Referee Report

Quantum Field Theory of Cosmological Perturbations Induced by Ultralight Dark Matter

Recommendation: Reject in the present form; a substantially rederived manuscript could be considered as a new submission. The problem is timely and the attempt to retain the quantum state of ultralight dark matter (ULDM) in a Schwinger–Keldysh/2PI treatment is potentially valuable. The present calculation, however, does not support its principal quantitative result. The oscillatory phase used to define the resonance frequency changes sign between equations (5.39) and (5.40), and the frequency assigned in equations (5.41)–(5.50) is not the derivative of that phase. In a matter-dominated background this alone shifts the leading resonance frequency by a factor of three. Independent errors occur in the non-local reduction, the inversion of the state spectrum, the scalar equations, the conversion of wave number to Mpc, and the normalization of Newton's constant. Consequently the stated upper bound in equation (5.55) and several broader conclusions cannot be assessed without a new, internally checked derivation.

Synopsis and overall assessment

The manuscript studies a minimally coupled massive scalar on a spatially flat FLRW background and treats both the scalar and metric perturbations as quantum fields. Section 2 constructs the graviton equation from the two-particle irreducible effective action on a closed time path. The scalar expectation value and connected two-point function enter, respectively, through a coherent condensate and a homogeneous, isotropic Gaussian state. The latter is parameterized by an occupation spectrum, a squeezing spectrum, and a squeezing phase. Matter perturbations are formally solved with a retarded Green function and inserted into a non-local graviton equation.

Section 3 imposes an adiabatic/WKB regime, specializes to matter domination, and evaluates the scalar energy-momentum tensor and the local and non-local parts of the one-loop graviton self-energy. The vacuum contribution is dimensionally regularized, while the state-dependent terms are reduced to families of momentum integrals. A midpoint prescription is used for non-local kernels. Section 4 fixes longitudinal gauge. In the transverse-traceless sector, the authors find a local effective mass from the occupation and squeezing integrals and argue that the homogeneous condensate cancels from the direct tensor propagation equation. Equations for the two Bardeen potentials are also displayed but not solved.

Section 5 assumes power-law occupation and squeezing spectra with a Gaussian cutoff. The occupation parameters are related to the present ULDM density and pressure. Four state choices are then discussed. For a nearly pure squeezed state, the local tensor equation is put in a time-dependent Mathieu form. A narrow-band Floquet estimate is used to infer a maximal relative enhancement of order 10^{-12} for ULDM masses 10^{-24}–10^{-21} eV. Appendices A and B collect vertices and self-energy algebra, while Appendices C and D estimate the local and non-local momentum integrals.

There is a worthwhile idea at the core of the paper. A state-resolved in-in calculation could connect semiclassical gravity to phenomenological descriptions of squeezed dark matter, and an explicit demonstration of the standard no-anisotropic-stress cancellation for a homogeneous coherent scalar would be useful. The paper also makes a serious effort to expose its long algebra. Relative to the classical coherent-field resonance studies cited in the Introduction, a controlled state-dependent loop calculation would be a meaningful advance. At present, however, the calculation is neither algebraically stable nor accompanied by the Ward-identity and numerical checks needed for a gravitational self-energy. The potentially novel result—the squeezing-induced tensor resonance—rests precisely on the inconsistent steps identified below. This falls short of the standard of correctness and reproducibility expected by PRD or JHEP.

Major comments

1. **The phase and resonance frequency are internally inconsistent.** Appendix C provides a useful place to see the problem. Expanding the exact WKB phase gives the current-time contribution

$$\left(2m - \frac{3p^2}{a^2 m}\right) t$$

in equation (C.11), yet the asymptotic result (C.18) replaces the correction by a positive, spectrum-averaged term. Equation (5.5) inherits that plus sign. More decisively, equation (5.39) contains $2m+3(n_N+5)k_{\mathrm{UV}}^2/(2ma^2)$, whereas substitution of equation (5.23) produces the negative corrections in equation (5.40). Direct substitution gives positive corrections; the two displayed equations therefore cannot both be true.

There is a separate frequency error even if one chooses a sign. Equation (5.42) has a driving phase $\Theta(\widetilde{\eta}) = 2m_S(\widetilde{\eta})\widetilde{\eta}$. Its instantaneous half-frequency is

$$\frac{\Theta'}{2} = m_S + \widetilde{\eta}\, m_S',$$

not m_S. At leading order equation (5.41) gives $m_S = am/3$, and matter domination has $a \propto \widetilde{\eta}^2$; hence $m_S + \widetilde{\eta} m_S' = am$, three times the frequency used in the resonance condition. The text immediately below equation (5.41) also says that $m_S - m$ is small, although $m_S \simeq am/3$ is a conformal frequency and is not close to the cosmic-time mass m. The band center, detuning, crossing time, Floquet exponent, and enhancement in equations (5.44)–(5.55) must all be rederived from the actual derivative of a single, consistently expanded phase. Until this is done, the headline 10^{-12} bound is unsupported.

2. **The quoted physical wavelengths are wrong by more than six orders of magnitude.** Equation (5.53) states that $k = 10^{-24}$–$10^{-27}\,\mathrm{eV}$ corresponds to $\lambda \lesssim 10^2$–$10^5\,\mathrm{Mpc}$. Using $\lambda = 2\pi/k$ and $1\,\mathrm{eV}^{-1} = 1.9733 \times 10^{-7}\,\mathrm{m}$ instead gives approximately

$$\lambda = 4.0 \times 10^{-5}\text{–}4.0 \times 10^{-2}\,\mathrm{Mpc}.$$

If $1/k$ rather than $2\pi/k$ is intended, the range is 6.4×10^{-6}–$6.4 \times 10^{-3}\,\mathrm{Mpc}$. Neither convention is remotely compatible with the printed values. This changes the observational scales associated with the proposed band and invalidates the surrounding CMB-scale interpretation. All conversions should be recomputed from a declared comoving convention, including the normalization of a_0.

The numerical normalization is also inconsistent. Equation (2.1) defines $\kappa^2 = 16\pi G$, and Section 5.2.2 accordingly uses $3.4 \times 10^{-55}\,\mathrm{eV}^{-2}$. The estimate after equation (5.53) instead uses $1.7 \times 10^{-55}\,\mathrm{eV}^{-2}$, corresponding to $8\pi G$. Since the exponent is proportional to κ^4, this is not an innocuous convention change. A transparent unit table and a reproducible numerical evaluation of every factor entering equations (5.52)–(5.55) are required.

3. **The analytic treatment of the non-local kernel is not controlled in the regime where resonance is sought.** Equations (5.6)–(5.10) and Appendix D freeze the kernel frequency at the final time, replace the phase coefficient at $t - s$ by its value at t, and Taylor expand the amplitude about $s = 0$, although the retarded integral extends from equality to the observation time. Near a frequency match, the contribution is coherent over a long interval; this is exactly where a short-memory expansion about $s = 0$ is least justified. No remainder or comparison with the unexpanded integral is supplied.

Moreover, equation (5.6) makes $\mathcal{W}(t)t$ the accumulated phase. Differentiation gives the instantaneous frequency $\Omega_0 a^\alpha(t)$, not $\mathcal{W}(t)$. Comparing $\mathcal{W}$ itself with the matter-kernel frequencies in equations (5.11)–(5.13) introduces the factor $3/(2\alpha+3)$; for a freely propagating subhorizon wave with $\alpha = -1$, the discrepancy is a factor of three. The same conceptual error appears in Appendix D. The claimed suppression of the non-local terms therefore has not been established on or near the characteristic surfaces. The authors should evaluate the causal convolution numerically with the full time-dependent WKB phases and demonstrate convergence to any proposed local approximation over the complete parameter range used later.

4. **Appendix D contains explicit algebraic contradictions.** In equation (D.5), the first line is a sum of positive integrals for $N_p \geq 0$, but the next equality is negative and contains an extra power of the scale factor, while the final expression is positive again. The angular factor is printed as $(1-x)^2$. Directly combining $J_{40} - 2J_{22} + J_{04}$ at leading order gives $(1-x^2)^2$, not $(1-x)^2$. Thus the sign, dimensions, and angular coefficient in this defining quantity do not agree with one another.

   Equation (D.20) starts from a phase depending on the integration momentum p, but its approximation replaces p^2 by the external momentum k^2. The subsequent p-integration in equation (D.23) nevertheless yields the moment $I_3^S \propto \Gamma[(n_S+7)/2]$, which requires the phase correction to retain its p-dependence. Equation (D.24) then defines a positive correction to m_S, in conflict with the negative phase immediately preceding it. These are not typographical details: they determine both the power of the cutoff and the resonant phase. Appendix D should be rebuilt from the original loop integral, with every change of variables, angular integral, dimension, and asymptotic order checked independently.

5. **The state-dependent self-energy needs an algebraic and gauge-consistency verification.** The tensor expressions in equations (3.35)–(3.40) contain terms that cannot be accepted as printed. For example, a coefficient multiplying J_{00} includes $\kappa^2/4 - \bar{m}^2$, adding quantities of different dimensions; analogous combinations recur in the squeezing sector. The squeezing formulas also contain a sum beginning at $t = i$, repeated free indices such as a second σ where β is required, and unbarred occupation integrals inside expressions that should be built from the squeezed integrals. Appendix B contains further warning signs, including the use of a coincident propagator where a noncoincident product is required in the displayed formula for S_3, and an index mismatch in the decomposition of $\mathcal{A}^{\alpha\beta}$.

   A gravitational self-energy is constrained by the Noether/Ward identities. The manuscript invokes such an identity for the condensate cancellation but never demonstrates the corresponding transversality relations for the large state-dependent local and non-local tensors after the WKB and midpoint truncations. The authors should provide a machine-checkable ancillary derivation, verify the exchange symmetries and mass dimensions of each tensor coefficient, and explicitly contract the complete renormalized kernel with the relevant covariant derivatives. It should also be shown that the counterterm choice used in the vacuum sector is compatible with the same identities and with covariant stress-tensor conservation at the stated adiabatic order.

6. **The mapping from the Gaussian spectrum to cosmological parameters is incorrect.** Equations (5.15) and (5.18) depend on the combination

$$\frac{N_0}{k_*^{n_N}}\, k_{\mathrm{UV}}^{n_N+3}.$$

   Solving these equations for the dimensionless amplitude therefore requires a factor $(k_*/k_{\mathrm{UV}})^{n_N}$. It is absent from N_0 in equation (5.23). As printed, that result is dimensionful whenever $n_N \neq 0$, in direct conflict with the definition in equation (5.1). This missing factor propagates into the substitution leading to equation (5.40).

The independent power-law ansatz must also obey the Gaussian-state inequality $S_k^2 \le N_k(N_k+1)$ mode by mode, not just at one scale. The approximation $S_0 \simeq N_0$, $n_S \simeq n_N$ follows from purity only where $N_k \gg 1$; it cannot hold throughout a Gaussian ultraviolet tail where $N_k \to 0$. Equation (5.56) both weakens the exact inequality and contains n_n in place of n_N. The authors should construct a bona fide covariance matrix (or Bogoliubov parameters) that is positive for every mode, state which momenta are in the high-occupation regime, and redo the bounds without extending that approximation into the tail.

7. **The scalar system is neither dimensionally consistent nor closed under the stated initial data.** The condensate contributions in equations (4.13) and (4.14) are proportional to ϕ_0, while the condensate stress tensor and self-energy throughout Section 3 are proportional to ϕ_0^2. The linear terms have the wrong field dimension and must be corrected before any interpretation of the Bardeen equations.

   More conceptually, equation (2.27) retains a particular retarded response of the matter variables to the metric but omits the homogeneous solutions of the linearized matter equation. Those solutions encode independent initial density and velocity perturbations. Consequently, equations (4.13)–(4.15) are a closed system only after imposing an additional zero-homogeneous-perturbation condition, which is not an adequate description of general cosmological initial data. The authors should state and justify the initial conditions, restore the homogeneous modes, verify the scalar constraint equations and stress-tensor conservation, and only then compare with the pulsar-timing literature cited in the conclusion.

8. **The condensate result is stated too broadly and its novelty needs calibration.** For a homogeneous, minimally coupled scalar with no tensor anisotropic stress, use of the background Einstein equations removes apparent matter terms from the linear tensor propagation equation. The cancellation reported in Section 4 can be a useful in-in realization of this standard result, but it does not imply that a coherent scalar has no observable gravitational-wave imprint. The condensate changes the background expansion; inhomogeneous condensate perturbations can source tensors beyond linear order; and nonminimal couplings or gradients alter the conclusion. The first bullet of Section 6 should therefore be restricted to the direct, local, linear propagation of a tensor perturbation on the homogeneous background used here. The authors should compare the cancellation explicitly with the standard cosmological-perturbation argument and distinguish what the loop formalism adds beyond the classical literature cited in the Introduction.

9. **The cosmological matching and domain of approximation require repair.** Equations (5.27)–(5.34) match a radiation-era scale factor $a \propto \eta$ directly to a matter-era form $a \propto \eta^2$. A piecewise background with continuous a and a' requires a shift of the conformal-time origin in the matter solution; without it, the matching coefficients and phases are not those of a continuous cosmology. The manuscript introduces a shifted coordinate elsewhere but does not use a single consistent convention in these formulas.

   Finally, the upper estimate below equation (5.54) sets pressure as large as energy at equality, whereas the local expansion, the cold-matter interpretation, and the phase estimates assume a nonrelativistic state. An upper bound derived at the edge of or outside the expansion's domain is not controlled. A revised analysis should evolve a continuous radiation-to-matter background, track physical and comoving cutoffs separately, and demonstrate numerically that every mode used in the bound satisfies the adiabatic, subhorizon, nonrelativistic, narrow-band, and short-memory conditions simultaneously.

Minor comments

1. Equation (5.17) writes the condensate pressure phase as $\cos(2mt_0 - \theta_0)$, whereas equation (3.49) gives $\cos[2(mt - \theta_0)]$. The factor multiplying θ_0 must be made consistent.

2. The cutoff in equation (5.1) is called physical, although the momentum k used in the mode functions is comoving. State explicitly whether $k_{\rm UV}$ is comoving and constant or physical and redshifting; many powers of a depend on this choice.

3. Equation (A.6) defines an operator with arguments x, x' but contains $\delta^D(x-y)$. The coordinate arguments and derivative actions should be corrected throughout Appendix A.

4. The first and second lines of equation (B.7) do not carry the same overall power of κ. This should be reconciled with the vertex normalization in Appendix A.

5. The distance variable is introduced as y^2, while later formulas alternate among y, $y/4$, and powers of $\sqrt{|y|}$. A single dimensionless definition and branch prescription would make the vacuum calculation checkable.

6. In the radiation-era discussion below equation (5.29), the manuscript says that $B(k)$ vanishes, whereas the displayed solution and initial condition set $D(k) = 0$.

7. The phrase "the four remaining degrees of freedom are physical" in Section 4 should distinguish propagating degrees of freedom from constrained scalar potentials. In general relativity the lapse and shift constraints remain essential after longitudinal gauge fixing.

8. The state called "generic Gaussian" is homogeneous, isotropic, momentum diagonal, and initially uncorrelated with the graviton. These restrictions and the later choice of squeezing phase should be stated in the abstract and conclusions.

9. The article repeatedly calls its sections "chapters." There are also numerous duplicated words and notation changes, including $\Omega_{\rm cl}$ versus Ω_χ, n_N versus n_n, and t versus $\widetilde{\eta}$ in quantities defined as frequencies. A careful notation and language pass is needed after the derivation is fixed.

10. Several references are described as forthcoming or very recent. The final version should identify archival versions where available and add a clearer comparison with standard semiclassical-gravity renormalization, cosmological Ward identities, and the conventional no-anisotropic-stress tensor equation.

Publication assessment

The manuscript addresses a subject appropriate for a high-energy theory or gravitation journal, and a corrected state-dependent in-in calculation could be significant. The current version does not meet the required standard of correctness. The fatal issue is not merely that a numerical estimate needs refinement: the phase whose resonance is analyzed is algebraically inconsistent, its instantaneous frequency is misidentified, and the non-local terms are reduced by an uncontrolled approximation that makes the same frequency error. The explicit tensor, spectrum, scalar, and unit inconsistencies provide independent reasons not to rely on the final bound.

A viable new submission would need (i) a fresh derivation from the causal self-energy with verified Ward identities and dimensions; (ii) a physically admissible Gaussian covariance spectrum; (iii) numerical evaluation of the full time-dependent kernel on a continuously matched cosmological background; (iv) a resonance analysis based on the derivative of the exact phase; and (v) reproducible scale conversions and parameter choices. It should then separate the standard homogeneous-condensate cancellation from genuinely new quantum-state effects and moderate its phenomenological claims accordingly.