results. They do not, however, supply the missing physical identification of the auxiliary construction with a JT observable algebra, nor do they determine the structure of the relative commutant in equation (56).

Novelty and significance

The cleanest new contribution is Theorem 2.5: it turns the fundamental complement into an exact finite-multi-arc rule and handles both causal completion and the threshold equality case. Proposition 2.8 is also a neat geometric result, though its higher-dimensional significance is limited by the absence of a corresponding completion theorem.

Theorem 4.2 is a useful synthesis in the concrete shared-clock representation. Its modular-invariance/conditional-expectation content is standard Takesaki theory, while the equivalence with inclusion of the particular common spectral corners is the more distinctive part. Proposition 4.5 is an informative no-go application of standard AQFT correlations. Lemma 5.4 and Corollary 5.5 provide the most substantial new operator-theoretic input to the model itself.

By contrast, the type-II_1 corner in Proposition 3.15 is the positive-clock-spectrum mechanism already central to the de Sitter crossed-product literature, here written with a careful normalization. Proposition 4.9 follows standard tensor-product modular theory after choosing a split product state. Proposition 5.6 inserts the geometric theorem into the definition (56), and the full-algebra endpoint in Theorem 5.10 is fixed by the empty-complement convention. The manuscript should separate these standard or definitional ingredients from its genuinely new statements much more sharply. The combination is potentially interesting to the hep-th community, but the present JT interpretation goes beyond the proved model.

Major comments

1. **The manuscript must state unambiguously what has, and has not, been constructed.**

   The algebraic construction uses an abstract chiral conformal net, its vacuum modular operators, and a common auxiliary clock. The JT comparison instead uses a separate, anomaly-free full CFT. Equation (35) is only a classical constraint-surface equality between a one-sided matter charge and an endpoint term. Appendix C explicitly leaves the reduction, monotone gravitational clock, quantization, and regional embeddings undone. In particular, none of these steps identifies the auxiliary h, the chiral K_R, the full-CFT static generator, the one-sided matter charge, and the JT endpoint or York variables as operators in one theory.

   This separation is acknowledged in several remarks, but the title, abstract, and repeated language of an "observable algebra for JT-de Sitter" still suggest that the bridge has been made. The author should either construct a genuine non-chiral/gravitational realization or consistently reframe the result as a chiral conformal-net model motivated by JT-de Sitter. The latter would be a defensible and useful contribution, provided the scope is explicit from the title and abstract onward.

2. **The chiral Möbius circle and the spatial de Sitter circle need a precise relation.**

   Section 3.1 describes an interval-preserving Möbius flow on S^1 and relates it to equation (22), $\xi = \cos\chi\,\partial_\eta$ on the symmetric slice. But this vector is normal to Σ_0; it does not generate a flow of the spatial circle. The intended relation can be made precise by displaying the full de Sitter boost

   $$\xi = \cos\eta\cos\chi\,\partial_\eta - \sin\eta\sin\chi\,\partial_\chi$$

   and its action on compactified null coordinates $x_\pm = \eta \pm \chi$. Its two chiral components then act by the appropriate interval-preserving Möbius subgroups, with an orientation that should reproduce equation (24). This would also make clear that $H^{\mathrm{ch}}_{\mathrm{stat}}$ is one chiral model generator, not the physical full-CFT static charge in equation (28).

   There is a related operator-domain issue in equation (29). Assumption 3.10(b) only supplies stress-tensor smearings against smooth test functions, whereas the one-sided weight is effectively $\cos\chi\,\mathbf{1}_R$. It is continuous because the cosine vanishes at the endpoints, but its derivative jumps there. The manuscript should either keep H^R_{mat} explicitly classical/formal, define smooth regulators and prove an affiliated or self-adjoint limit with endpoint control, or strengthen the assumption. The warning in Remark 3.11 does not by itself define the operator used later.

3. **The support projection and the ambient commutant must be formulated consistently.**

   As represented on the uncompressed Hilbert space, $P\widehat{\mathcal{M}}(u)P$ has identity P, not the ambient identity. Under the standard convention it is a von Neumann algebra on the common Hilbert space $P(\mathcal{H}_{\mathrm{CFT}} \otimes L^2(\mathbb{R}))$, rather than a unital von Neumann subalgebra of the full $B(\mathcal{H}_{\mathrm{CFT}} \otimes L^2(\mathbb{R}))$. Lemma 3.19, Section 5.1, and equation (56) move between these viewpoints without declaring a convention, while the empty complement is assigned $\mathbb{C}\mathbf{1}$.

   The clean solution is to formulate all compressed algebras on the common space $P\mathcal{H}$, use $\mathbb{C}P$ for the empty algebra, and take the commutant in $B(P\mathcal{H})$. Alternatively, the author may explicitly adopt support-P, nonunital embeddings into the larger $B(\mathcal{H})$ and prove that the resulting intersection agrees with the compressed-space definition. The principal results should survive this repair, but well-definedness, units, and commutants cannot remain convention-dependent.

4. **Equation (56) needs a stronger structural and physical justification.**

   The central assignment is postulated rather than derived. It always lands in one fixed observer/reference factor, and equation (47) establishes covariance only when the reference arc and the input region are rotated simultaneously. With the reference fixed, the manuscript does not establish an intrinsic de Sitter-covariant region-to-algebra map. It should say whether $\mathcal{A}_{\mathrm{model}}(a)$ is intended to describe observables of a, observables of its hologram, or observables accessible to the reference observer after imposing a commutation condition.

   More importantly, the basic net properties are not yet known. Isotony is open when both fundamental complements are nonempty, locality is not formulated, and equation (57) is only a desired commutant-recovery statement. These are not secondary details for an observable-algebra proposal. The paper should test the assignment systematically against the axioms proposed in the cited generalized-entanglement-wedge literature and distinguish proved properties from targets.

   Corollary 5.5 shows only that the initial algebra in the activation example is proper. It could still be merely $\mathbb{C}P$; neither a non-scalar observable nor factoriality is established. A meaningful strengthening would be to compute the relative commutant for at least one tractable conformal net, or prove nontriviality/factoriality, an index statement, or another invariant in the general framework. Without such information, the physical size of the claimed jump is unknown.

5. **Theorem 5.10 does not yet establish a discontinuity.**

   The theorem proves that adding an antipodal component of arbitrarily small angular measure changes the output from a proper subalgebra to the full reference factor. No topology is given on admissible regions and no topology is given on von Neumann subalgebras. The family is small in symmetric-difference measure, but the added component remains antipodally far away and need not be small in a Hausdorff topology. Likewise, the distinction "proper versus full" does not determine convergence in the Effros–Marechal topology or any other standard topology on subalgebras.

   The author should specify the domain and codomain topologies and prove failure of continuity, or replace "discontinuity" throughout with a precise description such as "an algebraic jump under additions of arbitrarily small measure." The text should also stress that the full-algebra branch follows from the convention $\mathcal{M}(\varnothing) = \mathbb{C}\mathbf{1}$ in equation (56). Thus Theorem 5.10 is a consequence of the proposed model, the geometric threshold, and Corollary 5.5; it is not yet an independently derived feature of a gravitational observable algebra.

6. **Two foundational steps deserve fuller proofs or precise citations.**

   Equation (8), especially $D(a)' = D(a^{\perp})$ for disconnected finite unions with the stated endpoint convention, is a premise of Theorem 2.5 but is asserted rather than proved. A short proof on the causal cylinder, using the closed crossing interval that later appears in equation (17), should establish both the equality and the claim that the individual diamonds are exactly the connected components of the causal complement. This matters because the manuscript correctly emphasizes that deleting an endpoint can change a full diamond.

   Similarly, equation (44) is central to Proposition 3.15. Its sign and normalization are consistent with the preceding conventions, but the passage from the dual weight to $\widehat{\mathrm{Tr}}(f(h)) = \int e^{-x} f(x)\,dx$ is compressed into a reference. The author should cite the precise continuous-core theorem in the chosen convention or derive the operator-valued-weight identity $T(V(g)) = g(0)\mathbf{1}$ under the displayed Haar normalization. This would make the otherwise careful discussion of unbounded weights fully reproducible.

7. **The split-state construction is a limited compatibility example, not yet a physical repair.**

   Proposition 4.9 is correct: once the split isomorphism is fixed, the product modular group restricts coherently to subproducts. But the construction removes precisely the vacuum correlations responsible for Proposition 4.5. It therefore changes the state rather than providing compatible embeddings for the Bunch–Davies vacuum. It applies only to unions of one fixed separated atomic partition, and different partitions require new, potentially incompatible choices.

   The manuscript should explain what physical state or gravitational operation, if any, is represented by this replacement. Otherwise it should be presented solely as an existence proof that coordinated modular data can restore inclusion on a finite poset. Calling it a repair without this qualification risks obscuring the strength of the no-go result.

8. **The novelty relative to the cited operator-algebra literature must be isolated more sharply.**

   The modular-invariance and conditional-expectation equivalence in Theorem 4.2 is Takesaki's theorem; the dual fixed-point step and the construction of the continuous core are also standard. The paper's contribution is the concrete shared-clock formulation, the equivalence with inclusion after compression by the common P, the application in Proposition 4.5, and the compression result in Lemma 5.4. The introduction should say this directly and compare with modular-covariant, split, and cocycle-modified inclusions.

   Likewise, Proposition 3.15 should be identified explicitly as the positive-spectrum finite-corner mechanism of the cited de Sitter construction in the present normalization. Theorem 2.5 should be compared directly with the de Sitter examples in the cited fundamental-complement work, specifying what the arbitrary multi-arc statement adds. Finally, equation (56) should be compared property by property with the cited algebraic generalized-entanglement-wedge proposal, rather than being described mainly by analogy. This comparison is needed to assess novelty at the level expected by a high-energy theory journal.

9. **The recovery claim following equation (65) requires additional hypotheses.**

A state-independent edge term cancelling in modular-energy differences does not by itself establish equality of relative entropies or Petz sufficiency. The manuscript must specify the encoding or restriction channel, a reference state, the relevant family of code states, the entropy identity, and the equality of relative entropies required by the sufficiency theorem. Moreover, the $O(G_2)$ remainder would naturally lead to approximate rather than exact recovery. Equations (65) and (66) can certainly be presented as a semiclassical target, but the text should not say that Petz reconstruction already follows from equation (65) alone.

Minor comments

1. Definition 2.1 should state explicitly that the number of component arcs is positive. In Corollary 2.9, either define the higher-dimensional round measure denoted by $|a|$ or simply assume that a is nonempty; a nonempty open set cannot lie in the equator.

2. Lemma 3.7 proves trivial intersection of disjoint arc algebras. The term “statistical independence” is often reserved for a product-state or split property. The result should be renamed or the terminology explained.

3. In Proposition 4.9, the collection indexed by nonempty subsets omits the bottom element and is therefore not literally a Boolean algebra. Include the empty region, or call it the Boolean family with its bottom removed and specify that complements are relative to the union of the fixed atoms.

4. In the proof of Corollary 5.5, the general argument writes $\mathcal{M}(\widetilde{A}_1)$. This should be $\mathcal{M}(\widetilde{A})$.

5. Remark 5.8 should keep one explicit UV regulator and renormalization scheme throughout. Entanglement entropy for continuum type-III local algebras is not an unregulated finite quantity, and positivity of a cutoff entropy is not by itself a scheme-independent check of generalized entropy.

6. The ADM discussion should state whether the matter variables in Appendix C are classical fields, expectation values in semiclassical gravity, or operators. Assumption 3.10 is phrased in operator language, while Proposition 3.12 is correctly labelled classical.

7. The duplicated package-loading block in the preamble should be removed. The final heading should read “Acknowledgements” (or “Acknowledgment”) for consistency with standard usage.

Recommendation

I recommend **major revision**. The geometric gap theorem and the regional modular-inclusion obstruction are worthwhile results, and the nontrivial compression argument is a useful technical contribution. The manuscript could become publishable as a rigorous chiral-net model motivated by de Sitter gravity. Before publication, however, it should repair the support-P algebra convention, make the chiral/static-time relation and one-sided charge precise, define or rename the alleged discontinuity, add the missing recovery hypotheses, and bring the title and physical conclusions into line with the still-provisional JT realization.