Referee Report

“Nucleon spectra and wave functions from holographic models with dual Einstein-dilaton and Starobinsky-dilaton gravities”

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Recommendation: Major revision before reconsideration. The proposed application of an R^2-corrected confining background to radial nucleon excitations is potentially useful. The present manuscript is not yet suitable for publication in PRD, JHEP, or a comparable journal, because the claimed string-frame action is not the Weyl transform of the nonlinear $f(R)$ theory, the displayed background potential contains a consequential derivative error, and the effective fermion mass printed in two central equations is incompatible with the spectrum actually analyzed. A consistent rederivation, stability analysis, and reproducible numerical study are needed before the phenomenological conclusions can be assessed.

Summary and overall assessment

The manuscript investigates the first four positive-parity spin-1/2 nucleon states in bottom-up holographic models. Sections 2 and 3 begin from a five-dimensional Einstein-dilaton action and then replace R by $f(R) = R + \alpha R^2$. The inverse Einstein-frame scale factor is prescribed as

$$\zeta(z, k) = z \exp\left(\frac{2}{3} k z^2\right).$$

One combination of the modified gravitational equations is used to reconstruct the dilaton profile. The authors then define a string-frame scale factor, discuss the Wilson-loop confinement criterion, and present reconstructed dilaton potentials.

Section 4 introduces a five-dimensional Dirac field, decomposes it into left- and right-handed modes, and reduces its radial equations to the supersymmetric pair

$$\left[-\partial_z^2 + U_{R/L}\right] \Psi_{R/L} = M_{N^n}^2 \Psi_{R/L}, \qquad U_{R/L} = W^2 \pm W',$$

where $W = M_{\text{eff}}/\zeta_s$. Section 5 varies a parameter λ in the effective mass in the Einstein-dilaton background. For each chosen (λ, Δ), the scale k is fixed to reproduce the nucleon ground-state mass. Section 6 then varies the Starobinsky coefficient α, first at fixed $\lambda = 0.5$ and subsequently at $\lambda = 0.4, 0.6$. The reported effect of increasing α is to raise the more highly excited levels and to turn the approximately linear Einstein-dilaton radial trajectories into nonlinear ones.

There is a sensible phenomenological question here. The manuscript combines the confining nucleon construction of “Nucleons and vector mesons in a confining holographic QCD model” with the authors’ later $f(R)$-dilaton background developed for vector mesons. Extending that combination to nucleons and exploring the coefficient of the dilaton contribution are the main new elements. The mass tables are clearly organized, and the qualitative observation that an infrared deformation affects higher modes more strongly is plausible.

The central difficulty is that the equations displayed in the paper do not define the model whose numerical results are reported. Several problems are more than cosmetic: the nonlinear curvature term does not transform between frames as claimed; the reconstructed potential has an incorrect product of derivatives and consequently an unphysical odd-power UV term; and the printed effective mass would remove the required AdS $1/z^2$ centrifugal behavior. In addition, the chosen positive-α backgrounds cross $f_R = 0$ at finite radial coordinate. These issues directly affect the claimed gravitational consistency, confinement interpretation, and Schrodinger problem.

Correctness and logical structure

The Einstein-dilaton reconstruction through equation (3.12) is standard, and the $\alpha \to 0$ limit of equation (3.33) gives the expected derivative $\phi' = \sqrt{k}\sqrt{9+4kz^2}$. Likewise, equation (3.27) follows from the difference of equations (3.22) and (3.23). These parts provide a reasonable starting point once conventions and dimensions are fixed.

The transition to the Starobinsky-dilaton model is not consistent as written. With the paper's relation $g^s_{mn} = e^{4\phi/3} g_{mn}$, one has in five dimensions

$$\sqrt{-g}\, R^2 = \sqrt{-g_s}\, e^{-2\phi/3} \left[R_s + \frac{16}{3} \Box_s \phi - \frac{16}{3} (\partial\phi)^2_s \right]^2 .$$

It therefore cannot become $\sqrt{-g_s}\, e^{-2\phi} R_s^2$. Equations (2.4) and (3.15) omit the resulting derivative couplings and even have the wrong dilaton weight for the R_s^2 term. They also contain a $-4(\partial\phi)^2$ term, whereas the stated $\alpha = 0$ limit, equation (3.2), contains $+4(\partial\phi)^2$. Thus the phrase "string frame" has not been derived for the nonlinear theory. Since both the Wilson-loop discussion and the fermion action use this metric, the issue propagates beyond the gravitational preamble.

There is also a concrete algebraic error in the potential reconstruction. From equation (3.22),

$$\zeta^2 R'' = 8\zeta^3 \zeta'''' - 24\zeta^2 \zeta' \zeta''' - 32\zeta^2 (\zeta'')^2.$$

The first term in the second line of equation (3.28) is instead printed as $8\zeta''''\zeta'''$. This product has the wrong derivative dimension and is the source of the linear term in equation (3.36). For the even ansatz $\zeta/z = \exp(2kz^2/3)$, the reconstructed potential must be an even function of z. Direct expansion of equation (3.22) gives

$$V(z) = 12 - 80\alpha + 36k(1+8\alpha)z^2 + O(z^4)$$

when $\ell = 1$, with no term proportional to z. As printed, the potential does not satisfy the field equation from which it is said to follow.

The fermion sector contains a second decisive internal contradiction. Equations (2.6) and (4.17) state

$$M_{\text{eff}} = \lambda \phi' \zeta_s + m_5 \zeta_s,$$

so that $W = \lambda\phi' + m_5$ is finite at the AdS boundary. This would produce neither the $1/z^2$ term nor the $1/z$ term shown in equation (5.1). The cited parent nucleon model, and equation (5.1) itself, instead require

$$M_{\text{eff}} = \lambda \phi' \zeta_s + m_5 \zeta_s', \qquad W = \lambda\phi' + m_5 \frac{\zeta_s'}{\zeta_s}.$$

The missing prime changes the differential operator, boundary behavior, and spectrum. Moreover, the two first-order equations (4.10) and (4.11) display the same sign of M_{eff}, although chiral projection of equation (4.7) gives opposite mass signs. Equation (4.12) silently returns to the opposite-sign factorization. The intended operator may be recoverable, but it must be rederived from the action rather than inferred from later plots.

Novelty, significance, and relation to the literature

The Einstein-dilaton geometry, its confinement criterion, and the original fermion mass coupling are inherited from established work cited in Sections 3 and 4. The $R + \alpha R^2$ background and its application to hadron spectroscopy were already introduced in the authors' vector-meson study. The present paper's incremental contributions are the nucleon application, the replacement of the earlier fixed coefficient 1/2 by λ, and a scan over $(\lambda, \alpha, \Delta)$.

These extensions could be phenomenologically informative, but the paper currently presents λ as an extra fitting parameter without explaining which theoretical condition permits the original coefficient to vary. In the parent nucleon construction, the nonminimal mass coupling was chosen to recover a constant AdS mass in the UV and quadratic confinement in the IR, and the correlator normalization and Sturm-Liouville problem were derived in detail. The revised manuscript should show that the generalized coupling preserves the same UV dictionary, variational principle, spectral decomposition, and left-right degeneracy. It should also state clearly that the Starobinsky terminology denotes a five-dimensional bottom-up R^2 deformation; the success of four-dimensional positive-curvature inflation does not by itself validate the present negative-curvature holographic background.

If these foundations can be repaired, a controlled comparison of spectra could be useful. At present, however, the numerical improvement of selected levels does not establish an improved holographic model. The scan introduces additional freedom while using only three excited-state masses after the ground state has been fitted exactly. The reported nonlinear fits also use three parameters for four levels, leaving essentially no power to establish a new trajectory law.

Major comments

1. **Define a consistent frame and probe coupling for the $f(R)$ theory.**

   Equations (2.4) and (3.15) are not related to equations (2.1) and (3.14) by the Weyl transformation (2.3). A nonlinear function of the Ricci scalar does not transform as the Ricci scalar itself. In addition to the different dilaton weight displayed above, the transformed R^2 term contains $\Box\phi$, $(\partial\phi)^2$, and their products.

   The authors should choose one of two logically consistent routes. They may work entirely in the original metric $f(R)$ frame, introduce the scalaron, derive the appropriate Einstein-frame two-scalar theory, and specify to which metric the fundamental string and baryon probe couple. Alternatively, they may present $g_s = e^{4\phi/3} g$ and the probe actions as independent bottom-up ansatze, without claiming that equation (3.15) follows from equation (3.14). Either route requires a new justification of the Wilson-loop criterion in Section 3.4 and of the Dirac action in Section 4. The $\alpha = 0$ sign mismatch between equations (3.15) and (3.2) must also be corrected.

2. **Correct the reconstructed background and verify every field equation.**

   The factor $8\zeta''''\zeta'''$ in equation (3.28) must be replaced by the factor obtained from $\zeta^2 R''$, namely $8\zeta^3\zeta''''$. The accompanying terms should be checked at the same time. Equation (3.36) must then lose its odd term $(5120/3)k^3\alpha z$. Equation (3.30) separately has $\sqrt{9+4kz}$, while both equation (3.31) and the $\alpha \to 0$ limit of equation (3.33) require $\sqrt{9+4kz^2}$.

   After these corrections, the authors should substitute $\{\zeta, \phi, V\}$ into equations (3.16), (3.17), and at least two independent metric equations and report the residuals analytically or numerically.

The text should distinguish a potential $V(\phi)$ from an on-shell function $V(z)$, demonstrate that $\phi(z)$ is invertible on the domain used, and state all powers of ℓ. Figures 2 and 3 should then be regenerated from the corrected equations.

3. **Analyze the $f(R)$ stability conditions on the actual radial background.**

   For equation (3.29), equation (3.25) gives, in the conventions with $\ell = 1$,

   $$R(z) = -e^{4kz^2/3}\left(20 + \frac{64}{3}kz^2 + \frac{64}{3}k^2z^4\right).$$

   Thus R is negative and unbounded in magnitude. For every $\alpha > 0$, the effective gravitational coupling

   $$f_R = 1 + 2\alpha R$$

   crosses zero at a finite z. Positivity of α, borrowed from a positive-curvature cosmological discussion, is therefore not sufficient for ghost freedom on this solution. The scalar-tensor representation is singular at the crossing, while the spectral problem and the claimed confinement geometry are formulated on $0 < z < \infty$.

   The authors must locate this crossing for every (k, α) used in Tables 4, 6, and 7, check $f_R > 0$, $f_{RR} > 0$, and the scalaron fluctuation spectrum on the full physical domain, and explain how boundary conditions are imposed if the domain is truncated. They should show that the reported eigenfunctions are localized well inside a theoretically admissible interval and that the eigenvalues are insensitive to the cutoff. Without such an analysis, the abrupt infrared rise in Figure 8 cannot be interpreted as a controlled Starobinsky correction.

4. **Re-derive the fermion system with the intended effective mass.**

   The missing derivative on ζ_s in equations (2.6) and (4.17) is not a harmless notation issue: the equations as printed are mathematically incompatible with equation (5.1). The authors should state the generalized mass coupling in a single unambiguous formula, derive equations (4.7)–(4.17) from equation (4.1), and display the two first-order radial equations before decoupling them. The mass signs should reproduce the partner Hamiltonians, and the eigenvalue in equation (4.14) should be $M^2_{N^n}$, not M_{N^n}.

   The UV analysis should include $\Delta = 2 + |m_5\ell|$, the normalizable and non-normalizable asymptotics, and the effect of λ on the holographic two-point function. The normalization measure for $f^n_{R/L}$ and for $\Psi_{R/L}$ must be stated. Figures 7, 10, 12, and 13 should identify which of these two functions is plotted. Finally, the numerical equality of the left and right spectra should be shown as a check of the factorization.

5. **Make the spectral computation reproducible.**

   The manuscript does not give the UV starting point, IR boundary condition, radial cutoff, integration method, grid size, eigensolver, normalization condition, or convergence test. These omissions are especially serious because the SD potentials rise rapidly and the underlying f_R changes sign at finite radius. The revised paper should provide:

   - the complete dimensionful parameter set, including the units of α, m_5, z, and ℓ;
   - the value of k used in every SD column and a statement of whether it is held fixed or refitted as α changes;
   - UV and IR boundary conditions and mode-normalization formulas;

- cutoff, precision, and convergence studies for every reported level;
- enough code or tabulated potential data to reproduce the tables and figures.

At least several levels beyond $n = 3$ should be computed. Four points are insufficient to support claims about asymptotic Regge behavior.

6. **Replace selected-state error statements with a global, parameter-aware comparison.**

   Table 1 fits k separately for every (λ, Δ) so that the ground state is exact. The SD scan adds α, and the text highlights the smallest error of any single excited state. This does not measure overall agreement. For example, increasing α at $\lambda = 0.4$ improves one intermediate state while substantially overshooting $N(1880)$.

   The authors should define a global objective, such as a covariance-aware χ^2 or at minimum an RMS fractional error over a fixed set of states, fit all continuous parameters under the same protocol, propagate experimental and numerical uncertainties, and quote the number of fitted parameters and degrees of freedom. An out-of-sample state or a joint fit to another observable would be valuable. The comparison to the soft-wall model must use the same state assignments, conformal dimension, and calibration rules; the present footnote itself notes that a different conformal dimension gives the successful soft-wall description.

7. **Recompute the tables and trajectory fits.**

   There are checkable numerical inconsistencies. In Table 6, the $\Delta = 7/2, \lambda = 0.6, \alpha = 0$ entry 1.472 GeV differs from 1.710 GeV by 13.92%, not 13.2%. In Table 2, 1.853 GeV differs from 1.880 GeV by 1.44%, not 1.3%; Table 7 gives the latter value correctly. More substantially, the four masses in Table 2 for $\Delta = 9/2, \lambda = 0.5$ yield the least-squares line

   $$M_n^2 \simeq 0.794\, n + 0.881 \text{ GeV}^2,$$

   not $0.904\, n + 0.884 \text{ GeV}^2$ as printed in Tables 3 and 5.

   All mass errors and all fits should be regenerated from a common data file. For the nonlinear ansatz $a(n + b)^\nu$, three parameters are fitted to only four masses. The revised paper should report residuals and parameter uncertainties and should test the ansatz on a longer spectrum or replace it with an analytic WKB asymptotic result.

Minor comments

1. Equations (3.5)–(3.7) have an inconsistent normalization of the stress tensor. Since the scalar Lagrangian is inside the same overall $1/(16\pi G_5)$ factor as the curvature term, the displayed definition of T_{mn} does not combine with $8\pi G_5 T_{mn}$ as written. This should be aligned with the equations actually used to obtain (3.9)–(3.12).

2. Equation (4.15) writes $\zeta_s(z, \kappa)$, although the manuscript otherwise uses k, and equation (4.18) should display the AdS radius and the absolute value of the spinor mass.

3. Equation (5.1) should carry all powers of k, or the equation should state before the display that dimensionless variables with $k = 1$ are being used. The same requirement applies to equation (6.3).

4. The physical dimensions of α are never stated in the present manuscript. Since the tables mix GeV-valued k with numerical powers of ten for α, every table and figure caption should supply units.

5. The paragraph following Figure 7 says first that the red curves have the largest extrema and then that the same red curves have the smallest extrema. The intended ordering should be checked against the normalized functions.

6. Figure 11 lists 10^{-11}, whereas the text and tables use $10^{-11.1}$. Similar notation varies between $10^{-13.5}$, 10^{-12}, and $10^{-11.1}$ with no explanation of significant digits.

7. The caption of Figure 3 identifies the plotted potential with the near-boundary expansion (3.36). If the full reconstructed potential was plotted, the exact formula should be cited; an asymptotic series should not be used outside its controlled range.

8. The action (4.1) should write the Hermitian Dirac kinetic term explicitly rather than using "*c.c.*" inside the operator, and the orientation and sign of the boundary term should be specified.

9. The title and abstract refer broadly to "nucleon spectra", but the study is restricted to four $J^P = 1/2^+$ radial states. This scope should be stated in the abstract and conclusions, together with the specific PDG assignments.

10. The manuscript would benefit from a thorough language edit. Examples include "operator" misspelled as "operador", "vielbein" repeatedly written as "veilbein", "Nambu-Goto" written as "Nambu-Gotto", and several subject-verb and duplicated-word errors.

Conclusion

The paper asks a worthwhile question and presents a potentially useful set of phenomenological comparisons. Its strongest qualitative message is that an infrared deformation can shift higher radial modes more than the ground state. The manuscript does not yet establish that this deformation arises from a consistent Starobinsky-dilaton holographic model, nor that the quoted improvements survive a controlled global fit.

A publishable revision would require more than local typographical changes. The authors should first define a consistent gravitational frame and probe coupling, correct and verify the reconstructed background, establish an $f_R > 0$ radial domain, and rederive the fermion operator with the intended ζ_s' term. They should then rerun the complete spectrum with documented numerical controls and compare models using a common statistical criterion. If the principal conclusions remain after that program, the work could merit reconsideration.