Referee Report

"D-branes and nonlinear interactions
in the AdS pure spinor string"

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Recommendation: Major revision before reconsideration. The coordinate-independent organization of pure-spinor boundary conditions by a grading-reversing involution is promising, and the exponential-graph treatment of transverse fluctuations may become useful. The present manuscript is not yet ready for JHEP, PRD, or a comparable journal. One D3 fixed-algebra count is internally inconsistent, the central nonlinear lift is equated to its coset projection in equation (5.19), and equations (5.42)–(5.44) do not yet define dynamics for the physical superconnection and embedding: the existence, cone preservation, gauge compatibility, and physical content of the four ghost maps are left as assumptions. These points require a substantive rederivation and at least one explicit probe-level test.

Summary and overall assessment

The paper develops a local algebraic description of Lorentzian, half-BPS boundary conditions for the pure spinor string on $AdS_5 \times S^5$. Section 2 uses the flat bosonic string to motivate an involution R, its fixed and anti-fixed subspaces, and a first-order boundary field imposing the transverse position. Section 3 then applies the same organization to the $\mathbb{Z}_4$-graded supercoset. The ghost gluing exchanges grades one and three, the physical tangent and normal spaces are identified with $\mathfrak{g}_2^\bullet$ and $\mathfrak{g}_2^\circ$, and a local Cartan coordinate

$$Y^\circ = -\frac{1}{2}\operatorname{Log}(g^{-1}R(g))$$

is introduced to impose the fixed orbit by an ordinary variational principle. A boundary polarization for the pure-spinor ghosts completes the zero-field construction.

Section 4 classifies complex-linear, field-independent representatives satisfying

$$R\Sigma R^{-1} = \Sigma^{-1}.$$

Four diagonal inner representatives and three supertranspose-type outer representatives are assigned to D1, D3, D5, and D7 probe bodies. The inner sign test in equation (4.7) gives a clean obstruction to the proposed $3+1$ unitary splittings, while equations (4.19)–(4.24) organize the orthosymplectic cases.

Section 5 is intended to be the main dynamical advance. An Abelian superconnection $\mathcal{A}$ lives on the reference orbit, while a transverse section Φ° defines the local graph $g = g_\bullet e^{\Phi^\circ}$. Four linear maps $\mathcal{M}_{rs}$ deform the ghost gluing. The exact differential of the graph is separated into fixed and anti-fixed coset blocks $\mathcal{T}_\Phi$ and $\mathcal{N}_\Phi$, leading to $\mathcal{D}_\Phi = \mathcal{N}_\Phi\mathcal{T}_\Phi^{-1}$. Ordinary boundary stationarity gives equations (5.31)–(5.34), and cancellation of the BRST flux gives equations (5.42)–(5.44).

There is real merit in this program. The paper sharply distinguishes the fixed symmetry algebra from the physical coset tangent space, treats the local logarithmic branch explicitly, and keeps the noncommutative exponential geometry rather than truncating the embedding at first order. The distinction between an abstract half-BPS superalgebra and an actual world-sheet involution is also correctly emphasized.

The difficulty is that the two advertised results have not yet reached the same level of completeness. The zero-field classification contains a concrete Abelian-dimension mismatch in the D3 rows. More importantly, the interacting construction produces necessary relations involving four otherwise unspecified ghost maps, but does not establish that such maps exist on the pure-spinor cone, descend to the antighost quotient, or yield the physical world-volume multiplet. Calling the resulting relations "Dirac–Born–Infeld-like equations of motion" is therefore premature.

Correctness and logical structure

Several parts of the derivation are well organized. The fixed/anti-fixed orthogonality used in equations (3.31)–(3.37) is the natural coordinate-independent version of the zero-field gluing. The Cartan map in equations (3.45)–(3.54) correctly records its locality and the invertibility domain of the differential of the exponential. In Section 5, the parity expansions in equation (5.17) follow from the even and odd powers of ad_{Φ°, and the conversion of the odd currents in equations (5.35)–(5.39) is algebraically consistent with the closed-channel convention stated in the paper.

There is, however, an immediate contradiction in the D3 classification. The initial half-BPS list correctly contains

$$\mathfrak{psu}(1,1|2) \oplus \mathfrak{psu}(1,1|2) \oplus \mathfrak{u}(1)^2$$

and its compact counterpart with $\mathbb{R}^2$. The later list and equation (4.16) retain only one Abelian factor. Yet equations (4.17)–(4.18) explicitly exhibit two independent fixed generators, z_{AdS} and z_S, one in $\mathfrak{g}_0^\bullet$ and one in $\mathfrak{g}_2^\bullet$. Both matrices are traceless and survive the projective quotient. Equivalently, the even centralizer for the $(u_\mathrm{a}, u_\mathrm{s})$ row contains

$$\mathfrak{su}(1,1)^2 \oplus \mathfrak{su}(2)^2 \oplus \mathfrak{u}(1)^2.$$

The grade-zero Abelian generator belongs to the stabilizer in equation (3.76), while the other supplies the physical S^1. With only one Abelian factor in the fixed algebra, these two statements cannot both hold. The same issue occurs for the $\mathbb{R} \times S^3$ row.

The nonlinear graph construction has a second exact inconsistency. Equation (5.16) and the paragraph following it stress that Γ_Φ generally has grade-zero components. Equation (5.18) then defines

$$\mathcal{T}_\Phi = P_\mathfrak{m} P_\bullet \Gamma_\Phi, \qquad \mathcal{N}_\Phi = P_\mathfrak{m} P_\circ \Gamma_\Phi.$$

Consequently equation (5.19) should read

$$P_\mathfrak{m} \Gamma_\Phi(V^\bullet) = \mathcal{T}_\Phi(V^\bullet) + \mathcal{N}_\Phi(V^\bullet),$$

not the displayed equality with the unprojected Γ_Φ. Equation (5.34) itself later restores $A_\sigma = a_\sigma + P_0\Gamma_\Phi$, and the caption of Figure 1 again says that the grade-zero pieces are omitted. The error may be repairable inside pairings with the coset momentum, but it lies in the definition of the central all-orders operator and all subsequent "exact" statements must be checked with the projector restored.

Finally, equations (5.42)–(5.44) are not closed equations for $\mathcal{A}$ and Φ°. They determine only the two output combinations $\mathcal{M}^\bullet(\lambda_+^\bullet, \lambda_-^\circ)$ and $\mathcal{M}^\circ(\lambda_+^\bullet, \lambda_-^\circ)$ on a restricted set of pure-spinor data. The functional dependence of the four maps on the matter fields is deliberately left unspecified, and the manuscript does not prove that the required outputs extend to linear maps satisfying the grade, pure-spinor-cone, and antighost-gauge conditions. Thus the system can presently be read either as a definition of two combinations of auxiliary maps or as a set of necessary consistency conditions. It has not yet been shown to be a nonempty, predictive world-volume theory.

Novelty, significance, and relation to the literature

The manuscript is well motivated by three strands of prior work cited in the paper. The Berkovits–Pershin construction shows that equality of left- and right-moving pure-spinor BRST currents can encode supersymmetric Born–Infeld dynamics in flat space. Later work extended that analysis to Dp-branes and non-Abelian boundary degrees of freedom. For $AdS_5 \times S^5$, the earlier pure-spinor D-brane analysis classified half-BPS configurations in an explicit coset parametrization. The present fixed-subgroup and Cartan-map formulation is a potentially valuable coordinate-independent reorganization of that material.

The explicit inner and outer matrices also give a useful bridge to the classification of sixteen-supercharge subalgebras. This part is more than a change of notation if the real-form, grading, tangent-signature, and isotropy checks are made fully reproducible for every row. At present, several of those checks are summarized rather than displayed, and the D3 Abelian mismatch prevents the tables from being accepted as stated.

The proposed nonlinear extension would be the more significant result. Retaining every power of the local embedding coordinate is attractive, but the comparison with the flat-space Born–Infeld system in equations (5.23)–(5.24) is only schematic. No component multiplet, linearized equation, flat-radius limit, or explicit solution for the ghost maps is given. The paper therefore establishes a geometric ansatz and necessary projected relations, not yet a derivation of D-brane equations of motion. A worked example could change that assessment substantially.

Major comments

1. **Correct the two D3 fixed algebras and propagate the correction through the orbit discussion.**

   The manuscript starts from the conventional $\mathfrak{u}(1)^2$ and $\mathbb{R}^2$ entries but drops one factor in Section 3.3 and equation (4.16). Equations (4.17)–(4.18) then require two independent Abelian generators to explain the stabilizer and the physical S^1 or time direction.

   Please compute the full fixed centralizer of the two $2|2+2|2$ representatives, including the effect of the projective quotient, and give its even and odd dimensions. The corrected fixed group should then be used in equation (3.76), the D3 orbit quotients, and the text describing the preserved superalgebras. Suppressing finite centers is harmless here; suppressing a continuous Abelian generator is not.

2. **Repair the projected graph identity and recheck every all-orders use of Γ_Φ.**

   Equations (5.16)–(5.18) imply $P_\mathfrak{m}\Gamma_\Phi = \mathcal{T}_\Phi + \mathcal{N}_\Phi$. Equation (5.19) omits $P_\mathfrak{m}$, despite the explicit grade-zero terms discussed immediately before and after it. Please correct this identity and show in detail why the derivation of equations (5.30)–(5.34) is insensitive to $P_0\Gamma_\Phi$ where appropriate.

   The revision should also distinguish the full tangent lift from its coset projection in the definition and adjoint of $\mathcal{D}_\Phi$, in the pullback of the curvature, and in the residual-isotropy connection. A short commutative diagram of the relevant bundles and projections would make the construction much easier to verify.

3. **Turn the four $\mathcal{M}_{rs}$ maps into a well-defined, nonempty system.**

   The caveat following equation (5.44) is fundamental rather than optional: the manuscript has not shown that the prescribed output combinations extend to linear maps that preserve the two grades, send the reconstructed ghosts to the pure-spinor cones, and descend to the antighost quotient. Nor is their dependence on $\mathcal{A}$ and Φ° specified.

   At minimum, the authors should construct all four maps for one retained probe on a smooth pure-spinor patch and verify:

   - the grade-one/grade-three reconstruction in equation (5.11);
   - both quadratic pure-spinor constraints, not merely their assumption;
   - compatibility of the adjoints with the gauge shifts in equation (5.13);
   - residual-isotropy equivariance; and
   - existence away from the zero-field point to the order claimed.

   The number of independent map components, equations, and gauge identifications should be counted. Otherwise equations (5.42)–(5.43) can simply define two outputs on the cone and do not constitute dynamics.

4. **Justify the antighost-quotient argument leading to equations (5.44a)–(5.44b).**

   The text first says that the remaining term in equation (5.41) gives one equation, rather than independent coefficients of $\omega_-^\bullet$ and ω_+°, and then concludes that both parity components vanish by nondegeneracy. This requires an explicit description of the tangent space to the pure-spinor cone and its dual antighost quotient at the boundary.

   Please formulate the pairing without choosing an antighost gauge slice, show which combinations of $\omega_-^\bullet$ and ω_+° remain arbitrary after equation (5.10), and prove that orthogonality to all such classes implies the two displayed equations rather than equality modulo a pure-spinor gauge direction. The argument should also address singular strata of the cone or clearly restrict the claim to a specified smooth patch.

5. **Demonstrate BRST closure of the full interacting boundary problem.**

   The paper imposes ordinary stationarity through equation (5.33) and then sets the bulk BRST flux to zero. For the interacting theory, one must also show that the BRST transformation preserves the graph condition, the field-dependent ghost gluing, and the chosen antighost equivalence class. This is especially important because Q^2 closes only modulo a local H_0 transformation, equations of motion, and pure-spinor gauge symmetry.

   Please compute the BRST variation of the complete boundary action, including the induced transformations of $\mathcal{A}$, Φ°, and $\mathcal{M}_{rs}$, and relate it explicitly to equations (5.41)–(5.44). The claimed independence of $K_\sigma^\bullet$ and $\mathcal{P}^\circ$ after all graph, momentum, and world-sheet constraints should also be justified. This would establish that the final equations are sufficient for BRST conservation, rather than only formally necessary coefficients.

6. **Provide a concrete linearized probe test and a controlled known limit.**

   The present comparison with Berkovits–Pershin is the schematic statement $\mathcal{D}_\Phi \sim D\Phi + \Phi^2 D\Phi + \cdots$ in equation (5.24). That does not test the numerical factors, RR-bispinor contractions, torsion, or the physical spectrum. Choose one representative, preferably the $AdS_5 \times S^3$ D7 or

$AdS_4 \times S^2$ D5, and expand the full system to first order in $\mathcal{A}$ and Φ°. Identify the world-volume supermultiplet and derive its linearized scalar, gauge, and fermion equations.

The same example should be compared explicitly with the earlier coordinate-dependent $AdS_5 \times S^5$ gluing matrix and, where appropriate, with the flat-space or large-radius pure-spinor Born–Infeld equations. A successful check would both validate signs and show that the new formalism contains physical information not already encoded in the choice of involution.

7. **Make the seven matrix checks reproducible and separate symmetry from probe existence.**

   For the outer representatives, equations (4.22)–(4.24) are followed by the asserted real fixed algebras and tangent dimensions in equation (4.29). Appendix B.1 similarly states that all four checks have been performed without displaying the fixed bases or signatures. Please give, for each row, either an explicit basis or a concise verification table containing R^2, commutation with the ambient reality map, the $\mathbb{Z}_4$ sign, the dimensions and signatures of $\mathfrak{g}_2^\bullet$, the sixteen-real-odd-generator count, and $\mathfrak{g}_0^\bullet$.

   In addition, a fixed half-BPS superalgebra is necessary symmetry data but does not by itself prove that a zero-flux probe solves its DBI–Wess–Zumino equations. The paper recognizes this distinction when discussing the supergravity classification, but later says that the seven matrices "realize" the probes. Please either supply the world-sheet or kappa-symmetry map establishing each zero-flux embedding, or consistently describe the rows as candidate boundary involutions matched to known probe geometries.

Minor comments and presentation

1. The abstract and conclusion should call equations (5.42)–(5.44) necessary local boundary-BRST consistency conditions until existence, closure, and a physical component limit have been demonstrated. The stronger phrase "Dirac–Born–Infeld-like equations of motion" currently overstates the result.

2. Clarify the use of "zero-field." The reference configuration has vanishing world-volume field strength, while Section 5 permits fluctuating $\mathcal{A}$ with nonzero $\mathbf{F}_\Phi$. A phrase such as "fluctuations about the zero-flux branch" would avoid ambiguity.

3. Global-form conventions deserve one compact paragraph. The text alternates between PSU, SU, OSp, and their Lie algebras, while suppressing centers and covers. These choices are particularly relevant to the D3 Abelian factors and to the definition of G^R.

4. In equation (3.44), state explicitly that the pullback of $\star L$ is $L_\tau\, d\sigma$ in the conventions of Section 2. A one-line table for K_τ, K_σ, and the $z, \bar{z}$ components would make the signs in equations (5.35)–(5.37) transparent.

5. Number the two series expansions following equation (5.18), since they are used to motivate the central nonlinear claim. State also that Φ° is even in total Grassmann parity, although it contains odd superalgebra grades; this matters for adjoints and graded signs.

6. The notation $\mathfrak{sp}(4)$, $\mathfrak{usp}(4)$, and $\mathfrak{spp}(2)$ is used for the compact sphere algebra in different places. Adopt one convention and state whether the numeral denotes the defining complex dimension or the rank.

7. The manuscript should give a direct equation-level comparison with the earlier *AdS* pure-spinor D-brane classification, rather than only stating that the older coset parametrization obscures the invariant structure. A row mapping the gamma-matrix gluing representatives to equations (4.16) and (4.29) would suffice.

8. Correct the corrupted apostrophe in the disclaimer and make the PDF title metadata agree with the displayed title. The current metadata uses "nonlinear boundary equations," while the title page uses "nonlinear interactions."

Recommendation

I recommend **major revision before reconsideration**. The invariant zero-field framework and the exponential-graph geometry are potentially publishable contributions. The D3 fixed-algebra count and equation (5.19) must first be corrected, and the interacting construction must be shown to exist, close under BRST symmetry, and reproduce a concrete physical probe at linear order. If the authors can supply those checks and moderate the claims that remain only necessary or local, the work could become a useful addition to the pure-spinor D-brane literature.